

TEMPERATURE SENSITIVITY OF TIME-REVERSAL FOCUSING IN A COMPLEX MEDIUM

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Abstract

We present time-reversal focusing experiments in an open multiple scattering medium undergoing a temperature change. Multiply scattered waves are recorded once to define a reference, then time-reversed and continuously backpropagated into the medium while the temperature decreases slowly. The temporal shift of the time-reversed peak enables to retrieve changes in temperature as small as 0.02°C , while the peak amplitude decays with temperature in a manner which is well predicted by a simple “shot noise” model. We also show that the sensitivity to a change of temperature increases with the scattering order (i.e. time), which cannot be accounted for by the shot-noise model, and is probably due to the elastic resonances of the scatterers, which result in a time-dependent phase variance in the scattered signals.

Experimental set-up and result

We apologize to the readers for the brevity and incompleteness of this paper. We ran short of time and decided to present only the essential experimental results and hypotheses.

The experimental set-up is depicted in Fig. 1. A sub-wavelength piezoelectric element transmits a short ultrasonic pulse (two cycles of a 3.2 MHz sine wave) that propagates through water and encounters a multiple scattering slab. The slab is made of a random collection of parallel steel rods with density $18.75/\text{cm}^2$ and diameter 0.8 mm (for comparison, the average wavelength in water is 0.47 mm). The transport mean free path in this sample is 4 mm, while its thickness is $L=40\text{mm}$, which implies that the wave will undergo high-order multiple scattering as it traverses the slab. The receiving array has one hundred and twenty-eight 0.39-mm large elements. The vertical dimensions of the rods and of the array are sufficiently larger than the wavelength to consider the set-up as two-dimensional. Highly scattered waves emerge from the sample and the array records 128 time series (Fig. 2).

Then the array is used as a “time reversal-mirror” : the scattered signals are digitized and recorded into electronic memories, time-reversed, and then sent back by the same array through the same scattering medium. The piezoelectric element that was previously used as a source is now a receiver, and records the waveform generated at the source location after the time-reversal process. It was already shown

in earlier studies that the ultrasonic time-reversal is a fairly robust operation : unlike one could have expected given the high order of scattering involved and the sensitivity of classical systems to initial conditions, the long-lasting scattered waves ($\sim 400\ \mu\text{s}$) was found to converge back to the source and recover its original duration ($1\ \mu\text{s}$), with a spatial resolution that was even better than in a homogeneous medium, and is very robust to quantization errors since “one-bit” time-reversal focusing is very efficient [1,2].

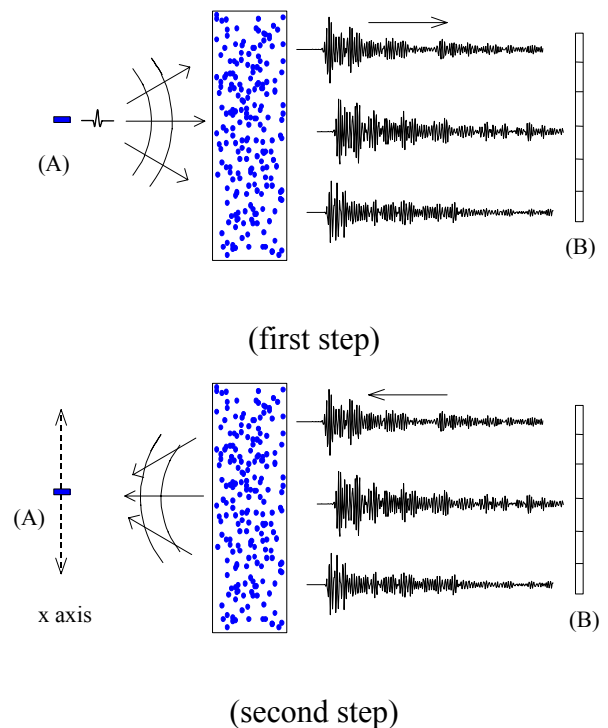


Fig. 1 : Time-reversal focusing. In the first step the source (A) transmits a short pulse that propagates through the rods. The scattered waves are recorded on a 128-element array (B). In the second step, the array retransmits the time-reversed signals through the rods. The piezoelectric element (A) is now used as a detector, and measures the signal reconstructed at the source position.

The scattered signal, as well as the signal recreated at the source after time-reversal are plotted in Fig. 2. A strong pulse compression is obtained. This was made at the original temperature of 29.5°C . Then we continuously retransmit the same time-reversed

waveforms while the water slowly cools down, and see how the signal recreated at the sources evolves. Two effects are made clear : the refocused peak is shifted in time, and its amplitude decays progressively as the temperature changes (Fig. 3, 4, 5).

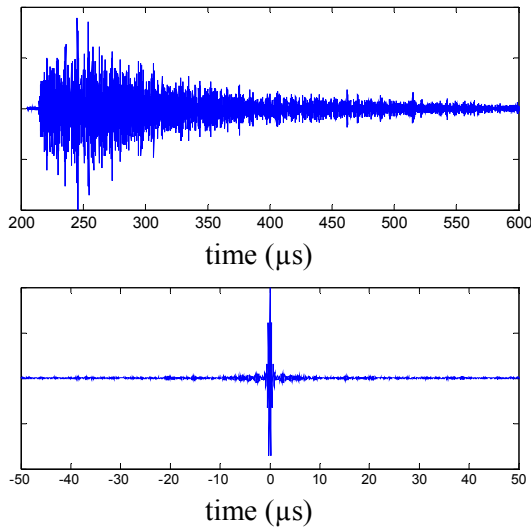


Fig. 2 : scattered waveform received on one of the array elements (top), and signal recreated at the source after time-reversal (bottom).

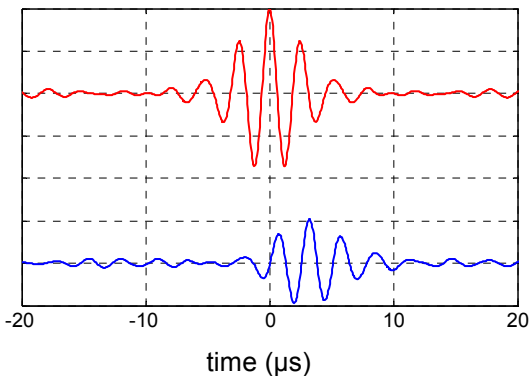


Fig. 3 : Signals obtained at the source at the original temperature (29.5°C, red line) and at 28.5°C (blue).

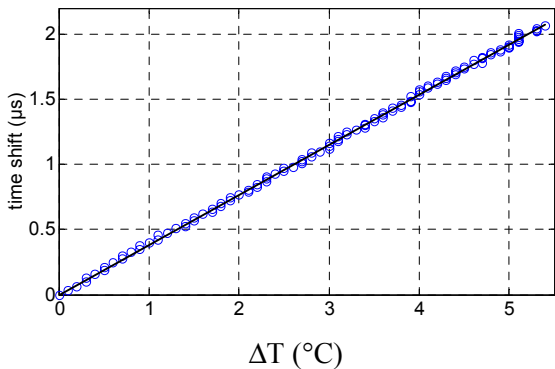


Fig. 4 : Time-shift of the recreated pulse as a function of the temperature change. (circles : experiments, line: prediction from the shot noise model)

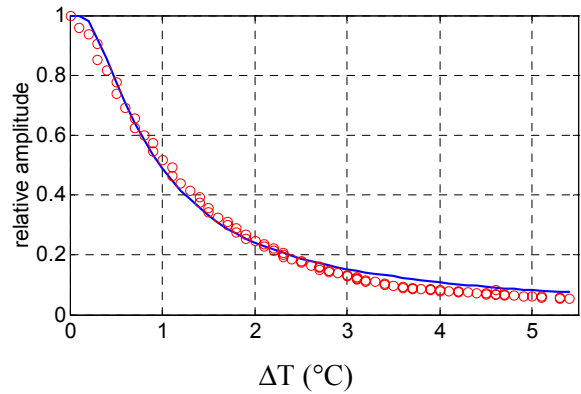


Fig. 5 : Amplitude decay of the recreated pulse as a function of the temperature change. (circles : experiments, line : prediction from the shot noise model)

These two results can be very well accounted for by a simple model, which amounts to considering the scattered wave forms as a “shot noise”, i.e. a series of replica of the incoming pulse, with random and independent arrival times and an envelope $A(t)$ a.k.a. the “time-of flight distribution”, which depends on the transport mean free path and diffusion constant in the forests of rods [3] :

$$h(t) = \Sigma A(t) \delta(t - t_n)$$

Since time-reversal can be seen as a correlation process, the peak recreated at the source is the temporal autocorrelation of the initial impulse response $h(t)$ and the modified impulse response due to the temperature change :

$$h^{\Delta T}(t) = \Sigma A(t) \delta(t - t'_n)$$

The change in temperature will affect the velocity of sound, the family of arrival times is simply stretched as :

$$t'_n \approx t_n \left(1 - \frac{\partial c}{\partial T} \frac{\Delta T}{c_0} \right)$$

Under this hypotheses, we find the following results. Firstly, the time-shift varies linearly with the temperature change ΔT : time-shift = $t_1 \frac{\partial c}{\partial T} \frac{\Delta T}{c_0}$, with t_1

the beginning of the time-reversal window. Secondly, the amplitude of the TR peak decreases as

$$\int_{t_1}^{t_{\max}} A^2(t) dt$$

with $t_{\max} = t_1 + \frac{\lambda/4}{\Delta T \frac{\partial c}{\partial T}}$ (t_{\max} is the time for which the

difference between the time-shifts $t'_n - t_n$ and $t'_1 - t_1$ becomes larger than a quarter of a period, so that $h^{\Delta T}$ and h are uncorrelated). Despite the simplicity of this approach, both predictions are in good agreement with the experimental observations.

However, if we perform a “dynamic” time-reversal, the model fails to describe the experimental observation. By “dynamic time-reversal,” we mean that a short time-window is selected among the scattered signals, and only this short time-window $[t, t+\delta t]$ is reversed and sent back. It can be shown that the “shot-noise” approach implies that the amplitude decay should depend on δt and not on t , which means that there should be no dynamic effect : the early arrival times and the late arrival time should be affected in the same way by the change in temperature. The experimental results presented on Fig. 6 show that this is wrong, at least for temperature changes larger than 2°C .

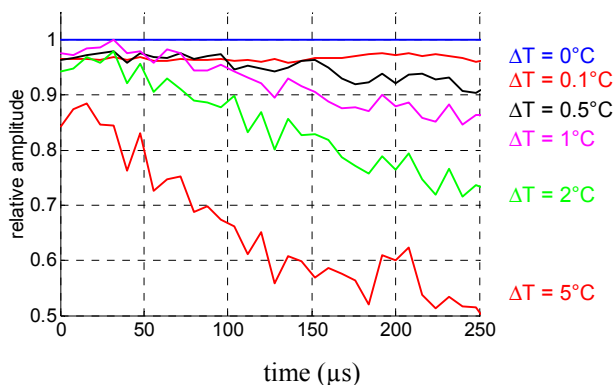


Fig. 6 Normalized amplitude decay of the peak recreated at the source for different temperature changes, as a function of the beginning time t of the time-reversal window. For large temperature changes (2°C , 5°C), there is a dynamic effects : late arrival times are more affected than early ones.

The possible origin of that discrepancy is that the shot noise model only takes into account the “geometric” phase (arrival time) $\mathcal{L}\omega/c$ (for a path length \mathcal{L}). But in addition to the geometric phase, one has to take into account the phase of the scattered wave relatively to the incident wave on a rod. This phase term can be neglected if the scatterers are purely rigid, but not if they are elastic, particularly around a resonance. Therefore two multiple-scattering paths, even if they have the same path length \mathcal{L} , will have different phases, and will be affected differently by a change in temperature. Since a multiple scattering path is the result of many independent scattering events, the variance of the total phase delay induced by a change in temperature can be expected to grow linearly with the order of scattering (i.e. time), which could account for the dynamic effect we pointed out.

Conclusions

The experimental results we presented show that time-reversal focusing is sensitive to small temperature

changes. The smallest temperature change we were able to measure from a time-shift was 0.02°C (with a sampling frequency 160 MHz, much larger than the central frequency 3.2 MHz). A very simple model allows to predict the main feature of the effect of a temperature change: a time-shift and an amplitude decay. However, the model fails to predict the effect on dynamic time-reversal and has to be refined to take into account the additional phase shift due to scattering on elastic targets, and possible resonances. The influence of temperature is crucial for applications of time-reversal focusing, like telecommunications for which the time-reversed peak has to be properly synchronized. The sensitivity of time-reversal focusing could be taken advantage of in order to detect small temperature changes in a strongly multiple scattering medium, in a manner similar to DWS (Diffuse Wave Spectroscopy) [4].

References

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