EXPERIMENTAL AND NUMERICAL FOCUSING OF SHOCK WAVES AT FOLD OR CUSP CAUSTICS

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Abstract
A future civil supersonic aircraft faces the formidable challenge to overcome the key environmental issue of reducing sonic boom. The most intense one results from its focusing on caustics (fold or cusp) produced by aircraft maneuvers like transonic acceleration. The classical modeling consists in introducing diffraction and local nonlinearities around the caustics. The pressure field is then governed by the nonlinear Tricomi equation for the fold caustic, and by the Khokhlov-Zabolotskaya equation for the cusp caustic. These two equations have been solved numerically recently. In order to confirm the theory and validate the numerical codes, experiments are necessary. An experimental set-up has been built to scale the focusing of sonic boom at 1:100 000 with ultrasonic shock waves in a water tank instead of sonic boom in air. In the vicinity of the caustic, the acoustic field is measured and compared with the numerical simulations.

Introduction

Focusing of shock waves at fold or cusp caustics has mostly been studied in the frame of sonic boom focusing. The sonic boom is an aero-acoustical phenomenon, which occurs when an aircraft flies at supersonic speed. Some aircraft manoeuvres or the atmospheric turbulence can cause a focusing of the sonic boom on special surfaces. These surfaces are called caustics and are classified by the theory of catastrophes[1], depending on their geometry. The two simplest caustics are the fold caustic and the cusp caustic. The classical theory of geometrical acoustics, usually describing the sonic boom, fails in the vicinity of a caustic for which it predicts infinite pressure. Introducing local diffraction effects, the pressure field can be found for smooth waves. Indeed, for monochromatic waves in linear regime, the pressure field is given by the Airy function[2] around a fold caustic, and by the Pearcey function[3] around a cusp caustic. Unfortunately, if the incoming waves are shock waves, taking into account the diffraction around the caustics is insufficient, as incoming shocks give rise to peaks of infinite amplitude. An additional physical mechanism must be taken into account to limit the amplitude of the peaks. In 1965, Guiraud[4] suggested to add local nonlinear effects around a fold caustic. Thus, the pressure field around a fold caustic is described by the nonlinear Tricomi equation. Recently, this equation was solved numerically by means of a fully validated algorithm, which provides us with a numerical solution of the problem [5,6]. Around a cusp caustic, adding nonlinear effects in the same way as for the fold caustic, Cramer and Seebass[7] showed that the field is governed by the Kholkhlov-Zabolotskaya (KZ) equation and Coulouvrat[8] made the problem numerically tractable by writing the boundary conditions associated with the KZ equation. A fully validated algorithm has been developed to simulate the focusing of shock waves at a cusp caustic numerically.

Experimental studies have already been performed. They can be classified into two categories. The first one deals with test flights[9,10]. The main advantage of these experiments is a direct measurement of the focusing of sonic boom, but quantitative comparisons with modelling are difficult due to uncertainties on meteorological fluctuations, atmospheric turbulence, ground effects, etc. Laboratory experiments have been carried out too [11,12], but with neither comparison with a theoretical modelling nor real scaling with the sonic boom so far. In this paper, we propose a new experiment to study the focusing of shock waves on caustics, which scales the focusing of sonic boom.
First of all, the experimental set-up is shown, and the experimental method to synthesize caustic is presented. Then, the experimental results for the fold caustic are compared to the numerical ones. Finally, the comparisons are made also for the cusp caustic.

Experimental set-up and method to synthesize a caustic

The three similitude parameters controlling the focusing of shock waves are associated with the diffraction, the nonlinearity and the absorption[13]. In our experiments, the focusing of sonic boom in air is scaled at 1:100 000 with ultrasonic shock waves in water. This choice ensures that the similitude parameters are constant for the both cases. Experiments are made in a water tank. The frequency of the waves is 1MHz (wavelength of 1.5mm). The waves are emitted by an array of 256 transducers, each one being rectangular (11×5 mm) so that the array is rectangular too (191×95 mm). A broadband amplifier controlled by a PC powers each transducer individually. So the amplitude, phase and shape of the signal emitted by each transducer are controlled. The pressure field is measured with a PVDF bilaminar

membrane hydrophone. The signals received by the hydrophone are first acquired by a digital oscilloscope and then stored in the PC. The hydrophone is set on a three-axes-motor system. The motors are controlled by the PC too.
Experiments are split into two parts. First, the caustic is created in linear regime, using the linear method of inverse filtering[14]. This technique is based on the knowledge of the propagation operator between the array of transducers and a set of control points. Once this operator is measured, it is inversed numerically. The inverse propagation operator is then used to compute the signals to be emitted by the transducers to create the prescribed pattern on the control points. So, the set of control points is chosen according to the geometry of the caustic to synthesize, as will be precised in the next section. The second stage of the experiment consists in emitting the signals, computed in linear regime, with a high amplitude. Then, nonlinear effects take place during the propagation until they reach the caustic. The nonlinear effects create a nonlinear steepening of the temporal profile of the waves up to the formation of shocks waves.

Experimental and numerical results for the fold caustic

First, the fold caustic is synthesized with the inverse filtering technique for monochromatic waves. 41 control points are defined along a control segment 6cm long and 1m away from the array of transducers. The center of this segment is not aligned with the center of the array of transducers but is shifted 2cm aside(Fig.1). The propagation operator is then measured. To apply the inverse technique method, the solution in linear regime for monochromatic waves (the Airy function) is imposed on the control points. This solution gives the pressure field on a line perpendicular to the caustic. This solution being imposed on the control segment, a caustic perpendicular to the segment of control is synthesized. This solution is a function of the frequency (already determined) and the radius of curvature, which is chosen equal to 10m. This choice is in agreement with the scaling ratio. Thanks to the inverse filtering method, the signals to emit by the transducers are determined and then emitted at weak amplitude.

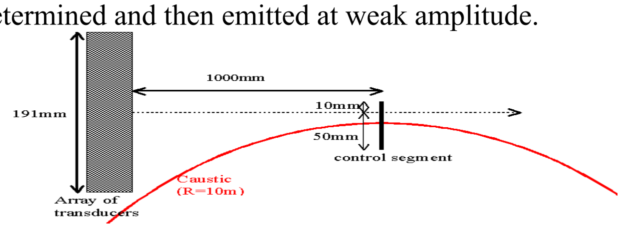

Figure 1 : Geometry of the experiment for the fold caustic

Fig 2 shows the spatio temporal representation of the Airy function (on the left) and the measured field (on the right). The two dotted lines delimit the control segment used in the inverse filtering stage. The horizontal axis represents the time and the vertical one the distance from the caustic on the segment of control. The pressure amplitude is coded with a color scale from blue to red. We note that the two fields are very similar. In particular, the zero amplitude between the two lobes is well recovered. After the dotted line the structure of the field is quite the same but not the amplitude. This difference is due to the fact that the aperture of the experimental wavefront is finite (because the finite size of the array), while the aperture in the theoretical modeling is infinite. But usually this has no impact for the nonlinear case because, according to the theory, the nonlinear effects are expected to take place mostly near the 1st lobe[4].

The signals are then emitted with a high amplitude to create shock waves. Fig.3 presents the spatio-temporal field measured on the segment of control every 0.5mm with a sampling of 1GHz. This pattern is completely different from the linear case. The lobes, characteristic for the Airy function, have disappeared. There is a strong dissymmetry between the compression phases (in red) and the expansion ones(in blue). Due to the presence of shock waves, the cusp shape of the wavefronts is clearly visible.

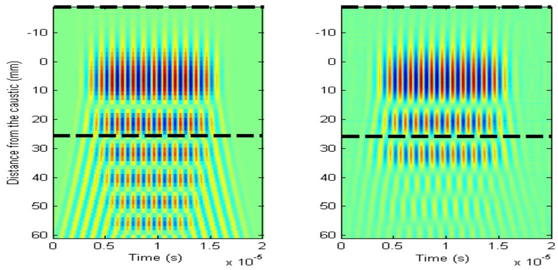

Figure 2 : Prescribed (left) and measured (right) linear field around the fold caustic

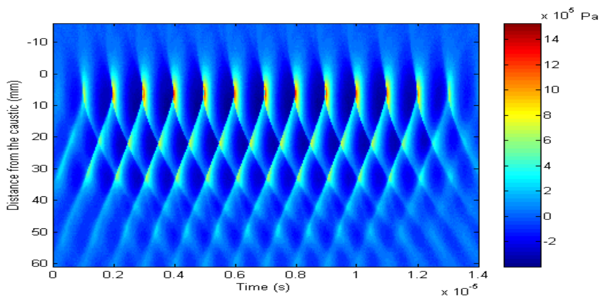

Figure 3 : Measured nonlinear field around the fold caustic

To compare these experimental results to the numerical ones, two data must be extracted from the experiments. The first one is the shape of the incoming wave and the second one is the value of the nonlinear parameter, which, in this case, is equal to 0.25. These two data are set as an input of the numerical code solving the nonlinear Tricomi equation and also as an input of the analytical linear solution in order to outline the role of local nonlinear effects. Fig. 4 presents the comparisons between these numerical results and the experimental ones at 5 different positions along the control segment. We can clearly see that the agreement between the experimental measurements (black solid lines) and the linear simulation (red dashed lines) is poor. The amplitude, the phase and the shape of the signals clearly do not match. On the other hand, the nonlinear simulations (blue dotted lines) are very similar to the experimental measurements. There is only one small difference: the second (outgoing) shock is a little bit underestimated by the numerical simulation. This is due to the difference between the experiment, where the field is not invariant along the caustic, and the theory (where it is). But these results show that it is now possible to simulate the focusing of shock waves at a fold caustic numerically, with a high precision.

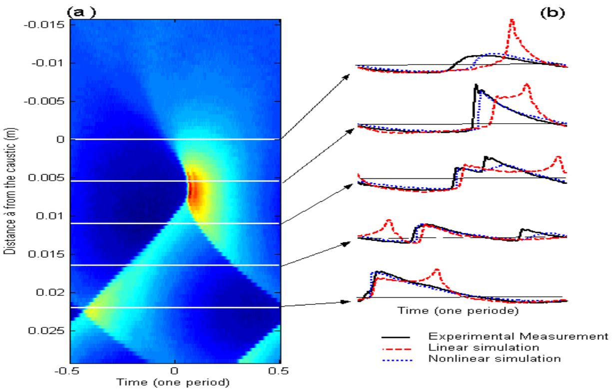

Figure 4 : Comparisons between measurement, linear and nonlinear numerical simulations.

Experimental and numerical results for the cusp caustic

As for the fold caustic, the first step of the experiment consists in synthesizing the cusp caustic in linear regime for monochromatic waves. We choose to synthesize a cusp with a cusp tip (focal) located 30cm away from the array (Fig 5). A set of control points is defined on a segment of 5.4cm length 37.5cm away from the array. In linear regime for monochromatic waves, the solution is provided by the Pearcey function. Even if the Pearcey function depends on two space variables (direction of propagation and transverse direction), we apply it only on the control segment. The signals to be emitted to synthesize the field on this segment are computed by the inverse filter technique and emitted at weak amplitude.

The field is measured not only on the control segment but also on a grid defined as follow : from -10cm to 14cm on the axis of propagation (the zero is defined as the geometrical position of the cusp tip) and from –2.75cm to 2.75cm on the transverse axis.

Fig. 6 presents the spatio temporal representation of the pressure field at three different distances from the caustic (z=-10cm, z=0 and z=10cm) in linear regime (left column). We can see the initial shape of the wavefront with its typical curvature producing the focusing at the cusp. Then we can see the focusing of the central part of the field at the cusp (z=0). Finally, after the focal point, we can see the inversion of the curvature, characteristic for a diverging wavefront. Fig. 7 shows the intensity distribution on each point of the grid of measurement. The white lines are the contours of the Pearcey function (analytical solution). We note that the agreement between the measurements and the Pearcey function is excellent, even if the field is imposed only along the control segment.

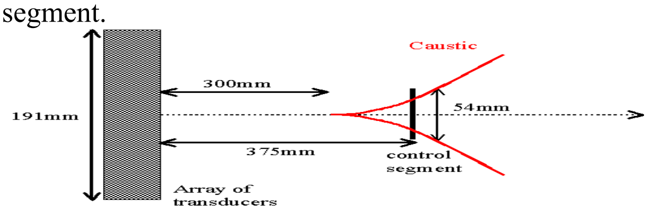

Figure 5 : Geometry of the experiment for the cusp caustic

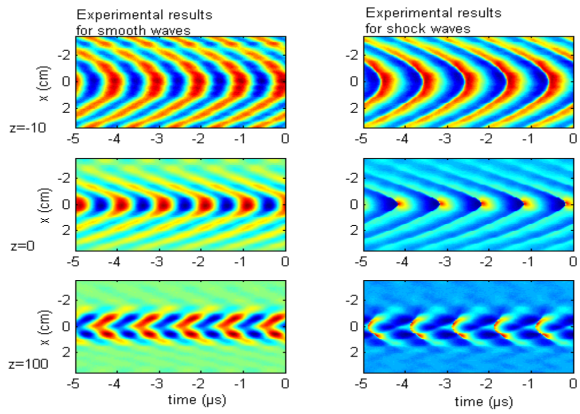

Figure 6 : Pressure field in the transverse direction at 3 distances from the cusp in linear (left) and nonlinear (right) regimes.

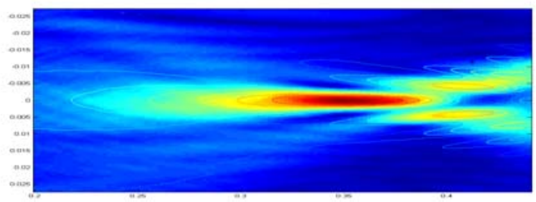

Figure 7 : Measured intensity pattern around the cusp caustic and the Pearcey function

To study the focusing of shock waves at cusp caustics, the signals are then emitted with a high amplitude. In this case, shock waves are produced at focus. Fig. 6 (right column) shows the nonlinear field in the transverse direction as a function of time at the same 3 different distances from the cusp as in linear regime (left column). In nonlinear regime the field is quite different from the linear one. The presence of shock waves is characterized by the quick transition between the compression (in red) and expansion (in blue) phases. At the cusp, the transverse size of the focal spot is smaller than in linear regime, moreover the compression phases are very sharp compared to the expansion ones. Finally, we can see the waves after the focal point. The shape of the wavefront is complex but thanks to the shock waves we can distinguish a swallow tail shape.

To simulate the focusing of shock waves numerically, two inputs are required as in the previous case : the temporal waveform of the incoming signals and the nonlinear parameter. The waveform used as input of the numerical code is the signal measured 10cm away from the cusp (Fig 8 the blue curve on the picture at the top right). The corresponding nonlinear parameter is 0.21. Fig. 8 (left column) presents the spatio-temporal representation of the field at the same three distances from the cusp as Fig.6 (right column) . We can see that the simulated field is very close to the measured one. Fig.8 shows a more quantitative comparison: the temporal profile at the 3 distances in x=0 obtained by experimental measurement (blue curves) and nonlinear simulation (red curves). We note the excellent agreement between the shapes of all three curves and especially the position of the shock. The amplitude is normalised for the 2 rows of curves by the amplitude at z=-10cm (boundary condition: the maximal amplitude on the curve at the top right is equal to one). So the amplitude is very well recovered by the simulation code which reproduces the amplification with a very good precision : there is only a small difference of about 5%.

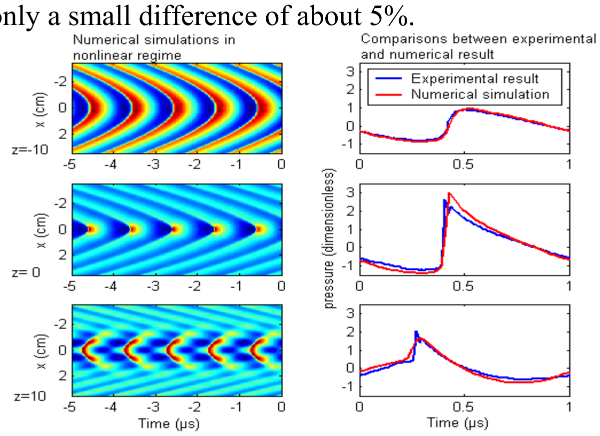

Figure 8 : Simulated fields at 3 distances from the cusp and comparisons on x=0 between numerical simulations and measurements.

Conclusion

The theoretical and numerical results show a very good agreement with the experimental measurements. This proves the validity of the theory, and especially the presence of local nonlinear effects in the neighborhood of caustics. Now that the physical mechanisms are better understood, the way of the reduction of focusing of sonic boom remains open and offers a formidable challenge.

Acknowledgements

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