SUPPRESSION OF HARMONICS BY SELECTIVE ABSORBERS IN THE RESONATORS

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Abstract

The application of nonlinear standing waves is connected with the high quality resonators that enable to accumulate the large amount of acoustics energy. This work deals with the possibilities of nonlinear attenuation by means of selective absorber in acoustic resonators. This passive method enables to enhance the quality factor Q. To increase the quality factor Q it is possible also using the method based on the active suppression of the second harmonic component of the sound wave. The inhomogenous Burgers equation is used for description of nonlinear standing waves in the resonator cavity. The numerical solution is computed in the frequency domain. It is presented the comparison between the passive and active methods for higher harmonics suppression which are generated in the course of cascade processes.

Introduction

Using of nonlinear standing waves is limited by the nonlinear attenuation that causes the acoustic saturation effects. The important characteristic of the resonator is the quality factor Q that shows how many times the amplitude of the steady-state wave is greater then the amplitude vibration of the exciting piston. The Q-factor depends on the amplitude of the vibrating piston due to nonlinear attenuation. The nonlinear attenuation is connected with nonlinear acoustic wave interactions when we can observe generation of higher harmonics. As the thermo-viscous attenuation is proportional to the square of frequency it is possible to decrease the nonlinear attenuation by suppression of the wave cascade processes. The resonators of the high Q-factor are used for thermo-viscous engines, acoustic compressors, chemical disintegrating devices.

Consistent with the second-order nonlinear theory, acoustic fields in the resonator can be represented by counter-propagating waves which are assumed to not interact in the resonator volume. These waves are coupled only by boundary conditions. If we suppose that the waves are slowly varying in space and in time it is possible to describe the waves by means of the inhomogeneous Burgers equation. When the exciting piston radiates more than one eigen-frequency of the resonator one can control generation of harmonics.

Model Equations for the active suppresion and the passive absorption

When describing the nonlinear plane standing waves in resonator of a constant radius it is possible to use the inhomogeneous Burgers equation in dimensionless form

$$\frac{\partial \overline{V}_\pm}{\partial \sigma} - \overline{V}_\pm \frac{\partial \overline{V}_\pm}{\partial \tau'_\pm} - \frac{1}{G_0}\frac{\partial^2 \overline{V}_\pm}{\partial \tau'^2_\pm} + D\frac{\partial^{\frac{1}{2}} \overline{V}_\pm}{\partial {\tau'_\pm}^{\frac{1}{2}}} = = K\sum_{n=1}^{N}(K_n + L_n V_n)\sin\left(n\tau'_\pm + \varphi^{(n)}_\pm\right), \quad (1)$$

where the dimensionless coordinates are defined as

$$\sigma = \frac{\beta v_0 \omega t}{c_0},\ \tau'_\pm = \omega\tau_\pm,\ \overline{V}_\pm = \frac{\overline{v}_\pm}{v_0}, \quad (2)$$

where t is time, c_0 is the small signal sound speed, β is parameter of nonlinearity, ω is the angular frequency, v_0 is the velocity amplitude, τ_+ and τ_- are the retarded times

$$\tau_+ = t - \frac{x}{c_0},\ \tau_- = t + \frac{x}{c_0}, \quad (3)$$

where x is the space coordinate in the direction of the resonator axis.

$$\overline{v}_\pm(t,\tau_\pm) = v_\pm(t,\tau_\pm) \pm \sum_{n=1}^{N}\frac{v_n x}{2L}\sin\left(n\omega\tau_\pm + \varphi^{(n)}_\pm\right), \quad (4)$$

where L is the resonator length. V_n is the amplitude of the n-th harmonic of velocity $\overline{V}_\pm$

$$V_n = \frac{2}{T}\int_0^T \overline{V}_\pm \sin(n\tau'_\pm). \quad (5)$$

G_0 is the Goldberg number, D is the boundary layer coefficient

$$D = \frac{Bc_0^2}{R_0\sqrt{\omega}v_n\beta}, \quad (6)$$

$$B = \sqrt{\frac{\nu}{2c_0^2}}\left(1 + \frac{\gamma - 1}{\sqrt{Pr}}\right), \quad (7)$$

R_0 is the resonator radius, ν is the kinematic viscosity coefficient, γ is the adiabatic exponent, Pr is the Prandtl's number. The fractional derivative can be expressed as

$$\frac{\partial^{\frac{1}{2}} f(\tau)}{\partial \tau^{\frac{1}{2}}} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\tau}\frac{\partial f(\tau')}{\partial \tau'}\frac{\mathrm{d}\tau'}{\sqrt{\tau - \tau'}}. \quad (8)$$

K_n are the amplitudes of active suppression harmonics, L_n are the amplitudes of selective absorption harmonics.

We can write for an acoustic velocity

$$v = v_+ - v_- \ , \tag{9}$$

Eq. (1) is valid for angular eigenfrequencies $\omega = \omega_n$ that

$$\omega_n = \frac{n\pi c_0}{L} \ , \quad n = 1, 2, 3, \dots \ . \tag{10}$$

In the case that we consider the harmonic excitation of the standing waves with the piston at the position $x = L$, we can express the boundary and initial conditions as follows

$$v = (v_+ - v_-)_{x=0} = 0 \ , \quad v_\pm(t = 0) = 0 \ , \tag{11}$$

$$v = (v_+ - v_-)_{x=L} = \sum_{n=1}^{N} v_n \sin(n\omega t + \varphi_n) \ , \tag{12}$$

where v_n are acoustic velocity amplitudes of the piston and φ_n are the phase shifts. We assume that a piston vibrates with the angular frequency ω which is equal to $(2n + 1)$-th eigenfrequency of the given resonator, it means that $\omega = \omega_{2n+1}$. This assumption causes that higher harmonic components of an acoustic velocity are in coincidence

Results

In this section we deal with comparison between the solutions of eq. (1) taking into account the active suppression of the second harmonic and the selective absorption in the second harmonic. The inhomogeneous Burgers equation (1) was solved by means of the standard Runge-Kutta method of the fourth order in the frequency domain (the first 100 harmonics were used). The numerical oscillations were damped by

$$\Psi_n = \frac{\sin(nH)}{nH}, \tag{13}$$

where H is the frequency damping coefficient. Each harmonic was multiplied by the coefficient Ψ_n. It causes the additional artificial attenuation of the solution. The value H was chosen so that the numerical oscillations practically did not arise.

The results are calculated for G_0=1000, $D = 0.01$, $K = 500$.

The calculation with no attenuation is made with parameters N=1, K_1=1, L_1=0, $\varphi_\pm^{(1)}$=0, Taking into account the active suppression of the second harmonic the parameters were N=2, K_1=1, $K_2 > 0$, $L_1 = L_2 = 0$, $\varphi_\pm^{(1)} = 0$, $\varphi_\pm^{(2)} = \pi$. To describe the selective absorption in the second harmonic, we set N=2, K_1=1, $K_2 = 0$ $L_1 = 0$, $L_2 > 0$, $\varphi_\pm^{(1)} = 0$, $\varphi_\pm^{(2)} = \pi$.

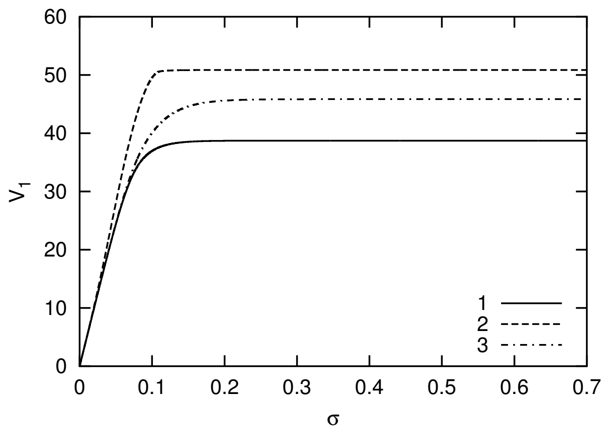

Figure 1: Time development of the first harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

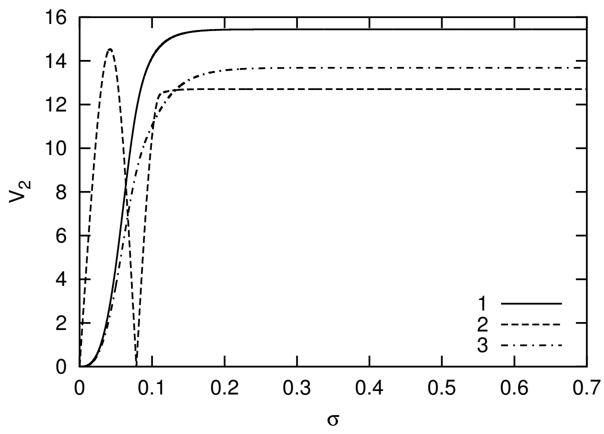

Figure 2: Time development of the second harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

The first set of figures 1-5 is made for $K_2 = 1$, V_2=20, whereas the second set of figures 6-10 is made for $K_2 = 25$, V_2=500.

We see that the energy transfer from the fundamental harmonics into higher ones is reduced and the "subharmonic" is generated. For this reason the acoustic saturation effects are also suppressed. The suppression of acoustic saturation causes both the amplitude of the steady-state wave and Q-factor increases. The higher Q-factor means that more acoustic energy is accumulated in the resonators. The effect of active suppression of the second harmonic is similar to the selective absorption in the second harmonic. Form the numerical results it is evident, that the effect of active suppression of the second harmonic is much more significant.

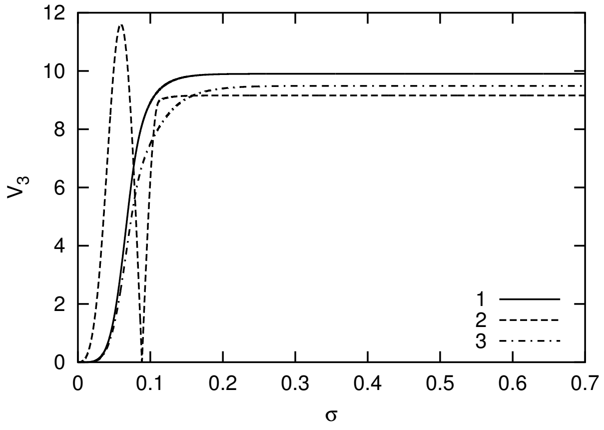

Figure 3: Time development of the third harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

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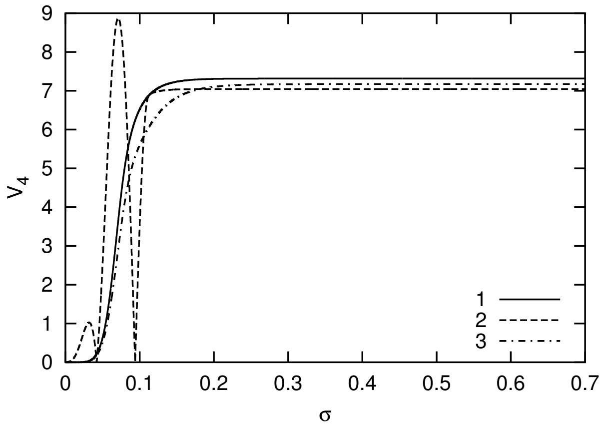

Figure 4: Time development of the fourth harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

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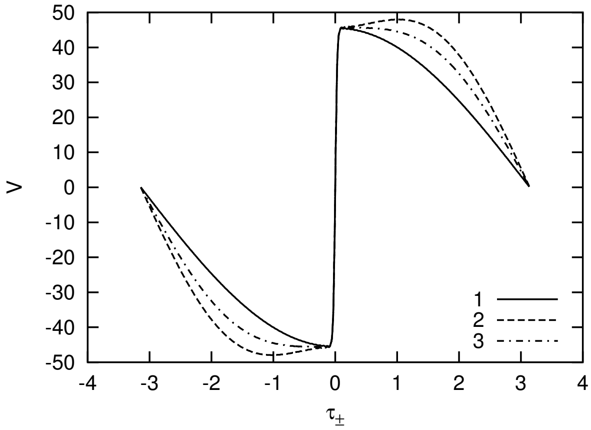

Figure 5: Time evolution of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

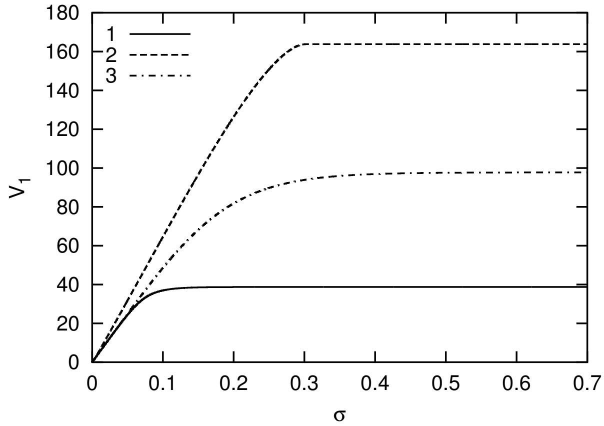

Figure 6: Time development of the first harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

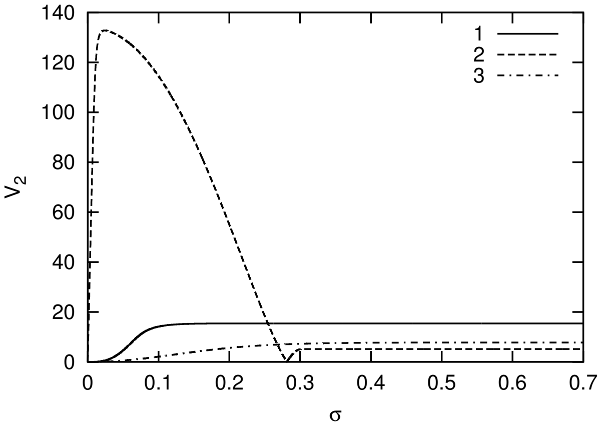

Figure 7: Time development of the second harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

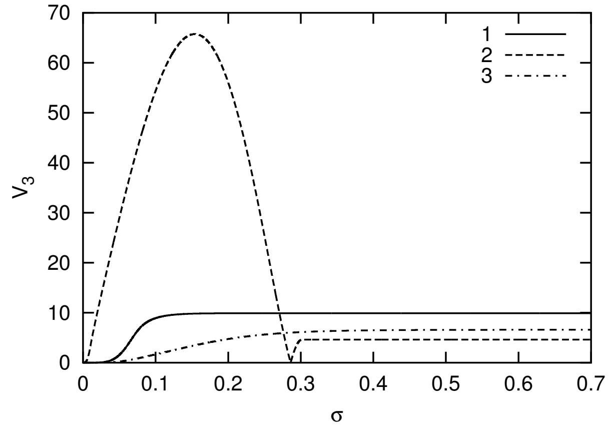

Figure 8: Time development of the third harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

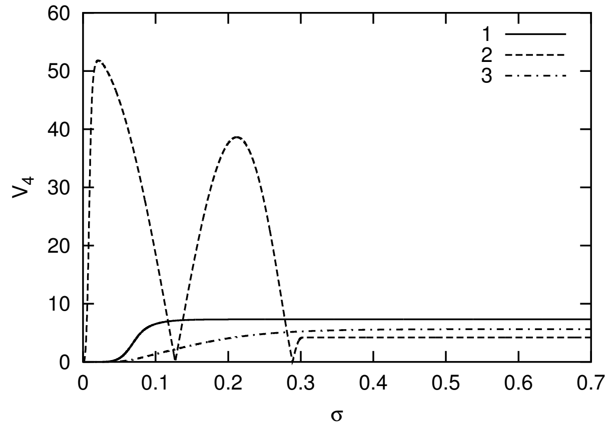

Figure 9: Time development of the fourth harmonic of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

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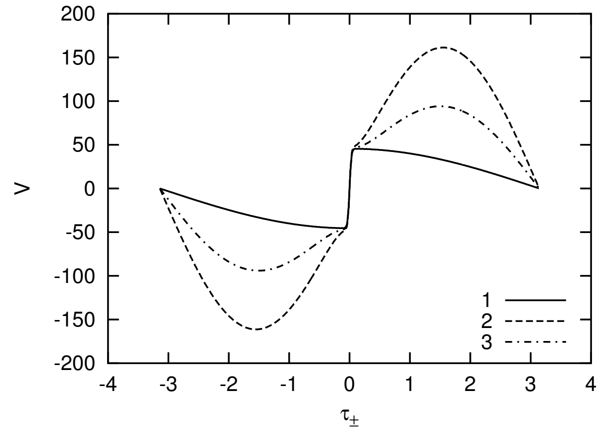

Figure 10: Time evolution of velocity. Comparison of the solution with no attenuation (solid line), with active suppression (dashed line), and with the selective absorption (dashed-dotted line).

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Acknowledgements

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