Short Space-Time-Wave Number-Frequency analysis of Lamb wave propagation and conversion at the edge of a plane plate.

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Abstract

A new 4D Space-Time-Wave Number-Frequency representation Z(x,t,k,ω) is proposed. This representation is an extension along the time dimension of the Space-Wave Number-Frequency representation. The Z(x,t,k,ω) representation is obtained by Short Time- Short Space 2D Fourier transforming the space-time signal collection. The Z(x,t,k,ω) representation is used to experimentally investigate Lamb wave propagation along a finite plane plate immersed in water. The space-time signal collection is recorded along the propagation direction using a laser vibrometer. Both Lamb wave propagation and conversion aspects are explored by using the Z(x,t,k,ω) representation. Using the appropriate slices of the Z(x,t,k,ω) representation, the complex wave numbers and the complex frequencies are quantified, pointing out the propagation aspect. One of the new features of the Z(x,t,k,ω) representation is to localize the Lamb waves in the space-time plane: for each Lamb wave, the mode conversion and reflection sequence is unambiguously revealed at the edges of the plate.

Introduction

The aim of this paper is to present new signal processing methods that allow the analysis of Surface Acoustic Waves (SAW) transient aspects in both time and space dimension. On the one hand, the time-frequency methods are efficient in localizing the frequency components of a 1D time signal [1, 2]. On the other hand the 2D Fourier transform of 2D space-time signals collections are very capable of extracting the propagation aspects involved in, but not of locating them in the space and time dimensions [3-12]. The methods proposed here are a fusion of both methods, extending the time-frequency method to 2D signals and leading to the Space Time-Wave number-Frequency representation Z(x,t,k,ω).

Space-time-wave number-frequency Z(x,t,k,ω) methods

Let's consider a wave propagating in a medium. For a one dimensional propagation along the x direction, the space-time collection s(x, t) is two dimensional (2D). Following the Fourier diamond (Fig. 1), by Fourier transforming s(x, t), three other spaces can be deduced from:
- S(x, f), the time Fourier transform of s(x, t),
- N(k, t), the space Fourier transform of s(x, t) and
- Ksi(k, f), the 2D Fourier transform of s(x, t) along the space and time dimensions.

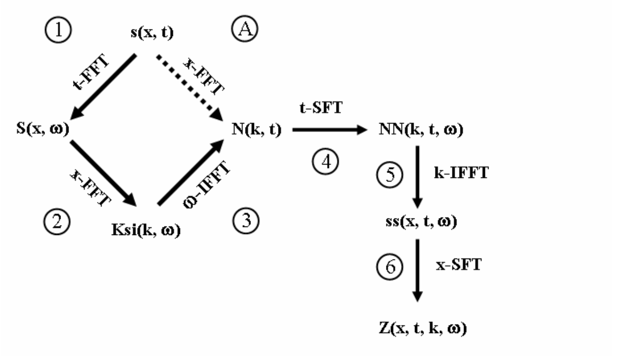

Figure 1 : Signal processing scheme.

These 3 spaces have interesting properties for wave propagation identification. The 2D Fourier transform of s(x, t) can be used to quantify, a posteriori, the waves properties, like their attenuation and phase velocity. However these existing methods suppose that each wave defined by its (k, ω) coordinates can occur only one time in the x-t plane: individual components of multiple generation are not separable by such classical methods. A way to separate these components is to use a small enough space and time Field Of View (FOV) before Fourier transforming.

The Space Time-Wave number-Frequency representation Z(x, t, k, ω) can be obtained through several equivalent paths. The first possible method is the straightforward extension of the time frequency methods to two dimensional signals: a small 2D window slides along the time and space dimensions, and the corresponding 2D spectrums Ksi(k, ω) are stacked, leading to the Z(x, t, k, ω) representation. However, this method needs a huge amount of computer resources to be computed and the resulting Z(x, t, k, ω) representation is not easily readable from a physical point of view.
Instead of achieving it in one step, the space and time localisation is achieved in two successive steps. This scheme is much more readable and less memory consuming. This second method uses the physical fact that energy is always spent, through the time dimension, from minus infinity to plus infinity, whereas, through the space dimensions, both directions are used. In addition, as the s(x, t) collection is real valued, loosing the negative frequencies that correspond to propagation towards

negative time, one can orient the space propagation and separate positive wave numbers from negative ones. This is the goal of following the path (1), (2) (3), instead of the path (A) in figure 1. The obtained Wave number- Time representation N(k,t) is then wave number oriented and the waves propagating along increasing x are separated from the one propagating in the opposite direction. The Short Time Sliding Fourier Transform (t-SFT) can now be applied to N(k,t) along the time dimension (Step 4). The 3D NN(k,t,ω) representation allows the localization of the Wave Number-Frequency (k, ω) aspect through time, but still not through space. The energy localisation through space is then done by the two following steps:

(5) NN(k,t,ω) is sent back to the space dimension by inverse space Fourier transform, leading to an intermediate Space-Time-Frequency representation ss(x,t,ω),

(6) the Z(x,t,k,ω) representation is the Short Space Sliding Fourier Transform (x-SFT) of the ss(x,t,ω) representation.

The advantage of using this path is that the steps (5) and (6) can be performed for an arbitrary set of selected frequencies instead of the whole frequency range. For example, as NN(k,t,ω) needs about 400MB of memory to be stored in the computer, using 128 space points would use 128 times 400MB of memory, whereas for a unique constant frequency ω0, NN(k,t,ω0) is 2D and Z(x,t,k,ω0) is a 3D cube that can be sliced and easily imaged as a time succession of Space-Wave Number (x, k) images or as a video sequence that can be directly stored on the hard drive of the computer.

Lamb wave propagation and mode conversion: Experimental study.

Experimental setup

The propagation of the surface waves is investigated on a plane plate (length L=60 mm and thickness e=2mm). The experimental setup shown in Fig. 2 is used to generate and detect the surface waves along their propagation along the shell.

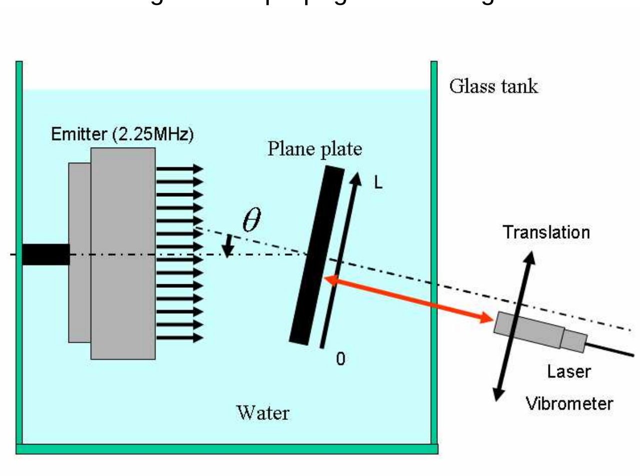

Figure 2 : Experimental setup.

The plate is immersed in a water tank. A pulse of 0.1µs long with 200V of amplitude is sent to a broadband transducer (1 MHz). In the enlightened zone of the plate, the plane bulk wave generates Lamb waves that propagate along the plate. A Polytec laser vibrometer is used for the vibration measurement.

The space time signal collection s(x, t) is presented in Fig. 3. The incidence angle used is 13°. Several surface waves are generated, mainly by the edges of the plate (x=0 and x=L).

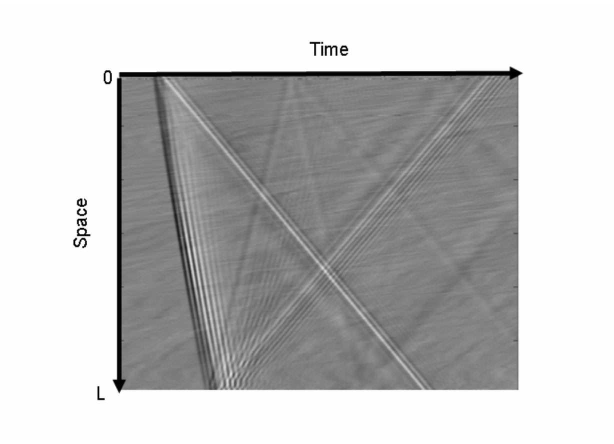

Figure 3 : Space time collection s(x, t).

Z(x, t, k, ω) analysis

Following the scheme presented in Fig. 1, the Z(x, t, k, ω0) analysis is performed on the space time collection s(x, t). The corresponding snapshots presented are presented in Figure 4.

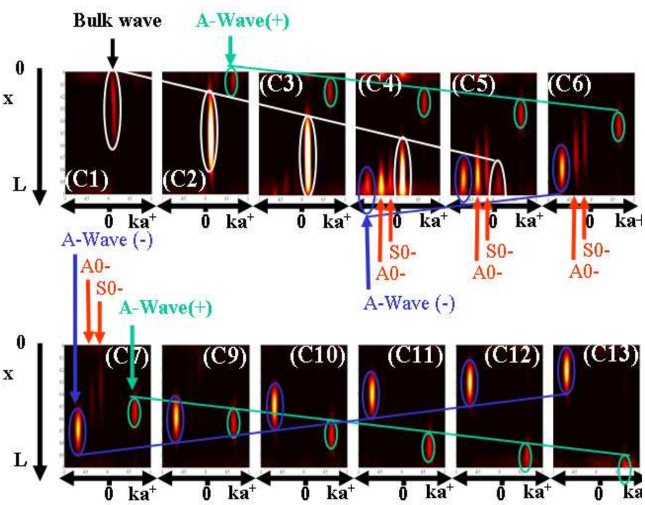

Figure 4 : Z(x, t, k, ω) analysis.

The selected frequency is 1MHz, and the time increases from left to right (from C1 to C13). From C1 to C3, the incident bulk wave passes trough the plate with an incident wave number equal to the one of the sound in water projected on the plate with the incidence angle. One can note that the Scholte like A-wave(+) is generated a little bit later and propagates at a slower speed from C2 to C13 [13, 14]. Due to the experimental geometry chosen, both waves propagates in the positive direction of propagation, from x=0 to x=L. One can note that the A-wave(+) is generated at the edge x=0 and propagates without attenuation till the other extremity, as predicted by the theory.

On the C4 slice, the incident bulk wave reaches the second edge of the plate (x=L) and converts itself in three back propagating modes: a A-wave(-), and two Lamb modes S0(-) and A0(-). These 2 last modes are highly attenuated and can hardly reach the other side of the plate at x=0.

Conclusion

A new 4D space-time-wave number-frequency representation Z(x, t, k, ω0) has been presented. This method is very capable of locating through the space and time dimensions the acoustic phonons by using the broadband experimental signals.

The 4D method has been successfully applied for the experimental study of wave propagation and conversion on a plane plate. Both the time and space transient aspect of wave generation, reflection and transmission have been observed. The representation also allows the analysis of the signal shape through time and space.

Acknowledgements

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