SUBHARMONIC THRESHOLD GENERATION IN ACOUSTIC VIBRATIONS OF PIEZOELECTRIC STRUCTURES

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Abstract

Some experimental results are reported on the generation of subharmonic vibrations in vibrating piezoelectric structures driven at a resonant mode frequency and its nearby frequencies. Particularly, attention has been addressed to the threshold phenomenon for half frequency subharmonic generation: by locally measuring the vibration amplitude of the samples' surfaces, it is found that the threshold value of the magnitude of the fundamental mode for the half frequency subharmonic generation depends only on the local oscillation amplitude of the structure and is independent both from the subharmonic wave space distribution and the frequency value. Some theoretical consideration are given to underpin this finding: by locally modelling the vibrating region with a Mathieu's equation, it is shown that the generation amplitude threshold is the same at all points of the structure.

Introduction

Subharmonic generation is an ubiquitous nonlinear effect taking place, under proper conditions, both in acoustic wave propagation and in the oscillatory behaviour of an anharmonic oscillator [1]. For it being a threshold phenomenon, the onset subharmonic oscillations, plays an important role in precahotic vibrations [2].

Some experimental results have been previously reported [3] on the generation of second and third harmonic frequency nonresonant modes in a finite piezoelectric structure. It was found that they completely match in space with the fundamental (driven) mode and do not follow their own space configuration, with higher frequency oscillations having a shorter space periodicity. That suggested us that, in our experiment, each point of the piezoelectric structure behaves independently from the others as if it were a simple one-dimensional nonlinear oscillator and, consequently, the nonlinear interaction were a strictly local effect. These findings are now confirmed by reporting new experimental results on the threshold value for half frequency subharmonic vibration mode in a thin plate and a hollow piezoelectric cylinder. The threshold value was found to depend uniquely from the "local" value of the amplitude of the fundamental mode and to be independent both from the frequency value of the fundamental mode and from the subharmonic mode space outline.

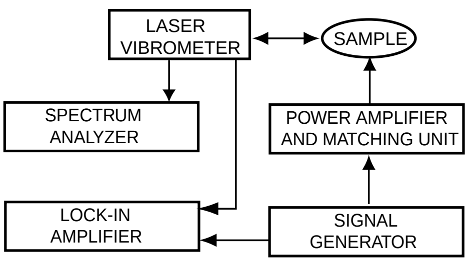

Figure 1: Experimental setup

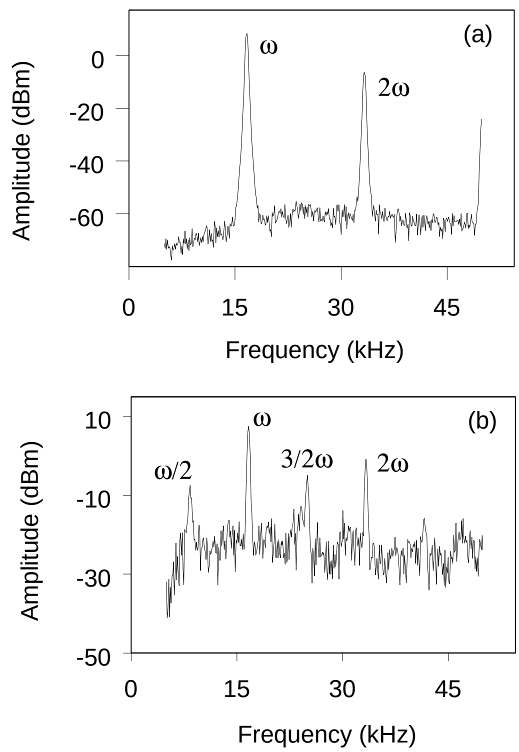

Figure 2: Frequency spectra, in nonlinear regime, of the vibration amplitude of the outer surface of the cylinder performed at the antinode and measured for two different values of the oscillation amplitude: below (a) and above (b) the threshold

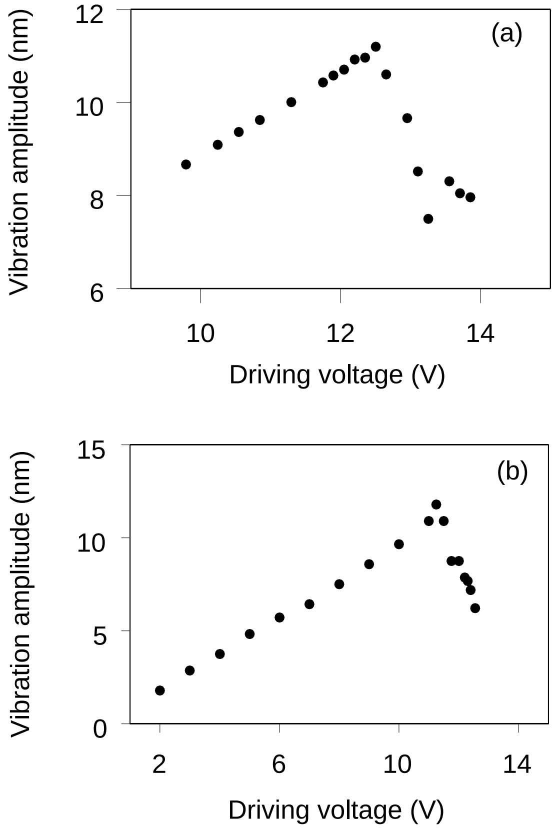

Figure 3: Surface vibration amplitude of a thin piezoelectric plate (a) and of a hollow cylinder (b) vs. the driving voltage amplitude at a fixed circular frequency: $\omega_0 = 12.9$ kHz and $\omega_0 = 16.8$ kHz for the plate and for the cylinder, respectively. Measurements were performed at the antinode of the resonant modes. The experimental error is ± 0.1 nm

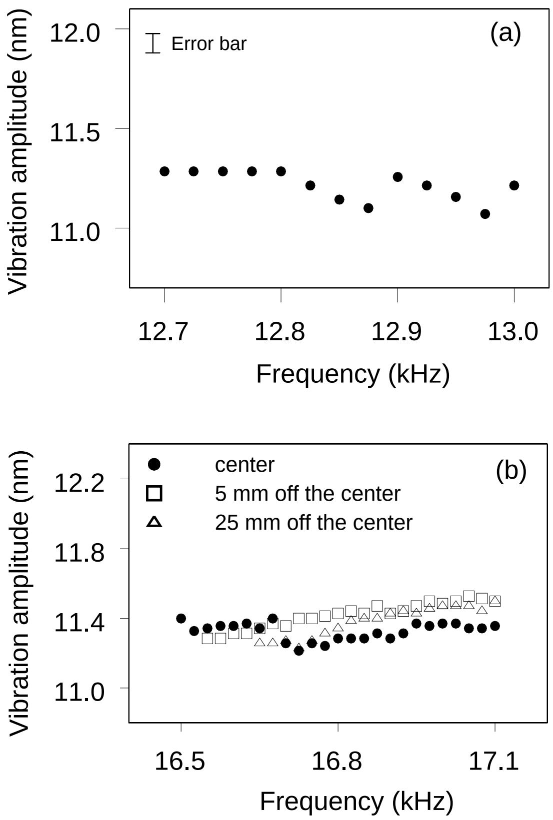

Figure 4: Threshold values of the surface vibration amplitude of the fundamental mode for half frequency subharmonic generation in a piezoelectric plate (a) and in a hollow piezoelectric cylinder (b) around the circular frequency ω_0 of the fundamental mode: $\omega_0 = 12.9$ kHz and $\omega_0 = 16.8$ kHz for the plate and the cylinder, respectively

Experimental Results

A thin piezoelectric plate (length $l = 20.0$ mm, width $w = 10.0$ mm, thickness $t = 1.0$ mm) and a piezoelectric hollow cylinder (length $l = 76.0$ mm, inner radius $r_i = 20.0$ mm, outer radius $r_o = 25.0$ mm) have been experimented on. Aluminum electrodes have been sputtered onto the major surfaces and onto the internal and external surfaces of the plate and hollow cylinder, respectively. The experimental setup is sketched in Figure 1. The samples were set into vibration by driving them with an a.c. voltage applied to the electrodes at the frequency of one of their normal modes (previously measured by an impedance meter): $\omega_0 = 12.9$ kHz for the plate, corresponding to its lowest Lamb mode, and $\omega_0 = 16.8$ kHz for the cylinder corresponding to its length (parallel to the axis) resonance. Both amplitude and phase of free surface oscillations were locally detected by an acoustooptical probe (laser vibrometer), while the frequency response of the vibration amplitude was obtained through a spectrum analyzer.

As an example, two typical frequency spectra, in nonlinear regime, of the vibration amplitude of the outer surface of the cylinder measured in the middle of the axis (the antinode), are reported in Figure 2 for two different values of the vibration amplitude: below the threshold, the spectra show only harmonic vibrations of the resonant mode at frequency equal to 16.8 kHz, (a); above the threshold, a half frequency (and 3/2) subharmonic appears, (b).

The growth of the vibration amplitude at the excitation circular frequency ω_0 vs. the driving voltage has been preliminary tested, fot both samples, in case that the driving voltage does not exceed the value where a sudden drop takes place: this is the threshold value where the generation of half frequency subharmonic takes place. The measurements, reported in Figure 3, were performed at the antinode of the excited eigenmodes of the sample (where the vibration amplitude attains it maximum value): the fundamental vibration amplitude steadily grows with the driving voltage (indeed, a slight increase of energy towards multiple harmonic waves is taking place) up to a value where energy

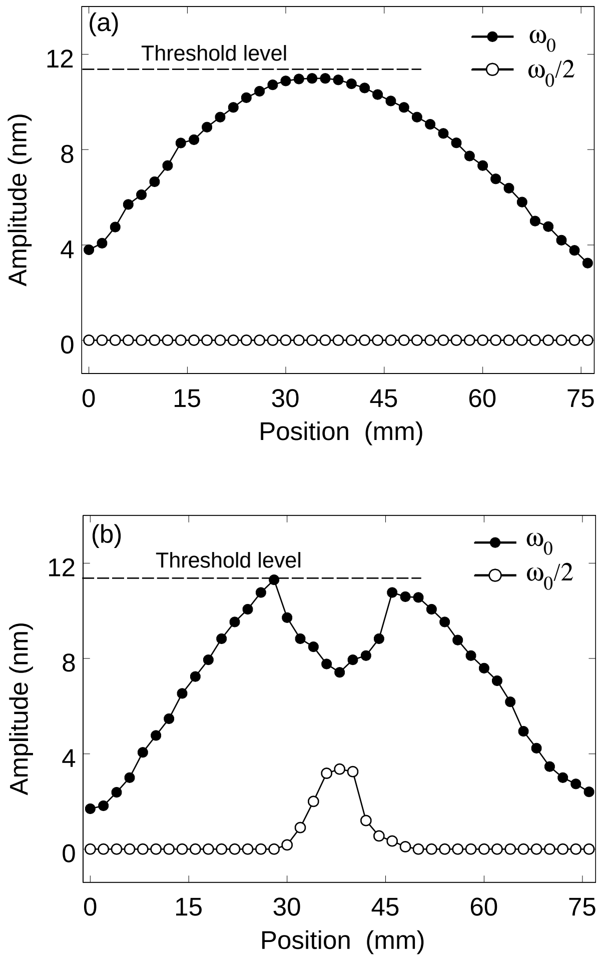

Figure 5: Space distribution along the axis of the cylinder of the amplitude of the fundamental mode (solid circle) and of the half frequency subharmonic vibration (open circle) for two different values of the driving voltage: (a) 11 V, (b) 14 V

abruptly starts flowing from the fundamental mode to half frequency subharmonic oscillation.

The independence of the threshold from the frequency of the fundamental mode, is clearly shown in Figure 4 ((a) plate, (b) cylinder) that reports, for driving frequencies around ω_0, the surface vibration amplitude of the fundamental mode for the generation of half frequency subharmonic oscillations. In the case of the plate the measurements were performed in a single point at the antinode of the resonant mode, while for the cylinder they were taken along its axis both in middle point, that is at the antinode, (solid circle) and at ± 5 mm (empty box) and at ± 25 mm (triangle) off the middle point.

Figure 4 (b) (in order to make it more clear, only data taken at +5 and + 25 mm, were reported) clearly shows the independence of the threshold from the subharmonic mode space distribution. This is further on confirmed by the data reported in Figure 5 that shows, for two different values of the driving voltage, the vibration amplitude, measured along the axis of the cylinder, of both the fundamental (length resonance) and subharmonic mode. The subharmonic do actually exist only in the region where the amplitude of the fundamental mode overcomes the threshold: the subharmonic vibration mode is strictly localized in a selected region of the cylinder's axis.

Model

From the experimental results it was found that the threshold value for half frequency subharmonic generation results uniquely dependent upon the local vibration amplitude of the fundamental and is not affected by the excitation frequency mode (this is, by the way, an indirect evidence of the independence from subharmonic wave space distribution). This suggests a kind of "independent oscillator" model for subharmonic generation phenomenon in a finite structure.

Consequently, in order to explain the experimental results, a simple model was considered based on a nonlinear forced point mass in a cubic potential well as the oscillation of order 1/2 is apt to occur when the nonlinearity is unsymmetrical [4], [5]. Therefore we started from the rate equation

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx(1 + cx) = F\cos(\omega t) \qquad (1)$$

with constants a, b, c, and F; assuming that the displacement values are sufficiently small such as the oscillator only experiences restoring forces toward its equilibrium position, the solution of Eq. (1) was considered in the form of $x = x' + \xi$, where x' is the solution of the linear equation

$$\frac{d^2x'}{dt^2} + a\frac{dx'}{dt} + bx' = F\cos(\omega t) \qquad (2)$$

and ξ is a small perturbation term. Substituting the solution in Eq. 1, a Mathieu's equation in ξ is obtained: the oscillation threshold of the resonant mode for one half frequency subharmonic oscillation, can be easily obtained in this case by recursively solving the Mathieu's equation at circular frequency $\omega/2$ and looking for amplification effect [6]: simple considerations show that the threshold is almost constant in the frequency range used in the experiments.

Conclusions

The values of the vibration amplitude of the fundamental frequency mode for half frequency subharmonic generation (the threshold values), have been measured in some points of the vibrating surfaces of two piezoelectric structures (a thin plate and a hollow cylinder) forced into vibration at one of the resonant frequencies and its nearby frequencies. It is found that, for each sample, the threshold values are the same at different frequencies in different points.

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