WIDE BAND QUANTITATIVE IMAGING OF HIGH CONTRAST OBJECTS BY A CANONICAL APPROXIMATION

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Abstract

We consider the inverse scattering problem for three dimensional cylindrical objects which are infinite in the axial direction. Our aim being to achieve quantitative ultrasound imaging of high contrast objects, we focus at first on the simplified problem of reconstructing quantitatively the physical parameters (velocity, density) and the shape of a section of the cylinder perpendicular to its axis. We present in this paper a reconstruction method based on an approximation of the geometry by a canonical circular cylinder according to ICBA (Intercepting Canonical Body Approximation)[1] applied in a large frequency range[2]. More precisely, we propose a modification of the ICBA method which takes into account the acquisition protocol and show that the inverse problem can be solved in two steps by reconstructing first the shape of the object and then its physical parameters. Numerical results show the feasibility of this approach.

Introduction

We are concerned by the ultrasound characterization of the physical parameters and the shape of high contrast objects. We assume that the object is a non-absorbing and homogeneous fluid embedded in water. Consequently, the inverse problem to solve reduces to reconstructing a generally non-circular section of a cylinder and estimating its mean acoustical parameters.

One of the main difficulties in reconstructing objects having a high impedance contrast with the surrounding medium is the consequent disruption of the ultrasound beam at the interface. Therefore traditional imaging methods used for soft contrast (echography[3], tomography[4]) will fail in this case, at least without any particular treatment for high contrast[5]. On the other hand, an approximation of the non-penetrating body type does not allow any reconstruction of the physical parameters.

Our strategy for solving the inverse problem is based on an approximation of the real object by a canonical one. The field diffracted by this canonical body can be computed analytically using a Rayleigh-Fourier method to obtain a modal representation. The main advantage of this method is that it can be used even in the case of high impedance contrast. The inversion process consists in estimating the minimum of a cost function defined as the difference (using a convenient norm) between the estimated and the measured scattered fields.

By analyzing the cost function, we examine the advantages and drawbacks linked to the data type and the acquisition protocol. In particular, we briefly discuss the advantages of taking into account the whole frequency range. We also show the influence on the reconstruction of the diffusion angle, defined as the difference between the angle of measurements and the incident angle. Finally, results concerning both qualitative and quantitative parameters estimation are presented.

Measurements

The incident and predicted fields

The cylindrical object is illuminated by a plane wave propagating in a direction perpendicular to its axis. In the time domain, the incident wave is a second derivative of a Gaussian whose spectrum extends from 0 to 3 MHz and has a maximum at 1 MHz (this type of impulse is close to the experimental ones used in our laboratory). We will examine here the reconstruction of an object with an elliptic section ($2\ mm$ small radius in the $0rd$ direction and $2.75\ mm$ big radius in the $\frac{\pi}{2}rd$ direction). We measure the scattered field in the time domain at several points located on a circle of radius $6\ mm$ whose center coincides with the center of the object (near field) and for several angles of diffusion $\Phi = \theta_i - \theta_m$. Those measurements are synthetic data obtained by solving numerically the wave propagation problem in the time domain[6]. A Fast Fourier Transform is used to convert the data from the time to the frequency domain. These measurements will be called "predictions" in the following.

Canonical estimation

The starting point is the ICBA method: during the inversion, the prediction is substituted by the scattered field from an equivalent circular cylinder of infinite extend in its axial direction. This field can be defined analytically in the case of a fluid circular cylinder having penetrable boundary conditions.

In the following we consider an incident field which is an acoustic pressure plane wave at frequency ω with complex amplitude P_0. Omitting the $e^{-i\omega t}$ time-dependence, the incident field can be expressed by $P^i(\mathbf{x_m}, \omega, \Phi) = P_0 e^{i\mathbf{k_0}\cdot\mathbf{x_m}}$, with $\mathbf{x_m} = (r_m, \theta_m)$ the observation point and $\mathbf{k_0}$ the wave vector in the surrounding medium Ω_0 ($\mathbf{k_0} = \frac{\omega}{c_0} \cdot (\cos(\theta_i), \sin(\theta_i))$, c_0

the velocity in Ω_0). We write the total pressure field in Ω_0 as $P^t(\mathbf{x_m}, \omega, \Phi) = P^i(\mathbf{x_m}, \omega, \Phi) + P^s(\mathbf{x_m}, \omega, \Phi)$, where $P^s(\mathbf{x_m}, \omega, \Phi)$ denotes the scattered field in Ω_0. The acoustic fields are governed by the Helmholtz equation and the scattered field satisfies the usual conditions (Sommerfeld radiation condition, continuity of pressure and normal velocity at the interface). For a circular cylinder of radius a, we use a modal decomposition in terms of Bessel and Neumann basis functions (denoted respectively J_n and N_n at order n) and we factorize the scattered field $P^s(\mathbf{x_m}, \omega, \Phi)$ in outgoing waves:

$$P^s(\mathbf{x_m}, \omega, \Phi) = P_0 \sum_{n=0}^{N} b_n i^n \epsilon_n H_n^{(1)}(k_0 r_m) \cos(n\Phi)$$

where $H_n^{(1)}$ is the n^{th} order Hankel function of the first kind, ϵ_n is the Neumann factor ($\epsilon_0 = 1$, $\epsilon_n = 2$ for $n > 0$). The limit N of the factorization is chosen by checking the stabilization of the b_n series. The coefficients b_n are calculated by

$$b_n = -\frac{J_n(k_0 a) J_n{}'(k_1 a) - \frac{\rho_1 c_1}{\rho_0 c_0} J_n{}'(k_0 a) J_n(k_1 a)}{H_n^{(1)}(k_0 a) J_n{}'(k_1 a) - \frac{\rho_1 c_1}{\rho_0 c_0} H_n^{(1)'}(k_0 a) J_n(k_1 a)}$$

These coefficients are strongly non linear with respect to the acoustical parameters : $\rho_2,\ c_2$.
Note that the ICBA method does not require any hypothesis on the contrast, the equations of propagation, or the frequency range, but only on the geometry of the object. It is thus, particularly well adapted for wideband analysis of cylindrical objects.

Reconstruction

We compare the prediction with the equivalent circular cylinder estimation by defining a cost function in the following way,

$$\mathcal{F}(\tau_\mathbf{m}; \mathbf{x_m}, \omega, \Phi) = |P^p(\mathbf{x_m}, \omega, \Phi) - P^e(\tau_\mathbf{m}; \mathbf{x_m}, \omega, \Phi)|^2, \tag{1}$$

with $P^p(\mathbf{x_m}, \omega, \Phi)$ the predicted and $P^e(\tau_\mathbf{m}; \mathbf{x_m}, \omega, \Phi)$ the estimated scattered field. The estimated scattered field depends on the unknowns: velocity, density and local radius of the section (r_m) we seek to reconstruct. We denote these unknowns in a more general way by the vector $\tau_\mathbf{m}$. The original ICBA method is based on the following local reconstruction: at a minimum of the cost function according to $\tau_\mathbf{m}$ corresponds an admissible estimation of the unknowns along the direction θ_m. This implies in particular that the estimated local radius r_m corresponds to the radius of the object in the direction of measurement θ_m.

Remark that $\mathcal{F}$ can be defined for several frequencies and diffusion angles. These parameters can be used to over-determine the problem. In the following we will define different ways of taking into account these parameters.

Angles of observation

For local unknowns (ie., unknowns varying with θ) such as the radius of the section, the decoupled equation (1) is well adapted. The acoustical parameters however, have a homogeneous distribution. For reconstructing these global unknowns we prefer representing the problem as a system having as many equations as angles of observation. We then minimize this system in the least square sense. Assume that the shape of the object is known and let τ denote the vector of unknowns (ρ_2, c_2), for fixed ω and Φ we have,

$$\left\{ \begin{array}{lcl} \mathcal{F}_1(\tau) & = & |P^p(\mathbf{x_1}) - P^e(\tau; \mathbf{x_1})|^2 \\ \mathcal{F}_2(\tau) & = & |P^p(\mathbf{x_2}) - P^e(\tau; \mathbf{x_2})|^2 \\ \vdots & = & \vdots \\ \mathcal{F}_M(\tau) & = & |P^p(\mathbf{x_M}) - P^e(\tau; \mathbf{x_M})|^2 \end{array} \right. \tag{2}$$

In practice we want to determine both local and global unknowns. So we have the choice between:

- Decoupling the reconstruction problem : solve first for one type of unknowns (local or global) with arbitrary a priori on the second type. Then solve for the second type using the solution obtained in the first step. This assumes that the reconstruction of the first type of unknowns is independent of the reconstruction of the second type.
- Solving globally the system, with some local and some global unknowns in the estimation vector τ. Even if this method looks more attractive, it is less stable and the number of admissible solutions increases.

We will propose in the following a reconstruction method based on the decoupling approach.

Frequency

It is commonly admitted that lower frequencies (according to the size of the object) give better quantitative results but with low resolution. On the contrary, higher frequencies provide better resolution but do not permit quantitative imaging. Using the whole frequency domain of the incident wave is thus motivated by joining these two aspects.

Figure 1 represents the cost function for each frequency in the frequency domain of the incident wave. Each minimum denotes an admissible solution. Both measurements and estimation correspond to the backscattered field at $\theta_m = \frac{\pi}{6}$. The distribution of the minima presents a periodicity which depends on the frequency and one of the minima is always close to the

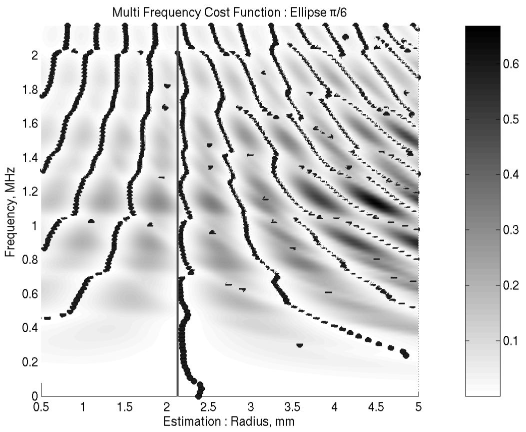

Figure 1: The cost function (in gray level) as a function of the local radius $r(\frac{\pi}{6})$ and the frequency. The straight line corresponds to the exact solution and dots represent the minima at each frequency.

exact solution. To take into account the whole frequency range we define the "mean cost function" (denoted MCF in the following): the mean over frequency of the previously defined one. We present in Fig.2 the MCF for back-scattered reconstruction corresponding to several local radius of the ellipsoid. We can see that the estimated solution is unique and that the quantitative value is given with a mean relative error of 2.0%. Comparing the results displayed in figures Fig.1 and Fig.2 we remark that summing the cost function over the frequency domain has a constructive effect on the minima close to the exact solution and a destructive effect on the other ones.

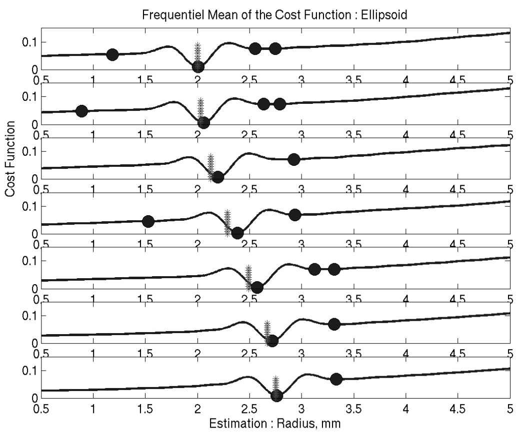

Figure 2: MCF as a function of the local radius $r(\theta_m)$. From top to bottom $\theta_m = 0$, $\frac{\pi}{12}$, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{5\pi}{12}$, $\frac{\pi}{2}$. Dots represent the minima.

In figure Fig.3 (resp. Fig.4) we display the MCF for back-scattered data as a function of the local radius $r(\frac{\pi}{4})$ and the velocity (resp. density), the density (resp. velocity) being fixed to the exact value. In both cases the MCF is almost constant as a function of the corresponding variable acoustical parameter. Consequently, the admissible solution for the local radius seems to be independent of the acoustical parameters. Therefore, section reconstruction is possible without ambiguity, even if we use a wrong a priori on the acoustical parameters. On the other hand, this also implies that back-scattered reconstruction does not permit estimation of the acoustical parameters.

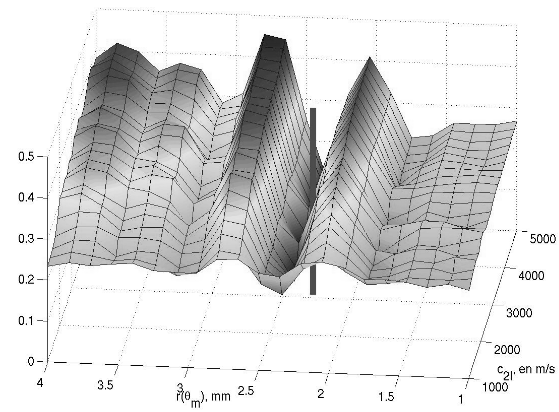

Figure 3: MCF for back-scattered data as a function of the local radius $r(\frac{\pi}{4})$ and c_2.

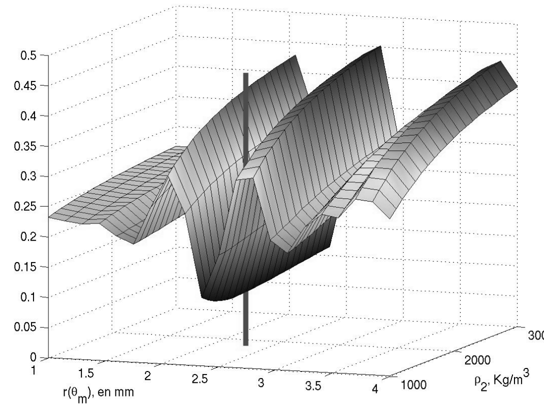

Figure 4: MCF for back-scattered data as a function of the local radius $r(\frac{\pi}{4})$ and ρ_2.

Shape reconstruction for various angles of diffusion

In this section we want to study the influence of the diffusion angle on the shape reconstruction. To do so we fix the acoustical parameters to the exact ones and use the MCF to reconstruct the shape of the object for different diffusion angles. We present the results obtained using the original ICBA method on the top line of Fig.5. Before explaining these results let us first recall that in the original ICBA method the local radius estimation is assigned to the radius of the object in the direction of measurement (θ_m). As we can see from the results, this assumption is true only for the back-scattered measurements. For non-back-scattered measurements, the obtained image is a simple rotation of the real object by $\beta = \frac{\Phi-\pi}{2}$. This angle corresponds to the bisector between the direction of the source $\theta_i - \pi$ and the observation direction θ_m.

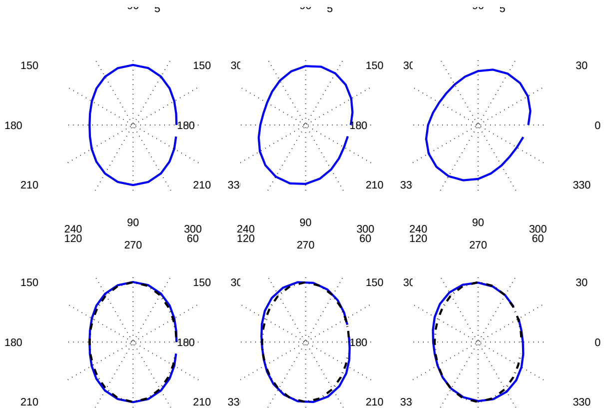

Figure 5: Shape reconstruction using MCF for several diffusion angles (from left to right $\Phi = \pi, \frac{3\pi}{4}, \frac{\pi}{2}$), with the original (top) and the modified (bottom) ICBA method. The dashed line represents the real section.

These results suggest the following modification of the ICBA method: the local radius estimation should be assigned to the radius of the object in the direction $\theta_m + \beta$ (instead of θ_m). Using this modification we obtain similar images for the different angles of diffusion $\Phi = \pi$, $\frac{3\pi}{4}$ $\frac{\pi}{2}$ (see bottom line of Fig.5). Similar results concerning the dependence of the shape reconstruction on the direction $\theta_m + \beta$ instead of θ_m were previously obtained for the Born approximation method in tomography [7].

Quantitative reconstruction

If we fix the exact section, the reconstruction of the acoustical parameters using back-scattered measurements does not converge. This result can be also linked to the Born approximation in tomography where quantitative reconstruction is not available for wavelengths smaller than the object. For non-back-scattered measurements quantitative reconstructions converges when we use the modified ICBA method. Results are given in table 1. The estimation of c_2 is satisfying, but ρ_2 is always under-estimated. Note that without this modification no quantitative reconstruction is possible.

Conclusion

We presented an algorithm for solving the inverse scattering problem in the case of a cylindrical object which is infinite in its axial direction. The proposed algorithm is based on a canonical approximation of the geometry and permits both qualitative and quantitative reconstruction using the whole frequency domain of the scattered field. By studying several acquisition geometries we observed that the shape reconstruction is independent of the acoustical parameters for back-scattered data which also implies that such data cannot be used for estimating the acoustical parameters. For non-back-scattered data we proposed a modification of the ICBA method which provides better shape reconstruction and permits quantitative estimation of the acoustical parameters. These preliminary results are encouraging. We are currently working on a more detailed analysis of the proposed method.

Table 1: Estimation of the acoustical parameters, for several angles of diffusion:

Φ	ρ_2 $[kg/m^3]$		c_2 $[m/s]$	
	mean	rel. err.	mean	rel. err.
$\frac{\pi}{2}$	1 650	-8%	3 180	3%
$\frac{3\pi}{4}$	1 620	-10%	3 000	-3%
π	diverge		diverge	

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