

## DEFINITION OF ELECTROMECHANICAL COUPLING COEFFICIENT OF BULK ACOUSTIC WAVES FROM ENERGY CONSIDERATION

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**Abstract** – Till recently it was considered that velocity of piezoactive acoustic waves propagating in piezoelectric materials decreases under total or partial “switching off” of piezoeffect. However, at present it has been shown that there are such materials and crystallographic orientations for which the velocity of bulk or surface acoustic waves in presence of piezoeffect ( $v_{pz}$ ) may be less than the velocity of the same waves under total or partial “switching off” of piezoeffect ( $v$ ). This fact leads to necessity of more precise definition of such important conception as electromechanical coupling coefficient, which is the measure of piezoactivity of acoustic waves. At present in spite of its traditional estimation there exist at least two different definitions from energy consideration. In this paper on example of the plane bulk acoustic waves propagating in some piezoelectric materials we first carried out the comparative analysis of different approaches to definition of this coefficient.

### I. INTRODUCTION

As is well known [1-3] electromechanical coupling coefficient is extremely important parameter, which defines the measure of acoustic wave piezoactivity and allows comparison of various types of waves from united consideration. Apparently this parameter must be common and universal for all types of waves and for all materials and structures. At present referred above coefficient  $K^2$  is most frequently defined as fractional change in the square of wave velocity due to total or partial “switching off” of piezoeffect [1,2]

$$K^2 = (V_{pz}^2 - V^2)/V_{pz}^2. \quad (1)$$

Here  $V_{pz}$  is the wave velocity in the presence of piezoeffect, and  $V$  is the wave velocity under total (for bulk acoustic waves) or partial (for surface acoustic waves) “switching off” of piezoeffect. At that for bulk acoustic waves the total “switching off” of piezoeffect means that all piezoelectric constants are considered vanishing at calculation [1]. As for surface acoustic waves, the partial “switching off” of piezoeffect may be reached by electrical shorting of the surface [1, 2]. If the wave is characterized by weak piezoactivity, i.e.  $V_{pz} - V \ll V_{pz}$ , expression (1) may be written as [3]

$$K^2 \approx 2(V_{pz} - V)/V_{pz}. \quad (2)$$

However in recent years it was reported that use of such traditional definition becomes unacceptable or inadequate in some cases. It was theoretically and experimentally shown that there exist such materials and crystallographic orientations, for which the presence of piezoeffect decreases the velocity of bulk

[4] and surface [4-7] acoustic waves. This fact is sufficiently unexpected because it was always considered that the velocity of piezoactive acoustic waves propagating in piezoelectric materials decreases under total or partial “switching off” of piezoeffect [1-3,8]. Besides for some orientations in piezoelectric materials the fast shear bulk acoustic wave can transform into the slow wave and vice versa due to “switching off” of piezoeffect [4] and after electrical shorting of the surface generalized Rayleigh wave may turn out into generalized Bleustein-Gulyaev wave [9]. And at last the estimation of electromechanical coupling coefficient for weakly inhomogeneous surface waves is embarrassed by existence of anomalous resisto-acoustic effect [7].

Aforementioned facts mean that traditional and widely used definition of electromechanical coupling coefficient in accordance with (1) lost their universality. In this case there appear the question: How should this coefficient be defined to be united and universal for all types of acoustic waves? As is known there exist at least two generally used definitions of this coefficient from energy consideration [1, 10-12]. It is apparently that these definitions are more universal because they do not suppose the total or partial “switching off” of piezoeffect, which can lead to significant change of wave type. In order to understand, which definition is more preferable it is necessary to make corresponding comparative analysis because so far this problem was disregarded.

Thus, as it was mentioned at present there exist two generally accepted definitions of electromechanical coupling coefficient from energy consideration. They are [10]

$$K_1^2 = P_{el}/(P_{el} + P_{mech}) \quad (3)$$

and [3, 11, 12]

$$K_2^2 = P_{elmech}^2/(P_{el} \times P_{mech}). \quad (4)$$

Here  $P_{mech}$ ,  $P_{el}$ ,  $P_{elmech}$  are mechanical, electrical, and electromechanical contributions in the time-averaged total power flow of acoustic wave.

In this paper we first carried out the comparative analysis of aforementioned definitions of electromechanical coupling coefficient on example of plane bulk acoustic waves propagating in crystals of quartz, lithium niobate, lithium tantalate, and potassium niobate. All types of waves (quasi-longitudinal, fast and slow quasi-shear) for all propagation directions in X, Y, and Z cuts have been included in our analysis.

## II. THEORETICAL ANALYSIS AND RESULTS

Below we consider the propagation of plane bulk acoustic wave in piezoelectric media along the direction defined by the unit vector  $\mathbf{n} = \{n_1, n_2, n_3\}$  in the following form

$$\begin{aligned} u_i &= U_i \exp[j\omega (t - n_i x_i / V)], \\ \Phi &= \Phi_0 \exp[j\omega (t - n_i x_i / V)]. \end{aligned} \quad (5)$$

Here,  $U_i$  is the amplitude of  $i$ -component of mechanical displacement  $u_i$ ,  $\Phi_0$  is the amplitude of electric potential  $\Phi$ ,  $\omega$  is the angular frequency,  $t$  is time,  $x_i$  is the spatial coordinate,  $j$  is the imaginary unit, and  $V$  is the phase velocity. The phase velocities and relative values of wave amplitudes in presence and absence of piezoeffect were calculated by standard way based on numerical searching for the eigen-values and eigen-vectors of appropriate Christoffel tensors [3].

Apparently, that for plane bulk waves the ratios of mechanical, electrical, and mutual electromechanical power flows appearing in (3) and (4) are exactly equal to ratios of corresponding contributions to the total energy density. Therefore in our calculations we used in (3) and (4) instead the contributions to power flow the corresponding contributions to energy density, which can be written as [8,10]

$$W_{\text{mech}} = 1/2 \text{Re}\{C_{ijkl} S_{kl} S_{ij}^*\}, \quad (6)$$

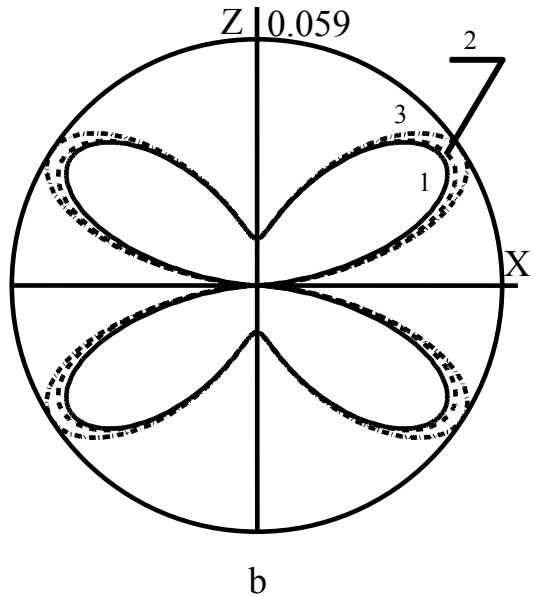
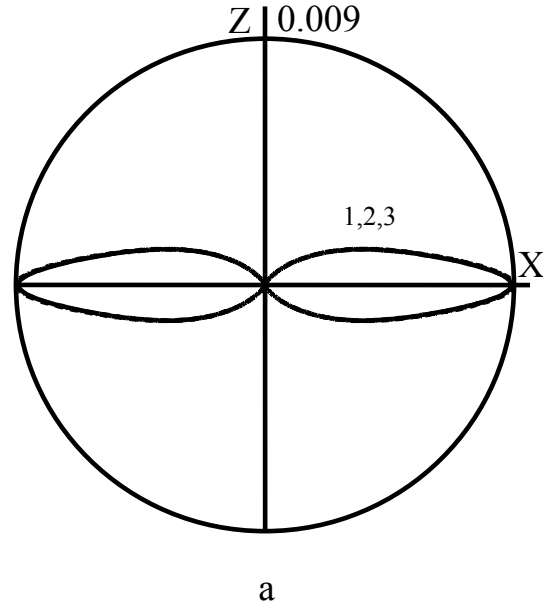
$$W_{\text{el}} = 1/2 \text{Re}\{\epsilon_{ij} E_i E_j^*\}, \quad (7)$$

$$W_{\text{elmech}} = -1/2 \text{Re}\{e_{kij} E_k S_{ij}^*\}. \quad (8)$$

Here,  $W_{\text{mech}}$ ,  $W_{\text{el}}$ , and  $W_{\text{elmech}}$  are the time-averaged densities of mechanical, electrical, and mutual electromechanical energies,  $S_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2$  is the mechanical strain tensor,  $E_k = -\partial \Phi / \partial x_k$  is the electric field intensity,  $C_{ijkl}$ ,  $e_{kij}$ , and  $\epsilon_{ij}$  are elastic, piezoelectric, and dielectric constants of media, \* denotes the complex conjugation.

At first, it has been shown that for every type of plane bulk acoustic wave for arbitrary crystallographic orientation the density of electromechanical energy is identical with density of electrical energy. Then in accordance with (1), (3), and (4) the coefficients  $K^2$ ,  $K_1^2$ , and  $K_2^2$  for all types of plane bulk acoustic waves as function of propagation direction for referred above materials and crystallographic cuts have been calculated. It has been shown that values of these coefficients are close each other for any wave with weak electromechanical coupling. However, in general case the values of these coefficients of course are different. At that the closest coefficients are  $K_1^2$  and  $K^2$ , which are exactly equal each other for a number of crystallographic orientations. This can be confirmed by Fig. 1, which

shows as example dependencies of these coefficients on propagation direction for quasi-longitudinal bulk wave in Y cut of quartz (a), lithium tantalite (b), lithium niobate (c), and potassium niobate (d).



### III. SUMMARY

As a whole the obtained results allows to make the following conclusion. Seemingly, the energy approach based on expression (3) is more expedient for definition of electromechanical coupling coefficient. In this case the electromechanical coupling coefficient lies always in interval  $[0 - 1]$  and this allows to define how the real coefficient is close to its limit value. Besides the definition based on (3) gives the numerical values, which are the most close to values obtained from traditionally used definition (1). It means that there is no need to recalculate all well-known cumulative values of electromechanical coupling coefficient for various materials. If the wave is characterized by weak piezoactivity the electromechanical coupling coefficient may be found by using of any expression (1), (2), (3) or (4).

### IV. ACKNOWLEDGEMENT

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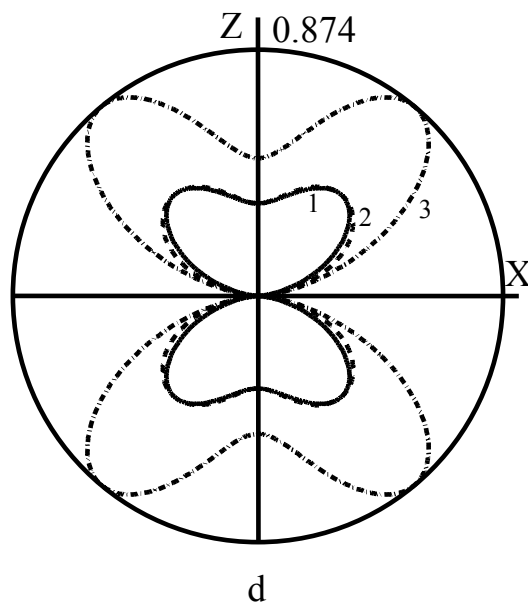
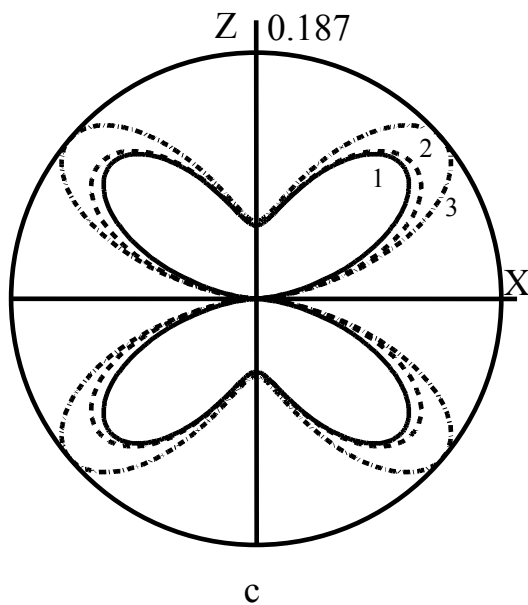


Figure 1. Dependencies of electro-mechanical coupling coefficients  $K^2_1$  (1),  $K^2_2$  (2), and  $K^2_3$  (3) for quasi-longitudinal bulk acoustic waves on propagation direction in Y-cut of crystals of quartz (a), lithium tantalite (b), lithium niobate (c), and potassium niobate (d).

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