2D AND 3D FINITE ELEMENT /BOUNDARY ELEMENT COMPUTATIONS OF PERIODIC PIEZOELECTRIC TRANSDUCERS RADIATING IN STRATIFIED MEDIA

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Abstract

A model base on the combination of finite element analysis and boundary element method is presented to address the problem of periodic transducers radiating in stratified media mixing fluids and solids. The mathematic principle of the computation is described, and numerical results are reported for standard 2 and 3D transducers as well as micromachined ultrasonic transducers (MUT).

Introduction

Ultrasound arrays for medical imaging applications are mainly based on composite structures associating materials of various acoustical properties. The actuation principle generally consists in the vibration of a PZT ridge glued on a backing, with one or two matching layers covered by a mylar and eventually an acoustic lens. These transducers may exhibit up to 192 single transducers for 1D probes or more than 64×64 transducers for 2D devices devoted to 3D imaging. Devices based on piezocomposite have also been introduced to improve the characteristics of classical acoustic probes, and the new concept of Micromachined Ultrasound Transducer (MUT) gives rise to new opportunities in the development of high density integrated imaging devices.

The design of these transducers requires powerful and flexible tools, able to accurately simulate complex combinations of materials exhibiting acoustic and dielectric losses, with a reliable representation of acoustic radiation in semi-infinite fluids or in stratified radiation media. Furthermore, the periodicity of the probes has to be taken into account to correctly predict their capability to convert bulk vibrations into acoustic radiation, avoiding any parasitic effects due to wave guiding along the array. The proposed paper is devoted to the development of a computation tool based on a finite element analysis (FEA) and a boundary element method (BEM) to address these problems. A very classical mechanical displacement – electrical potential formulation has been first implemented as a starting base of further developments. It is based on an harmonic analysis of the admittance (or impedance) of any piezoelectric transducer. It was then associated with boundary element techniques to simulate acoustic radiation in different media assuming plane interfaces. Periodicity has been taken into account using a standard periodic finite element approach and Bloch-Floquet developments for the radiation medium. In the first section of the paper, the basic periodic FEA formulation is recalled. The associated BEM approach is then reported. Computation results are then presented for different kinds of piezoelectric ultrasound transducers to illustrate the efficiency of the proposed approach. It is particularly shown how to use these computations to predict cross talk effects in periodic structures radiating in water. Further developments of the proposed work are discussed in conclusion.

Fundamentals of the model

The developments of finite elements calculations to address piezoelectricity problems have been widely reported in many articles (see for instance ref. [1]). The one adopted for the present work relies on a mechanical displacement and electrical potential formulation and only the final form of the problem is recalled here. Let us consider an elementary cell of a quasi-periodic transducer which geometry conforms to the representation of fig.1.

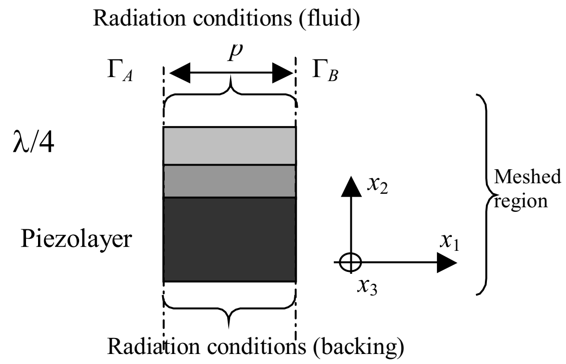


Fig.1 Definition of an elementary cell of an infinite periodic array of transducer

One of the boundary conditions consists in the electrical excitation of the piezoelectric layer, which is assumed to be governed by an harmonic relation [2] as follows :

$$\phi_n = \phi_o e^{-j2\pi\gamma n} \quad (1)$$

which signifies that the nth active electrode is excited by a potential of magnitude ϕ_o modulated by a phase proportional to its distance from the 0th electrode. The excitation parameter γ denotes the way the structure is excited. The counter electrode is set to the reference potential (0 V.). In the same approach, the mechanical displacements and the stresses obey this quasi periodicity law :

$$\begin{aligned} u_i(x_1+np) &= u_i(x_1)e^{-j2\pi\gamma n} = u_{i_n}(x_1) \\ T_{ij}(x_1+np) &= T_{ij}(x_1)e^{-j2\pi\gamma n} = T_{ij_n}(x_1) \end{aligned} \quad (2)$$

These relations yield the definition of specific boundary conditions at the limits Γ_A and Γ_B of the elementary cell of fig. 1. These conditions simply deduced from eqs (1) and (2) are given in eq. (3), and directly concern the degrees of freedom (dof) at the corresponding boundary :

$$\begin{Bmatrix} u\ \Gamma_B \\ \phi\ \Gamma_B \end{Bmatrix} = \begin{Bmatrix} u\ \Gamma_A \\ \phi\ \Gamma_A \end{Bmatrix} e^{-j2\pi\gamma} \quad (3)$$

Note that the spatial distribution of nodes (supporting the dof) on Γ_A and Γ_B must be identical to ensure the coherence of eq. (3). This relation is then used according to ref. [3] to simplify the linear algebraic system obtained after discretization and integration of the piezoelectric Lagrangian expression. In this approach, the following variable change matrix $[C_u]$ is introduced :

$$\begin{Bmatrix} u,\phi_{\Gamma_A} \\ u,\phi_\Omega \\ u,\phi_{\Gamma_B} \end{Bmatrix} = \left[C_{u,\phi}\right] \begin{Bmatrix} v,\varphi_{\Gamma_A} \\ v,\varphi_\Omega \\ v,\varphi_{\Gamma_B} \end{Bmatrix} = \begin{bmatrix} I_{\Gamma_A} & 0 & 0 \\ 0 & I_\Omega & 0 \\ I_{\Gamma_A}e^{j2\pi\gamma} & 0 & I_{\Gamma_B} \end{bmatrix} \begin{Bmatrix} v,\varphi_{\Gamma_A} \\ v,\varphi_\Omega \\ v,\varphi_{\Gamma_B} \end{Bmatrix} \quad (4)$$

where u,ϕ_Ω corresponds to the dof of the inner meshed domain (Γ_A and Γ_B excluded). Equation (4) is then inserted in the standard discrete form of the FEA written as follows for a monochromatic variation of mechanical and electrical fields considered in the problem (time dependence in $e^{j\omega t}$) :

$$^t\begin{bmatrix} C_u^* & 0 \\ 0 & C_\phi^* \end{bmatrix} \begin{bmatrix} K_{uu}-\omega^2 M_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} C_u & 0 \\ 0 & C_\phi \end{bmatrix} \begin{Bmatrix} v \\ \varphi \end{Bmatrix} = {}^t\begin{bmatrix} C_u^* & 0 \\ 0 & C_\phi^* \end{bmatrix} \begin{Bmatrix} F \\ Q \end{Bmatrix} \quad (5)$$

where K_{uu} and M_{uu} correspond respectively to the stiffness and mass matrices of the purely elastic part of the problem, $K_{u\phi}$ is the piezoelectric coupling matrix and $K_{\phi\phi}$ represents its purely dielectric part. Superscripts t and * repectively indicates a matrix transposition and a complex conjugation. Equation (5) is solved by setting $v_{\Gamma B}$ and $\varphi_{\Gamma B}$ to zero in order to comply with boundary condition (3). In the right hand side of eq. (5), F and Q are respectively relative to nodal mechanical and electrical load. One should note that in the case of complex matrices K_{uu}, $K_{u\phi}$ and $K_{\phi\phi}$, the matrix product on the left hand side of (5) yields a general complex matrix with no particular property. Addressing the problem of radiation in adjacent media requires an effort in the description of radiation conditions. This achieved by using the periodic Green's function formalism in which the pressure is related to the normal displacement via a convolution integral as follows :

$$T_{ij}n_j = -P \text{ with}$$

$$P = \frac{1}{p}\int_{-p/2}^{+p/2} G_{22}^p(x_1 - x_1')u_2(x_1')dx_1' \text{ and}$$

$$G_{22}^p(x_1) = \sum_{l=-\infty}^{\infty} \tilde{G}_{22}(\gamma + l, \omega)e^{-j\frac{2\pi}{p}(\gamma+l)x_1} \quad (6)$$

$$\tilde{G}_{22}(s_1,\omega) = \frac{j\rho_f\omega}{\sqrt{s_p^2 - s_1^2}}$$

In eq.(6), the radiation axis was assumed arbitrarily along u2 without any loss of generality. This formulation only holds for flat radiation surfaces. In the case of 3D computation with two periodic boundary conditions, one has to take a particular care to the way the corners of the mesh are related one to the others (see ref.[3]). Inserting eq. (3) into the discrete FEA formulation via the transformation matrix given in eq.(4) or the 3D version [3,4] yields the problem to be dependent on ω and γ. Thus, eq. (5) must be solved for each couple (ω, γ) to determine the specific properties of a given structure and to calculate the harmonic admittance $Y(\omega, \gamma)$. Since the magnitude of the excitation ϕ_o is fixed to 1.V, $Y(\omega, \gamma)$ is directly given by the courant generated in the active electrode by the vibration of the structure. This courant is simply derived from the nodal charges on the active electrode using the following formula :

$$Y(\omega,\gamma) = I(\omega,\gamma) = j\omega \sum_{n=1}^{Ne} Q_n \quad (7)$$

where Ne is the total number of nodes at the considered electrode. Using the harmonic admittance as defined in eq.(7), one can easily define the mutual admittances [15] of the array by using the Fourier series properties as follows :

$$Y_n(\omega) = \int_0^1 Y(\omega,\gamma)e^{-j2\pi\gamma n}d\gamma \quad (8)$$

This equation gives access to the influence of the excited cell of the array (n = 0) on the others, enabling to estimate for instance the level of cross-talk between two adjacent cells or to point out propagation phenomena along the surface of the array. The integral of eq.(7) is performed using a Gauss numerical scheme for γ defined in the range (0;0.5), taking advantage of the symmetry of the harmonic admittance around 0.5. For each frequency point, one has to compute $Y(\omega, \gamma)$ at these integration points. On the other hand, the computation of $Y_n(\omega)$ is almost immediate using the Gauss integration scheme. This approach is very efficient for smooth contributions to the harmonic admittance. For sharp peaks, the proposed computation is valid only for a given range of neighbour cells. In the case of 2D periodic structures (3D computations), two excitation parameters have to be defined yielding the following definition of the mutual admittances:

$$Y_{nm}(\omega)=\int_0^1\int_0^1 Y(\omega,\gamma_1,\gamma_2)e^{-j2\pi\gamma_1 n}e^{-j2\pi\gamma_2 m}d\gamma_1 d\gamma_2 \qquad (8)$$

Exploitation

1D linear probe

The first proposed illustration of how the above-presented model can be exploited corresponds to a classical 1D antenna probe based on the following stacked structure :

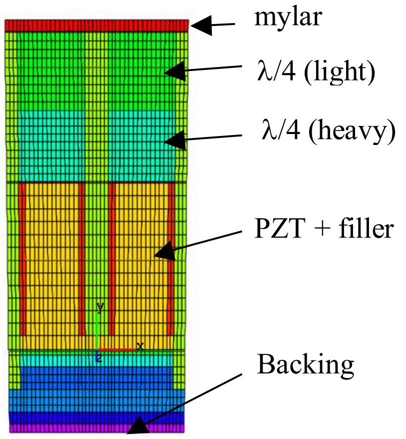
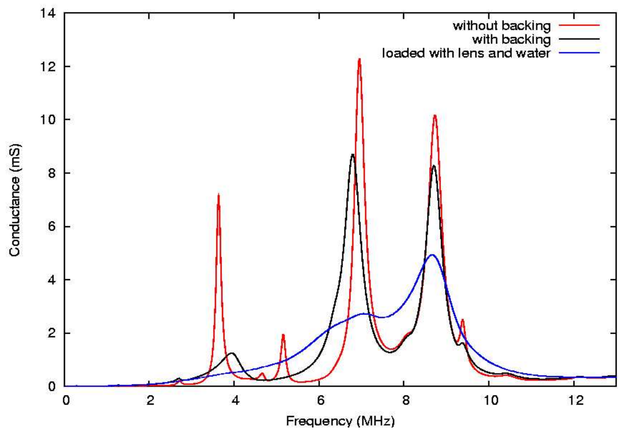
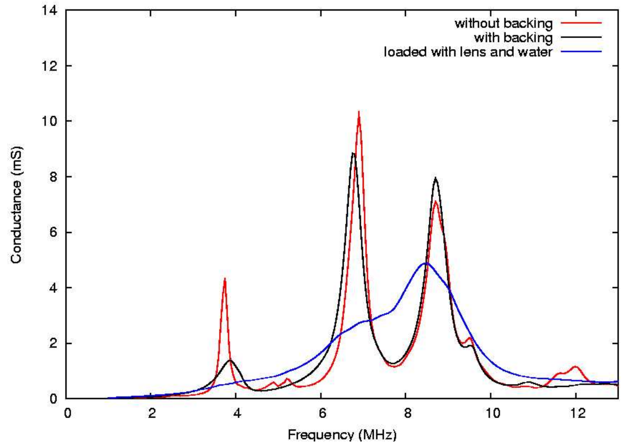


Fig.2 Example of one meshed period of a 1D classical probe.

In that problem, the top surface is loaded by the lens plus a semi-infinite water region and the back side of the stacked structure is also assumed semi-infinite (backing). For this particular design we had access to experimental measurement of the admittance of the device excited in phase (γ=0) at different stage of the fabrication, allowing for the following theory vs experiments assessment.



(a)



(b)

Fig.3 Comparison between theoretical (a) and experimental (b) admittance of a 1D linear probe

One still remarks some discrepancies between both results, mainly due to the difficulty to correctly identify the loss parameters. Actually, one can see that contributions in air without backing are not accurately predicted. Nevertheless, all the contributions are correctly taken into account when radiating in backing and/or in water, yielding a rather nice agreement between theory and measurements in both cases. In those situations, the mutual admittances have been finally computed, showing the very small amount of cross-talk effects in this particular design.

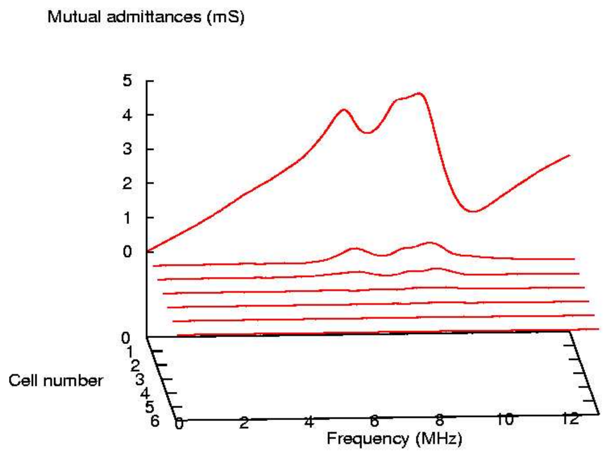


Fig.3 Computation results of the mutual admittance of the 1D probe, almost no contribution was found after the 7th neighbor.

2D array for 3D imaging

3D imaging systems require the implementation of 2D probes. Many developments have been initiated during the last decade yielding different kind of 2D arrays devoted to that application. In that issue, the 2D assumptions are no more valid and the simulation of actual elementary structures impose the use of 3D computations. The periodic computation then appears as an elegant way to reduce the mesh for FEA. However, one needs to adapt eq.(6) to account for a planar radiation area instead of a line. This can be easily performed for fluids because of their simple 3D spectral Green's function but requires more efforts for solids. Our modeling tools have been enhanced in that way yielding the following results for a 3D probe loaded by a semi-infinite backing on its bottom side (red area in fig.5). One can easily identify the efficient modes (normal radiation) using the deformed mesh :

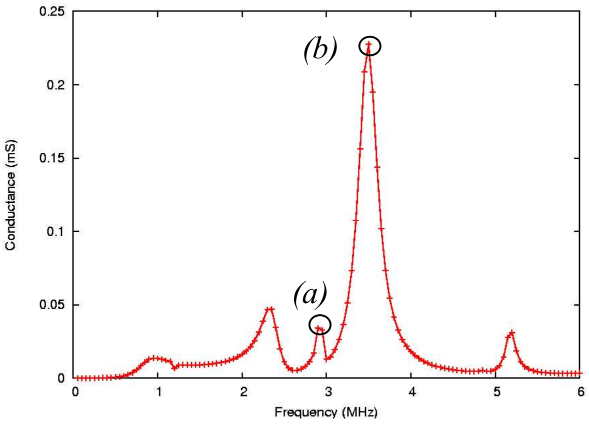


Fig.4 Admittance of the single cell at γ=0

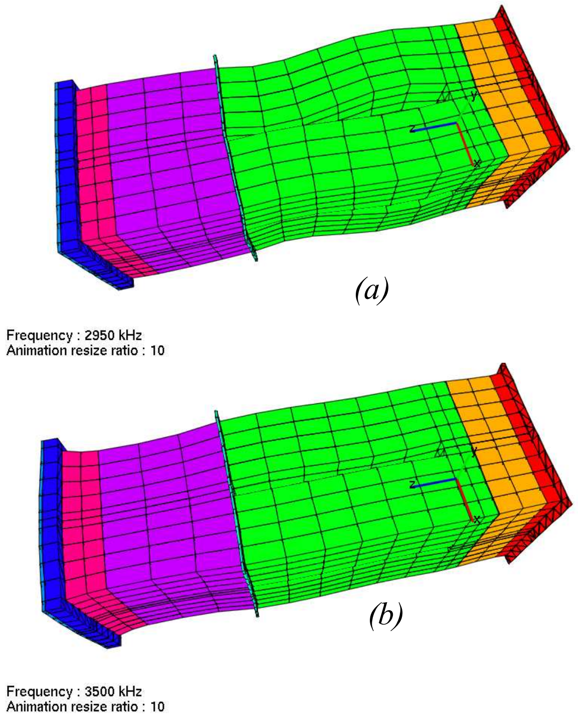


Fig.5 deformed mesh corresponding to fig.4
(a) parasitic mode (b) useful longitudinal mode

MUTs

The last example presented in this paper concerns micromachined ultrasonic transducers (the so called MUTs). Such transducers are massively periodic since each electrical pixel is composed of many elementary sub-wavelength acoustic cells. Such a structure exhibits a very particular behavior compared to standard resonant PZT-based transducers. One can show using 3D periodic FEA accounting for radiations in the substrate and in the water that the linear operation of electrostatic as well as piezoelectric MUTs can be represented using the so-called Wilm diagram reported here, which describes how the structure vibrates along the excitation parameter γ.

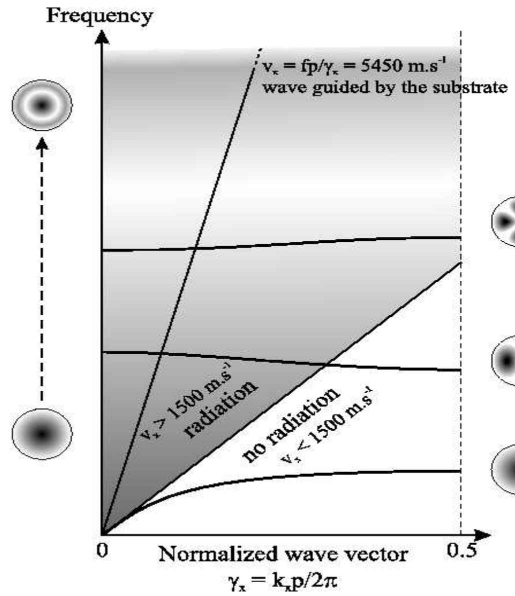


Fig.6 The so-called Wilm diagram showing the nature of the MUT vibration vs the excitation parameter

Conclusion

The use of periodic FEA associated with a BEM approach to simulate radiation in different kind of media enables a realistic description of the actual operation of ultrasonic transducers. Particularly for medical imaging applications or non-destructive evaluation, one can take advantage of such simulation tools to accurately design and analyze any kind of 2D as well as 3D devices. The possibility to extract mutual admittances or displacements from these periodic computations provides an efficient way to estimate cross-talk phenomena but also to simulate any excitation condition of the probe, yielding for instance the possibility to estimate directivity of a probe or to compute its actual transfer function.

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