A FEA/BEM APPROACH TO SIMULATE COMPLEX ELECTRODE STRUCTURES DEVOTED TO GUIDED ELASTIC WAVE PERIODIC TRANSDUCERS

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Abstract

A modelling approach able to address complicated SAW periodic structures with non homogeneous geometry has been developed and implemented. It is based on the combination of finite element analysis and a boundary element method. Validation of the computation is reported. An example of simulation of a passivated STW resonator is used for theory/experiment assessment.

Introduction

The simulation of periodic transducers devoted to surface acoustic wave (SAW) application has received a very high interest for more than 10 years. Very accurate models have been developed to compute SAW excitation and propagation parameters [1], efficiently used in COM or mixed matrix procedures devoted to SAW filter design. Moreover, the use of more advanced transducer structures has revealed many advantages for more robust devices exhibiting optimised properties (temperature compensation for instance). However, most of the above-mentioned simulation tools are implemented assuming a single electrode (or 3 electrodes max.) per period with vacuum as the adjacent medium. This assumption considerably simplifies the model (the electrode is assumed perfectly flat electrically) but is not suited to simulate devices based on more complicated electrode arrangements including dielectric layers deposited on the electrode array.

The present paper describes an approach mixing a Finite Element Analysis (FEA) and a Boundary Element Method (BEM) to simulate any periodic elastic wave guide. The basic idea consists in meshing the non homogeneous part of the transducer (typically the domain close to the electrodes) and a part of the substrate (typically one to two wavelengths). A radiation condition is then applied at the meshed boundary of the substrate based on boundary elements built using the Green's function of the substrate. The later can be composed of a single material or a layered structure (including fluids and vacuum) assuming flat boundaries. This approach can be applied on one or two sides of a given device to reduce computation duration or to address the problem of interface waves.

In the first section of the paper, theoretical developments are detailed together with numerical implementation considerations. Validation tests are then reported, performed for standard SAW devices on quartz. Experimental measurements have been also performed using fused silica deposited atop a surface transverse wave (STW) device. Comparison between theoretical and experimental characteristics of the STW is finally reported and discussed.

Fundamentals

As already reported in many references (see for instance [2]), FEA can be performed for periodic devices with rather simple modifications of the basic algebraic formula relating the displacement and electrical fields to the boundary solicitations. Figure 1 shows a general scheme of the considered periodic device geometry. Propagation is assumed taking place along x_1, inhomogeneous along x_2 and independent along x_3. As shown in this figure, the meshed region can be composed of various materials with arbitrary contours assuming they can be well represented using an elastic displacement FEA formulation. Material losses can also be considered.

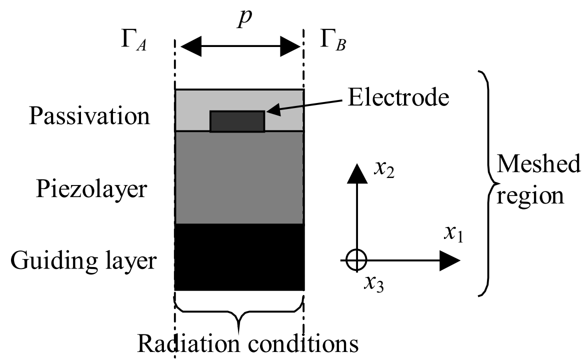

Fig. 1 General scheme of the addressed problem

The basic equations governing periodic FEA computations are now briefly recalled. It consists first in relating all the degree of freedom (dof) on boundary Γ_A to those on boundary Γ_B, yielding the following expression:

$$\begin{Bmatrix} u\ \Gamma_B \\ \phi\ \Gamma_B \end{Bmatrix} = \begin{Bmatrix} u\ \Gamma_A \\ \phi\ \Gamma_A \end{Bmatrix} e^{-j2\pi\gamma} \tag{1}$$

in which u and ϕ respectively hold for displacement and electrical potential, and γ is the excitation parameter. This relation is then used to reduce the number of independent dof of the FEA model. This is performed without changing the total number of dof of the problem, simply by using a variable change operator C. This provides the following form of the FEA algebraic system to be solved:

$$^t\begin{bmatrix} C_u^* & 0 \\ 0 & C_\phi^* \end{bmatrix}\begin{bmatrix} K_{uu}-\omega^2 M_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix}\begin{bmatrix} C_u & 0 \\ 0 & C_\phi \end{bmatrix}\begin{Bmatrix} v \\ \varphi \end{Bmatrix} = {}^t\begin{bmatrix} C_u^* & 0 \\ 0 & C_\phi^* \end{bmatrix}\begin{Bmatrix} F \\ Q \end{Bmatrix} \tag{2}$$

where K and M are respectively the FEA stiffness and mass matrices, ν and φ the independent dof of the problem and F and Q the right hand side boundary forces and electrical charges. Since K can be complex, the left hand side matrix in (2) is general (hermitic if $K \in \Re$), but sparse. These properties are considered when solving the problem.

Let us now consider the case of acoustic radiation on one border of the meshed domain. In that purpose, the general variationnal equation is considered, limited to the purely elastic problem without any loss of generality:

$$\iiint_\Omega \left(\rho\omega^2 u_i \delta u_i^* - \frac{\partial \delta u_i^*}{\partial x_j} C_{ijkl} \frac{\partial u_l}{\partial x_k}\right) dV = \iint_\Gamma \delta u_i^* T_{ij} n_j dS \tag{3}$$

in which δu_i is the variationnal unknown and n_j the normal to boundary Γ on which the radiation boundary condition is applied. Equation (3) is written in 3D but of course its restriction to 2D problems does not induce any fundamental difficulty. The right hand side of (3) is then considered separately. In this matter, one can relate the stress T_{ij} to the displacement u_k in the spectral domain (denoted by ~) by using a Green's tensor, which generalise the usual surface stress relation widely used in SAW modelling [4], as follows :

$$\tilde{T}_{ij} = \tilde{G}_{ijk}(s_1, \omega)\tilde{u}_k \tag{4}$$

This equation allows one to consider any flat boundary for the application of the radiation conditions, even if tilted in the plane (x_1, x_2). Using the now well-established periodic Green's function formalism, the right hand side of eq.(3) is expressed as :

$$\iint_\Gamma \delta u_i^* T_{ij} n_j dS = \iint_\Gamma \delta u_i^*(x) \sum_{l=-\infty}^{+\infty} G_{ijk}(\gamma + l, \omega) \frac{n_j}{p} e^{-j\frac{2\pi}{p}(\gamma+l)(x-x')} u_k(x') dx' dx \tag{5}$$

The classical FEA interpolation procedure is then applied to eq.(5), yielding the following expression of the boundary radiation operator :

$$\iint_\Gamma \delta u_i^* T_{ij} n_j dS = \sum_{e,\varepsilon=1}^{E} \sum_{l=-\infty}^{+\infty} \frac{n_j}{p} G_{ijk}(\gamma + l, \omega) \times \int_{\Gamma e} \sum_{n=1}^{Nd(e)} P_n(x) e^{-j\frac{2\pi}{p}(\gamma+l)x} dx \delta u_i^{*(n,e)} \int_{\Gamma\varepsilon} \sum_{m=1}^{Nd(\varepsilon)} P_m(x') e^{j\frac{2\pi}{p}(\gamma+l)x'} dx' u_k^{(m,\varepsilon)} \tag{6}$$

where E is the total number of "radiating" elements, and $P_n(x)$ are the FEA interpolation polynomials (1^{st} or 2^{nd} degree). This equation can be finally written as :

$$\iint_\Gamma \delta u_i T_{ij} n_j dS = \sum_{e,\varepsilon=1}^{E} \sum_{n=1}^{Nd(e)} \sum_{m=1}^{Nd(\varepsilon)} \sum_{l=-\infty}^{+\infty} \frac{n_j}{p} G_{ijk}(\gamma + l, \omega) u_k^{(m,\varepsilon)} \delta u_i^{*(n,e)} I_{\gamma+l}^{*(n,e)} I_{\gamma+l}^{(m,\varepsilon)} \tag{7}$$

$$\text{with } I_{\gamma+l}^{(m,\varepsilon)} = \int_{\Gamma\varepsilon} P_m(x) e^{j\frac{2\pi}{p}(\gamma+l)x} dx$$

The contribution of the radiating boundary to the global algebraic system to be solved consists then in a frequency and excitation parameter dependent matrix $X(\omega,\gamma)$ related to both dof and variationnal unknown and consequently computed in the left hand side of eq.(2).

Numerical implementation

1. General discussion

In this section, a particular interest is dedicated to the properties of the different terms involved in the computation of eq.(7) and a strategy is proposed to reduce computation duration. In eq.(7), it appears that the two boundary integrals are complex conjugated, potentially yielding hermitic properties of the matrix. However, the parity properties of the Green's function [3] prevent any simplification of the calculation. It is then necessary to compute all the terms of eq.(7). However, it is relevant to compute the integral $I_{(\gamma+l)}^{(n,e)}$ for all the possible values of $\gamma+l$ and for all the radiating elements, and also to compute the Green's tensor for all $\gamma+l$ and ω before assembling the radiation matrix $X(\omega,\gamma)$ which can be performed just before solving the system. Note that this assembly cannot be performed in the usual FEA approach in which elementary matrices are summed to built K and M. In the present case, the non symmetric sky line matrix is directly built computing all the cross-coupled terms induced by the convolution between the unknown fields and the Green's tensor.

In eq.(5) to (7), an infinite sum over the space harmonics is performed to compute the Green's function. Practically, the sum is reduced to a finite number of terms to ensure the convergence of the calculation. A specific treatment of the asymptotic behaviour of the Green's function is under development. Finally, the radiation medium can be composed of any combination of solids and fluids (assuming flat interfaces), taking advantage of the stabilisation of the Green's function described in [4].

2 Numerical tests

The proposed theoretical development has been implemented and a first set of computation tests has been performed considering the excitation of transverse waves on AT quartz and on Y+42° $LiTaO_3$. The later allows for considering generally polarized waves even if mainly transverse. The computation results are compared to those provided by the well-known mixed Finite Element/Boundary Integral approach proposed by Ventura *& al* [1]. The mesh used to compute the problem is plotted in fig.2. The period p of the grating was arbitrarily fixed and a metal ratio a/p=0.5. The number of elements has been kept identical for all the computations and only the aluminium thickness was set arbitrarily too. As shown in fig.2, only a small part of the semi-infinite medium had to be meshed, yielding rather short computation delays.

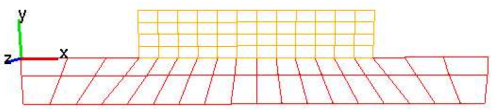

Fig.2 Typical mesh for modelling a standard SAW transducer (2nd degree interpolation)

The next figures shows quite nice comparison between the well established FEA/BIM model and the present work. Convergence was found for a small number of spatial harmonics and only to element layers were required to operate the matching between FEA and BEM formulations. Figure 3 shows the comparison between harmonic admittance of a STW device on (AT,Z) quartz computed along the two considered approaches for different frequencies and excitation parameters, with an almost perfect superimposition of both results.

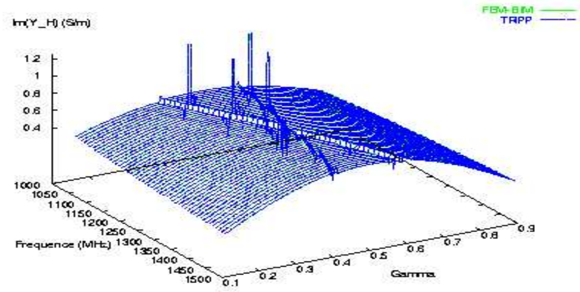

Fig.3 Imaginary part of the harmonic admittance of a STW on (AT,Z) quartz
solid green : FEA/BIM solid blue : this work

The next figure shows a superimposition of the harmonic admittance for a PSAW on Y+42° $LiTaO_3$. Here again, the two results perfectly matches one to the other.

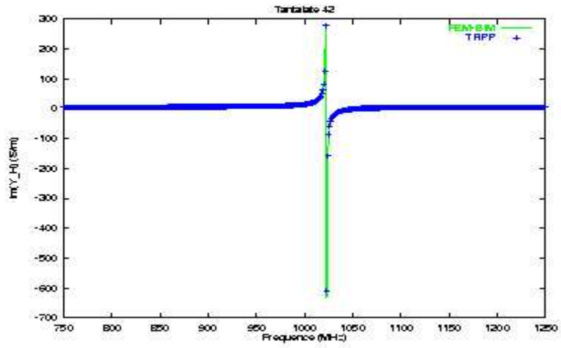

Fig.4 Imaginary part of the harmonic admittance of aPSAW on (Y+42) $LiTaO_3$
solid green : FEA/BIM dots blue : this work

The next figure shows that the dispersion curve and then propagation parameters can be efficiently extracted from the curves obtained using the proposed approach, comparatively to the extraction procedure proposed in ref. [1].

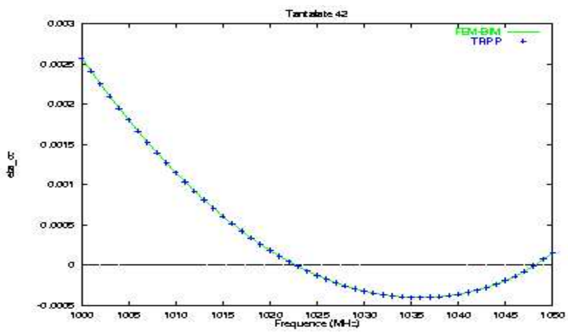

Fig.5 Comparison between dispersion curves provided by the two considered models

Application to interface waves

Interface acoustic waves (IAW) represents an attractive evolution of SAW devices for different reason. First, it is possible to excite high velocity waves [5] exhibiting properties close to bulk waves, yielding possibilities for improving device parameters such as coupling factor, temperature sensitivity, quality factor, and so on. Another advantage consists in the packaging of such device which becomes compatible with all those developed for standard microelectronics (no more air gap required). Whatever the advantages, one has to model these devices particularly to optimise their propagation loss which can be quite large [5]. The proposed development appears particularly well suited in that matter since the radiation condition can applied for each side of the considered domain. For instance, the next figure shows a typical mesh of and interfacial transducer built between to Y+128 $LiNbO_3$ plates glued using a polymer layer. The period is 6µm, the thickness of the electrode equals 200 nm and the expected polymer thickness was of the same scale order. Radiation boundary conditions are then applied on top and backside of the mesh with a particular care to the normal orientation imposing different mode selection for the Green's function computation on each side of the mesh.

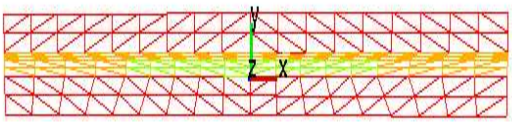

Fig.6 mesh of an interface wave transducer

Such a device has been built for an experimental assessment of the theory. A photo of the devic is reported in fig.7. It shows reduced $LINbO_3$ plates glued on an $LiNbO_3$ wafer atop single port resonators.

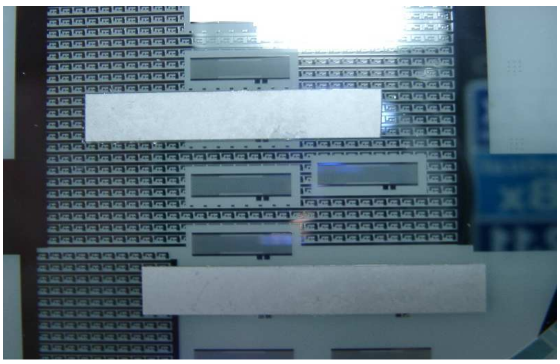

Fig.7 Photo of the test vehicle for the IAW experimental assessment

These devices have been probed to obtained their admittance which is compared to the corresponding computed harmonic admittance for an excitation parameter set to ½ (alternation of +V/-V excitation potential). The reported results show that the model accurately predict the arising of an IAW exhibiting large losses (the cut nor the structure were optimised in that sense).

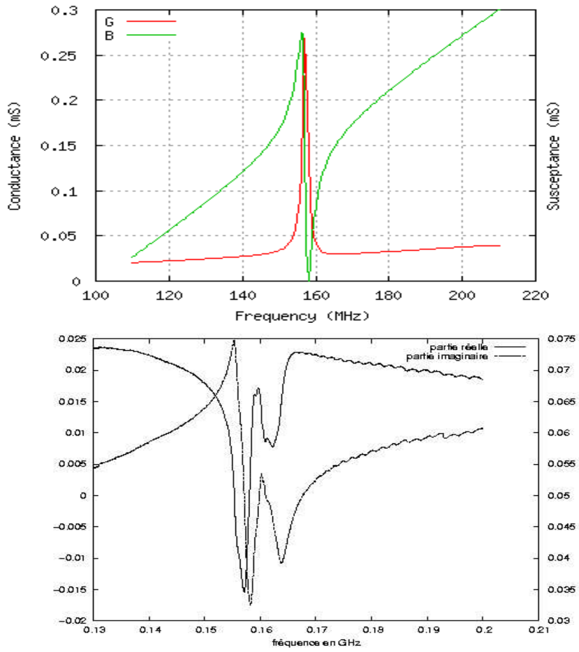

Fig.8 Comparison between theoretical (top) and experimental (bot.) IAW admittance signature

Conclusion

A model combining periodic FEA and a boundary element method has been successfully developed and implemented to address the problem of highly non homogeneous surfaces of elastic wave guides. The results provided by the proposed approach has been compared to well-established models and was found to provide accurate predictions whatever the polarisation of the wave is. This approach can be efficiently used for addressing many problems such as passivated SAW devices or Interface wave-guides.

References

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