FINITE DIFFERENCE MODELLING OF THE ULTRASONIC DIFFRACTION IN BIOLOGICAL TISSUES AND VISCOUS FLUID MEDIA

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Abstract
In this paper, Finite Difference (FD) formulations in cylindrical co-ordinates have been used to model the field radiated, in a biological tissue and a viscous fluid, by a circular transducer embedded in a rigid baffle . These two media have been described by models where the attenuation is respectively proportional to the frequency and to the square of the frequency.

Introduction

Ultrasound attenuation is of a considerable importance in theoretical acoustics, non-destructive evaluation and ultrasound tissue characterisation. For a wide variety of materials, the attenuation increases with the frequency according to a power law relation. In particular for soft biological tissues, the attenuation is approximately proportional to the frequency whereas it is obeying to a squared frequency law in viscous fluids.

Ultrasound fields propagating in such media will undergo changes in shape not only due to the frequency dependent attenuation but also due to the diffraction that can leads to wrong interpretations in the diagnosis if it is not taken into account.

Finite difference time domain (FDTD) approximation has been shown to be an interesting modelling technique for the acoustic wave propagation. Moreover, the FDTD is relatively simple to implement since it is a direct time domain method.

The purpose of this paper is to use the FDTD in the modelling of the field radiated, by an ultrasonic circular transducer, in biological tissues and viscous fluids. The numerical results, which are interpreted in terms of plane and edge waves, are presented in order to illustrate the absorption effect on the diffraction phenomenon.

Wave equations

Studies of acoustic phenomena generally employ an equation which governs the wave propagation.

While a lossless model may be adequate when absorption loss is very small, attenuation in viscous fluids and tissues characterisation must be taken into account and an equation that includes the loss due to the absorption has to be derivated.

Biological tissues

The model begins with a more complete expression involving mixed time and space derivatives. This expression simplifies to a simpler equation when one wishes to model the attenuation behaviour in tissues over a frequency range from 1 to 10 MHz . A tissue model, which provides an adequate description of the absorptive propagation of ultrasound has been postulated by Leeman [1]. The wave equation is modelled by the telegrapher's equation of the form :

$$\Delta P - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - 2A\frac{\partial P}{\partial t} = 0 \qquad (1)$$

where P is the pressure, c is the propagation velocity and A is a constant depending on the ultrasound absorption inside the medium since this later is supposed to be uniform and isotropic. The acoustic field will be restricted to the longitudinal waves since there is no evidence that shear waves can be supported over significant distances in soft tissues [2]. Hence, we can write:

$$P = \rho\frac{\partial \phi}{\partial t} \qquad (2)$$

Where ϕ is the velocity potential and ρ the density. So, the wave equation can be rewritten as follows:

$$\Delta\phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - 2A\frac{\partial \phi}{\partial t} = 0 \qquad (3)$$

Viscous media

In viscous fluid media, the propagation equation is given by [3]:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi + \beta\frac{\partial}{\partial t}\nabla^2\phi = 0 \qquad (4)$$

where β is a constant proportional to the viscosity coefficient.

Finite difference method

The physical problem to solve is dealing with the acoustic field radiated into a half space attenuating medium, by a circular transducer of radius a [Fig. 1], having a uniform axisymmetric surface velocity distribution .

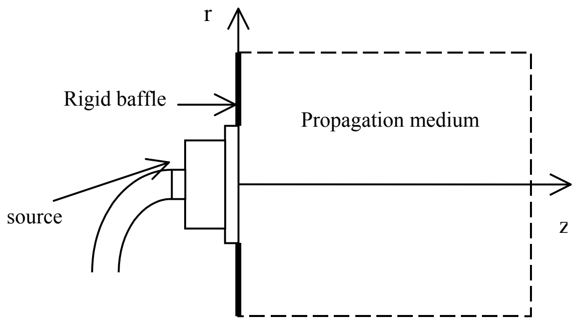

Figure1: Geometrical configuration of a transducer embedded in a rigid baffle

In such a problem having an axial symmetry, the wave equation will be formulated in cylindrical co-ordinates where $\frac{\partial}{\partial\theta}$ is close to zero making the problem two-dimensional.

Biological tissues

For a biological tissue, the wave equation becomes then :

$$\frac{\partial^2\phi}{\partial r^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-2A\frac{\partial\phi}{\partial t}=0 \qquad (5)$$

Viscous media

For a viscous medium the wave equation is given by:

$$\frac{\partial^2\phi}{\partial r^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} + \beta\frac{\partial}{\partial t}\left(\frac{\partial^2\phi}{\partial r^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}\right)=0 \qquad (6)$$

A square grid is imposed over r, z quarter and completed by using the symmetry of the problem. We let: $r = i\Delta r$, $z = j\Delta z$ and $t = k\Delta t$ where $1 \le i \le N_r, 1 \le j \le N_z, 1 \le k \le N_t$ and where $\Delta r, \Delta z, \Delta t$ are respectively the increments in r, z and t. Let us denote by $\phi(i, j, k)$ the approximated potential velocity at a point grid $(i\Delta r, j\Delta z)$ at time $k\Delta t$.

The stability of the adapted scheme is controlled by a proper choice of time and space steps. In order to minimise CPU time, calculations are incrementally limited for regions disturbed by the propagating ultrasonic pulse.

Biological tissues

By substituting centred difference formulae for all the derivatives in Equation (3), an explicit finite difference scheme is obtained [4]

Viscous media

In the case of viscous fluids, this substitution yields to an implicit scheme requiring resolution of a huge system of equations. Hence, an explicit FD scheme is deduced by substituting only regressive difference formulae for the temporal derivatives of the Laplacian in Equation (6) [4].

Numerical results

The waveform of the velocity of the source vibration is a pulse containing one cycle [Fig. 2].

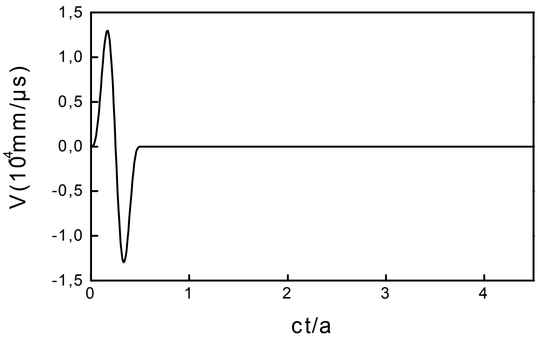

Figure 2: Velocity source vibration versus the time.

Biological tissues

In figure 3 which represents the pressure field on axis, the edge wave is more attenuated than the direct one. Hence, it is not an inverse replica of the plane pulse. This results from the fact that the attenuation of the wave emitted by a transducer vibrating in a biological tissue differs from a region to another since the travelling distance from the source is different.

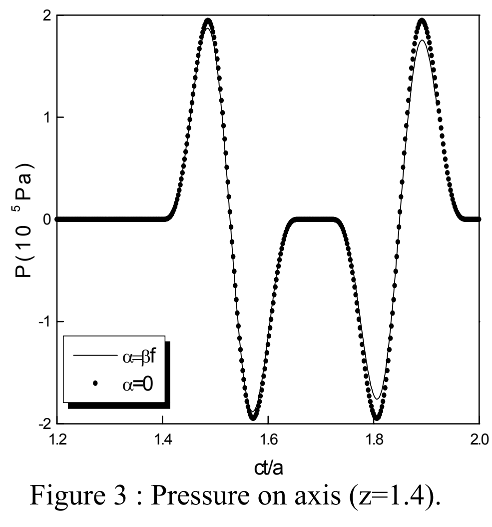

Figure 3 : Pressure on axis (z=1.4).

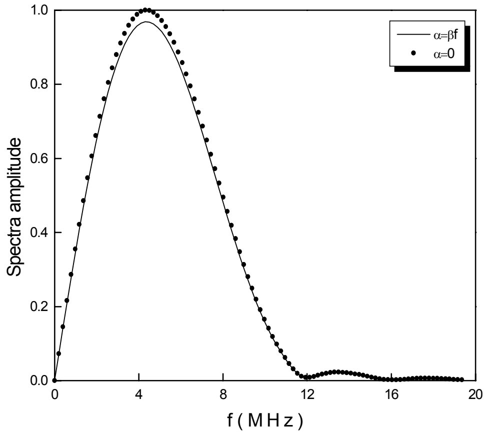

Figure 4 : Spectrum of the pressure on axis (z=1.4).

The result of the spectral analysis of the selected plane wave shows a diminution of the amplitude of the spectra [Fig.(4)].

Viscous media

Figure (5) represents the pressure field on the axis radiated in a viscous fluid. It consists of two distorted waves of inverted polarity. These distortions are due to the filtering of the high frequencies which results from the squared frequency dependency of the absorption. More the distance on the axis increases, more the pulses decrease in magnitude. In addition, since the edge wave travels a larger distance, it is the most attenuated [Fig.6].

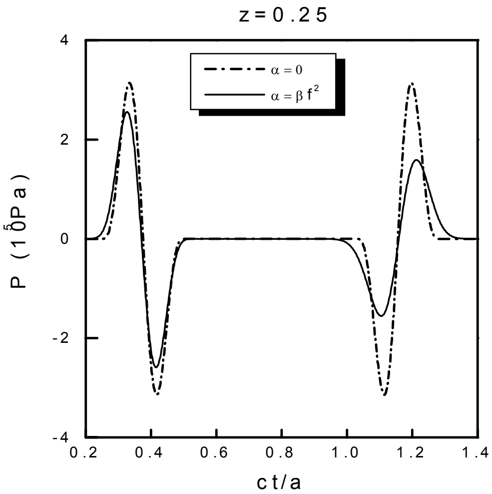

Figure 5 : Pressure on axis (z=0.25).

The spectral analysis of selected plane and edge waves shows a diminution of the spectra amplitude [Fig.7] and a squared law dependency [Fig. 8].

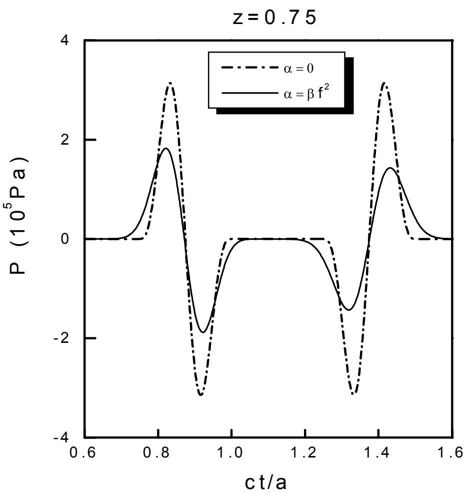

Figure 6 : Pressure on axis (z=0.75).

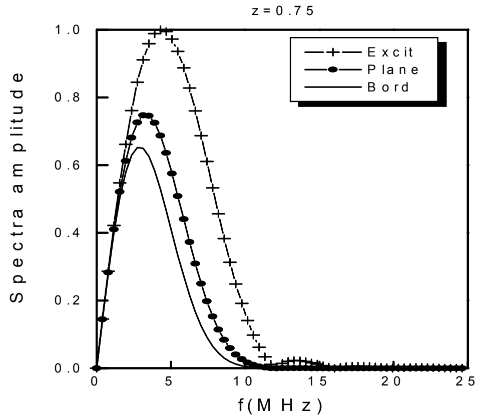

Figure 7 : Spectrum of the pressure on axis (z=0.75)

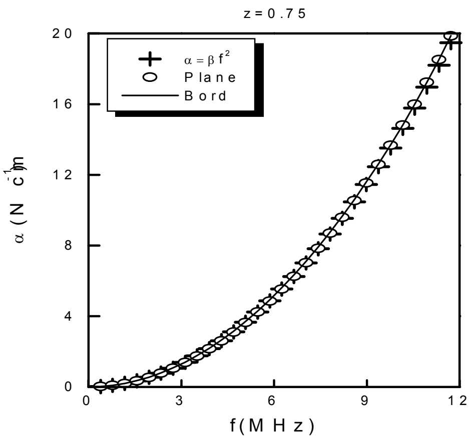

Figure 8 : Comparison of the law frequency dependency of the attenuation of plane and edge waves with the $\alpha=\beta f^2$ law

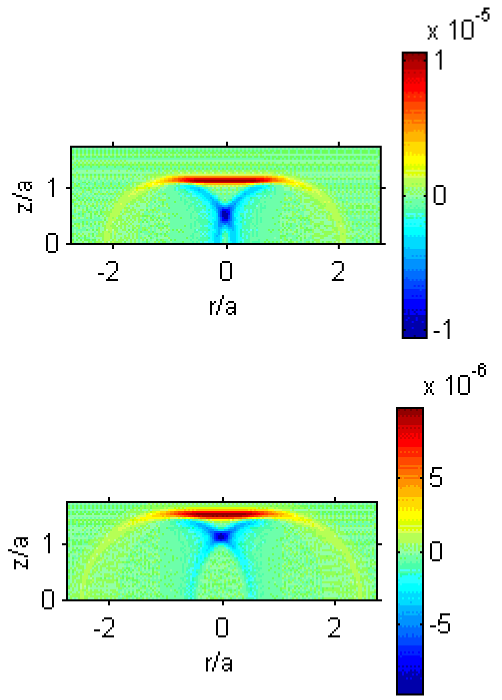

Figure 9 : Instantaneous spatial distribution of the velocity potential (ct/a=1.25, ct/a=1.625)

The spatial distribution of the ultrasonic potential field is given by figure 9. The plane wave seen as a line parallel to the source is followed by the edge wave appearing as two circular arcs. When the travelled distance increases, the magnitude of the field decreases [Fig 9.b].

Conclusion

In this work, FDM have been used to model the diffracted field by a circular transducer in biological and viscous media. These simulations show that, because of the absorption, the diffracted field is constituted by attenuated and distorted edge and plane waves.

References

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