Abstract
The work is devoted to theoretical and experimental study of processes of wave interaction in cavitational bubble cluster. The cavitational processes in the liquid have been generated experimentally with the help of an electromagnetic generator of acoustic shock waves. The modeling has been performed within the frame of a two-dimensional axial symmetry non-stationary approach on the base of conservation laws for the model of an ideal compressible liquid. Processes of a bubble growing, collapse, coalescence, fragmentation and bubble motion have been investigated in detail. It is shown that the possibility of bubble coalescence critically depends on amplitudes of falling rarefaction wave. Increasing of the amplitude of initiating pulse results in growing of internal pressure in cluster due to the wave transformation. Fading of rarefaction wave and appearance of compression waves brings about different bubble drift along the axis, resulting in increasing of a distance between them. It prevents bubble coalescence because of translational movement.

Introduction
A large number of experimental and theoretical works devoted to researches of wave processes in a bubbly media is carried out by now [1]. However, the detailed picture of bubble dynamics in a field of external pulse waves, when secondary waves inside the cluster are generated by bubbles of various sizes, is not quite clear yet. The problem of wave interaction inside the cavitational bubble cluster becomes more actual recently in connection with the problem of multi-bubble sonoluminescence [2]. Recent work is devoted to theoretical and experimental research of processes of a bubble generation and to the dynamics of bubbles under cavitation. Initial pulse represented an acoustic N-wave, consisting of the phase of compression and phase of rarefaction (or the phase of rarefaction only). Bubble oscillations in and behind the wave are the objects of the study.

Experimental
The cavitation area is formed in the center of cylindrical multi-spark electric discharge transducer [3], which was used as a source of compression and following rarefaction acoustic pulses with amplitude of 5-10 MPa (3-4 us duration on a half-height). The ends of transducer was closed with a glass windows and cavitation is photographed with a digital camera SensiCam Fast Shutter (PCO, Kelheim, Germany) with a light flash (duration 1μs).

Figure 1 shows cavitation in the center of the cylindrical transducer 180 (a) and 190 us (b) after discharge. The cavitation cluster has a form of a cylinder with a diameter of ~10 mm. The maximum bubble radius measured in the center of the cluster is equal to 2-3 mm. At the moment of observation (180-190 us) the radius reaches its maximum when peripheral bubbles (1-3 in Fig. 1a) undergo the first collapse and rebound. The bubble form is turned to be not spherical due to bubble-bubble interaction and influence of acoustic pulses from the transducer. Their collapse is accomplished by a cumulative jet development (2 in Fig. 1b), conical deformation (3 in Fig. 1a), and fragmentation (1, 2 in Fig. 1a and 1 in Fig. 1b). Larger bubbles in the center of the cluster (4 in Fig. 1a) are deformed by surface waves which give to the bubble a form of a star in their cross-section.
Numerical simulations

Let's consider the flow in the round tube filled with water, at initial pressure 0.1 MPa. In the center of the tube originally spherical gas bubbles are located, having initial diameter of 20 microns and the same initial pressure. The system a liquid - bubbles is originally in a condition of dynamic equilibrium state, velocity of the flow is equal to zero. The initiating pulse moved from the left closed end of the tube (along Z axis) and represented a plane sinusoidal wave rarefaction. The length of a wave is equal to 5 mm, amplitudes of waves are -11.5 MPa or -22.3 MPa. Through certain time the pulse passes through micro-bubbles, they begin to extend, and the system gets essentially nonequilibrium character with formation of secondary waves. It was supposed, that lateral walls of the tube are closed (a rigid wall), the right hand end is open.

The modeling has been performed within the frame of a two-dimensional axial symmetry non-stationary approach on the base of conservation laws for the model of an ideal compressible liquid.

The problem was solved numerically with the use of the method of individual particles (detailed mathematical formulation of the problem and the method of the solution could be seen in paper [4]), that is an improvement of Harlow method of particles in cells. Non-stationary fields of the basic thermodynamic parameters have been calculated both inside everyone bubble, and in external for them liquid flow. It was supposed, that the border between a liquid and a bubble represents contact discontinuity surface, where necessary conditions were satisfied: (i) equality of pressure on the different sides of the surface and (ii) continuity of normal component of a velocity vector of the flow. In course of time gas bubble may be deformed, break, drift and stick together to others.

The sizes of computed are is ZxR=35x4 mm.

In Figure 2 the pressure flow field is represented at instant t = 5.8 us after the beginning of movement of a rarefaction wave from the left end of the tube. Initial coordinates of the centers of micro-bubbles here are equal: z1 = 7.3 mm, z2 = 7.8 mm, z3 = 8.3 mm, z4 = 8.8 mm, z5 = 9.3 mm. It could be seen from Figure 2a, that by that moment gas bubbles gas have considerably grown in the wave of rarefaction. For example, average diameter of first bubble (from the left hand side) achieves 450 um. In Figure 2b, longitudinal and cross pressure profiles are given for R=1.4 mm and Z=11.1 mm accordingly (position of the profiles are marked by shaped lines in Figure 2a).

It is visible, that process of a bubble expansion is accompanied by formation of a secondary wave of the compression moving behind an initial wave of rarefaction. Intensity of the last one thus is reduced up to –16.1 MPa. The amplitude of a secondary wave is comparable to one of the initial pulse.

Detailed map of the pressure field around the bubble cluster behind a pulse wave and dynamics inter-cluster interactions are shown in Figure 3. Here the initial wave of rarefaction goes from below upwards. Parameters of a wave and initial conditions correspond to the case mentioned above. More dark of tone in figure answers waves of compression, light one - to waves of rarefaction. Intensity of waves coincides with intensity of tones. White spots represent the bubbles, which have arisen from five micro ones.
The first bubble becomes visible by instant $t = 3.4$ us, the fifth - by $t = 4.6$ us. It is clear from the figure, that character of bubble fluctuations, their form and the sizes are essentially various. The first bubble achieves the maximal sizes at $t = 7.0$ us (cross diameter thus is 0.61 mm), then it begins to collapse. Process of the collapse is accompanied by formation of micro-jets, especially visible, for example, at $t = 8.2$ us and $t = 11.4$ us. By the instant $t = 12.0$ us the first bubble breaks out, quickly collapsing affiliated bubble, having the toroidal form, is separated from it. The collapse of the first bubble occurs at $t = 13.4$ us, and further it slowly extends. Changes of the form of the second bubble are more significant, by the instant $t = 12.4$ us it gets the crosswise form, its collapse occurs at $t = 15.4$ us, and then it begins to extend. The third bubble, having achieved the maximal size at $t = 7.6$ us, collapses at $t = 10.6$ us, and further remains invisible up to $t = 16.0$ us. The fourth bubble makes the way a jet of the water going from third one, becomes toroidal at $t = 7.6$ us, collapses at $t = 10.4$ us. In the interval between $t = 11.2$ us and $t = 15.2$ us it gets flat form, and then again becomes toroidal. The fifth bubble, having maximal average diameter of 0.32 mm at $t = 5.2$ us, collapses at $t = 9.0$ us, remains invisible up to $t = 14.0$ us, and then quickly extends. Thus it achieves maximal size for the group of bubbles of 0.74 mm. We shall note, that process of a fragmentation of bubbles and their collapse is accompanied by local increase of pressure up to the values exceeding 0.2 GPa.

By the instant $t=16.0$ us essential shift of bubbles in a direction of an $Z$ axis of symmetry is observed. The distance between bubbles is increased both in an initial wave of rarefaction, and in secondary cavitation wave. The distance between the first and fifth micro-bubbles is 2mm at $t = 0$ us, but the distance between centers of the first and the fifth bubbles is equal to 2.86 mm at $t = 16.0$ us. Thus, internal pressure of the cluster interferes bubble coalescence.

Bubble deformation is often accompanied by generation of so-called “noses”, which are gas jets directed outward with accordance with the initial wave direction [5]. It is found numerically and confirmed experimentally that the “noses” appear in turn on the opposite sides of a bubble, and the collapse of one nose results in generation of another
due to inward shock wave within the bubble. Similar mechanism is described in paper [6] (“counter-jets”). The collapse of the “noses” is accompanied by their fragmentations and generations of micro-bubbles on the opposite sides of the initial bubble (see marks 1 and 2 in Figure 1).

**Conclusion**

Thus it is revealed, that in a generating bubble cluster at passage of the external pulse consisting of a phase of compression and a phase of rarefaction the bubble dynamics is essentially various. The bubble form is far from spherical, their fluctuations have irregular mode. At a fragmentation of a bubble, collapse of the bubble or its fragment there is a significant local increase of pressure in the flow that may result in the phenomena of sonoluminescence, observed in experiment.

It is shown, that the opportunity of coalescence critically depends on amplitude of a falling wave of rarefaction. Attenuation of a rarefaction wave and the generation of secondary acoustic waves result in unequal bubble displacement along the axis, the distance between them increases. At increase of intensity of an initiating pulse internal pressure in the cluster grows due to transformation of waves that interferes bubble coalescence.

Bubble fragmentation may occur in accordance with “counter-jets” mechanism with generation of two micro bubbles on the opposite sides of the initial bubble.

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**References**


