A NUMERICAL COMPARISON OF NONLINEAR INDICATORS FOR DIAGNOSTIC ULTRASOUND FIELDS

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Abstract

Nine measures of nonlinear propagation have been evaluated according to how they describe the transfer of energy from a diagnostic ultrasound beam's fundamental frequency to higher harmonics.

Fields produced by 30 transducer configurations and four scanner models were simulated in water using a finite difference model. Nine parameters proposed in the literature to indicate the degree of nonlinear distortion were calculated from measurements on the modelled fields.

The most robust indicators were the nonlinear propagation parameter σ_m, the spectral index, and two simple local distortion parameters incorporating the measured pressure, the distance from the source, and the local transducer gain.

Nonlinear Indicators

This study seeks a measurable parameter which can be used for a threshold under which fields measured in water are sufficiently 'quasi-linear' to predict *in situ* exposures using scaling and linear derating.

The nonlinear indicators fall into three types: those derived from the waveform spectrum, pressure, or shape.

Spectral Parameters (SI & H2)

The *Spectral Index* SI is the ratio of the power $P(f)$ above the fundamental frequency to the total power in the spectrum at a point in the beam:

$$SI = \frac{\int_{f_a}^{\infty} P(f)df}{\int_0^{\infty} P(f)df}. \tag{1}$$

The cutoff point f_a between the fundamental and higher harmonics is $1.5f_c$, where f_c is the peak frequency of the original spectrum.

The *Second Harmonic Ratio* $H2$ is the ratio of the power under the second harmonic peak to that under the fundamental peak of the spectrum. The powers were calculated between $[0 - f_a)$ and $[f_a, 2.5f_c)$, where f_a and f_c are as defined in equation 1. Hence

$$H2 = \frac{\int_{f_a}^{2.5f_c} P_f df}{\int_0^{f_a} P_f df}. \tag{2}$$

Pressure Parameters (σ_m, σ_z & p_m)

The *Nonlinear Propagation Parameter* σ_m is defined only at the focus and was derived assuming a Gaussian–shaded, focused, circular source [1]. It is given by

$$\sigma_m = zp_m \frac{2\pi f_c \beta}{\rho_0 c^3} \frac{ln(G + \sqrt{G^2 - 1})}{\sqrt{G^2 - 1}}. \tag{3}$$

where z is the distance from the source, p_m is half the peak–to–peak pressure, f_c is the fundamental frequency, and β, ρ_0, and c_0 are the nonlinearity parameter, density, and speed of sound in the medium. The local area ratio G is used to approximate the transducer gain. It is the square root of the ratio of the beam $1/e$ areas at the source and focus, including side lobes.

$$G = \sqrt{\frac{area_{source}}{area_{focus}}} \tag{4}$$

Although σ_m is only defined at the beam focus, this study calculated σ_m at all points along the axis. The local area ratio G was recalculated at each point by assuming a beam area proportional to the product of the x– and y– $1/e$ widths.

Field Sigma σ_z is equivalent to σ_m with the gain term omitted so that

$$\sigma_z = zp_m \frac{2\pi f_c \beta}{\rho c^3}. \tag{5}$$

The *Maximum On–axis Pressure* p_m is the maximum pressure measured along the axis in the field and is half the peak–to–peak pressure:

$$p_m = \frac{p_c + p_r}{2}. \tag{6}$$

Waveform Shape Parameters (p_{asym} & σ_s)

The *Asymmetrical Ratio* p_{asym} is the ratio of the peak positive to peak negative pressure:

$$p_{asym} = \frac{p_c}{p_r}. \tag{7}$$

The expression for the *Ostrovskii/Sutin propagation parameter,* σ_s used here is obtained by substituting the asymmetrical ratio into the expression for asymmetrical distortion proposed by Ostrovskii and Sutin [4] and gives

$$\sigma_s = \frac{p_{asym} - 1}{p_{asym} + 1} = \frac{p_c - p_r}{p_c + p_r}. \tag{8}$$

Alternative Pressure Parameters (σ_r & σ_q)

Analytical expressions for waveform distortion have not been published for complicated transducer configurations such as the astigmatically-focused rectangular transducers studied here. However, as in the analytical expressions for distortion in Gaussian–shaded beams [1] [2], the waveform distortion should be a function of the the path integral of some quantity through the field and the transducer gain. For this study we define two parameters incorporating the simple gain terms $1/G$ and $1/\sqrt{G}$.

The first proposed parameter, σ_r, is given by

$$\sigma_r = z p_m \frac{2\pi f_c \beta}{\rho c^3} \frac{1}{G} \tag{9}$$

where G is defined in equation 4. The second proposed parameter, σ_q, is defined as

$$\sigma_q = z p_m \frac{2\pi f_c \beta}{\rho c^3} \frac{1}{\sqrt{G}}. \tag{10}$$

The use of the $\frac{1}{\sqrt{G}}$ term follows from the observation that, with the gain range used in this study, this term closely approximates the gain term used to calculate σ_m. Both parameters are defined everywhere in the field.

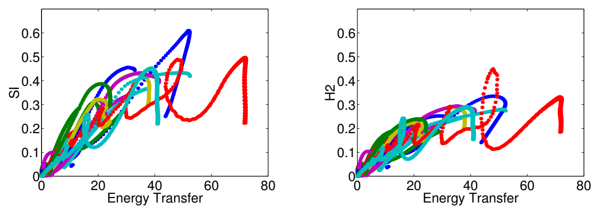
Energy Transfer

The parameters were evaluated according to how accurately they measure the energy transferred from the fundamental frequency to higher harmonics. Some of this transferred energy is quickly dissipated. The remaining high harmonics form the shock front of the pressure waveform.

For each field, energy transfer was calculated using an artificially–linear field which was calculated for identical conditions except that the nonlinearity coefficient β was set to zero. The energy transfer $\frac{\Delta E}{E}$ is the relative difference in power under the fundamental frequency peaks between the linear and nonlinear fields. Hence

$$\frac{\Delta E}{E} = \frac{\int_0^{1.5 f_c} P_{lin}(f) df - \int_0^{1.5 f_c} P_{nonlin}(f) df}{\int_0^{1.5 f_c} P_{lin}(f) df}. \tag{11}$$

Typically, the energy transfer profile of a beam oscillates slightly in the near field, increases sharply near the focus, then flattens out past the focus as waveform pressure and nonlinear effects decrease. Maximum energy transfers for the modelled beams range from 0.7% in very low–amplitude pulsed and continuous beams, to 42% in high–gain runs, and to 70% in a low–gain run with a deep focus.



(a) SI (b) H2

Figure 1: Spectral Parameters

Methods

Ultrasound fields were simulated using the Bergen frequency-space implementation of the KZK equation [5], modified to handle pulsed excitation [6] and rectangularly-symmetric transducers [7].

Thirty fields are included here. They include measured waveforms from 16 transducer settings on three models of ultrasound systems, two theoretical transducer configurations using short pulses, one chirped pulse, and continuous runs. Frequencies used ranged from 2 MHz to 6 MHz. The maximum gains ranged from 2 to 12. The transducers in over half of the runs had astigmatic foci.

Results

In figures 1 to 4, each of the parameters is plotted against energy transfer. Each colored line in the graphs represents one modelled field. An ideal nonlinearity indicator would be tightly distributed and proportional to energy transfer.

Spectral Parameters

Figure 1 shows that the spectral index (SI) and second harmonic ratio (H2) increase similarly up to 10% energy transfer. Above this, the H2 curve flattens out, likely because energy that has been transferred to the third and higher harmonics is not reflected in H2.

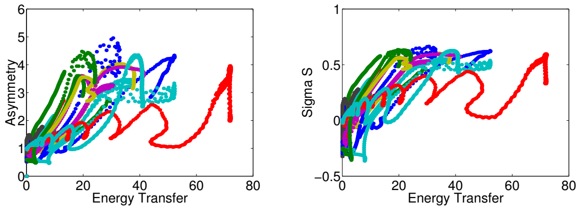
The vertical regions at the right of the SI and H2 curves correspond to the regions of the field after the focus, where the energy transfer from the fundamental remains constant but the value of the nonlinear indicators decreases.

Waveform Shape Parameters

Figure 2 shows that neither of the shape parameters reflects energy transfer to higher harmonics.

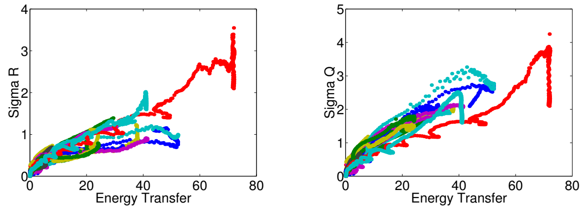
Pressure Parameters

Figure 3 shows that σ_m is the only pressure parameter which adequately represents the energy transfer. Field sigma (σ_z) also generally increases with energy transfer but massively overestimates nonlinearity in high-gain runs. The measured pressure (p_m) alone



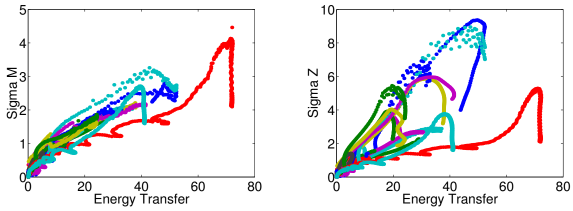
(a) p_{asymm} (b) σ_s

Figure 2: Waveform Shape Parameters

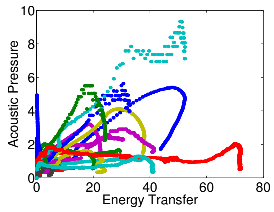


(a) σ_r (b) σ_q

Figure 4: Alternative Parameters

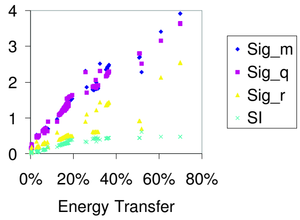


(a) σ_m (b) σ_z

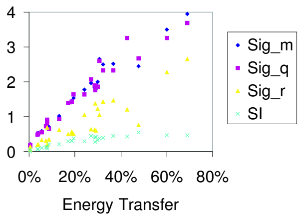


(c) p_m

Figure 3: Pressure Parameters



(a) Max Gain



(b) Max Pressure

Figure 5: Derived Parameters at Axial Maxima

does not represent the energy transfer at all. Clearly, a nonlinear indicator derived from pressure measurements must include a gain term.

On the σ_m and σ_z charts, the curves for higher-gain fields lie at the top left, while the low-gain runs lie at the bottom right. The vertical portions of the curves correspond to values on the beam axes after the focus, as in the previous section.

Alternative Parameters

Figure 4 shows the performance of the parameters σ_r and σ_q. The parameter σ_q performs similarly to σ_m because the gain term is similar.

Inspection of the distribution of curves on the charts shows the effect of the different gain terms. As on previous charts, the curves for high–gain runs on the σ_q lie at the top left of the graph, with the low–gain runs at the bottom. In the σ_r chart this trend is reversed, suggesting that σ_r slightly overcorrects for the gain while σ_q under–corrects.

Behavior at Maximum Gain, Pressure

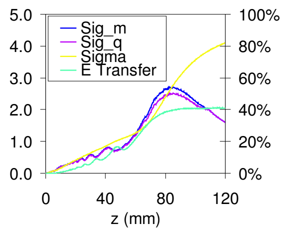
It should not be necessary to measure nonlinear parameters everywhere in a field, and in fact σ_m is only defined at the focus. The charts in figure 5 show the best–performing indicators plotted only at the locations of maximum gain (approximated by the local area ratio, G) or pressure in the field. The behavior of the parameters correlates much better with energy transfer to higher harmonics than the all-points graphs.

The plots of the parameters at the gain and pressure maxima are similar, but the pressure maximum is much easier to find experimentally.

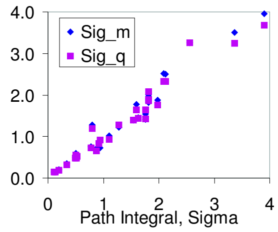
Figure 5 shows the parameters σ_m, σ_q, σ_r, and SI as a function of energy loss at low to medium energy transfer. It is clear that, in this region, σ_m and σ_q reliably follow the energy transfer, where σ_r does a plausible job. The spectral index may also be a useful indicator, but its coefficient of variance is large, so SI will be a very conservative indicator.

Comparisons with σ

An alternate criterion for the nonlinearity indicators is to compare them to Blackstock's dimensionless non-



(a)



(b)

Figure 6: Nonlinearity Criterion σ

linearity parameter

$$\sigma = \beta k \int_0^z \epsilon(l) dl \qquad (12)$$

where $\epsilon = \frac{u}{c_0}$ is the acoustic Mach number [3]. Figure 6(a) shows the relationship along the axis of a typical ultrasound field of this parameter σ, the measured parameters σ_m and σ_q, and the energy transfer. The axial maximum of this field lies at 80 mm, and the curves of σ, σ_m, and σ_q cross at this point. Before the focus, the derived parameters tend to overestimate this parameter, and after the focus they underestimate it while σ keeps increasing. This is typical of the fields in modelled in this study. Figure 6(b) shows the values of the derived parameters plotted vs $sigma$ at the axial maximum of all 30 fields. In each field, the energy transfer correlates linearly with the path integral σ at the axial maximum (not shown).

Conclusions

A nonlinear indicator which incorporates the distance from the transducer, the measured pressure, and the local area ratio (calculated from the beam areas of the source and measurement point) is the best choice to represent an ultrasound beam's energy transfer into higher harmonics. The parameters σ_m, or σ_q are the leading candidates.

A protocol for ensuring that an ultrasound field in water is 'quasi-linear' would require determining the maximum pressure along the beam axis, measuring the x and y 6dB or $1/e$ beam widths at the axial maximum and the source, and calculating σ_m or σ_q.

If the nonlinearity parameter lies below a threshold value, it is certain that the energy transfer from the fundamental to higher harmonics is lower than the threshold for 'quasi-linear' propagation. Possible thresholds are shown in figure 7.

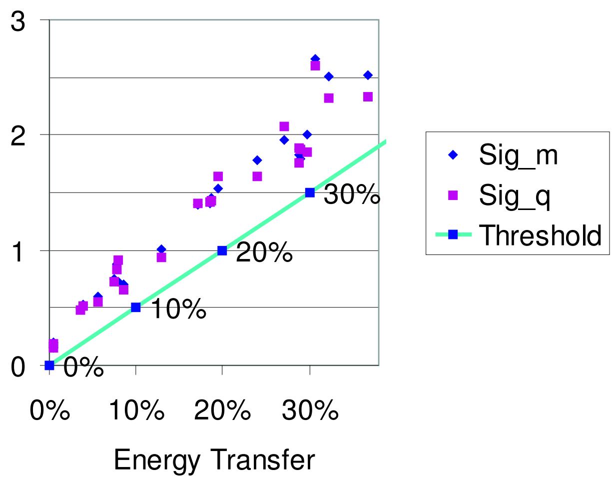


Figure 7: Possible Thresholds

References

[1] D.R. Bacon, "Finite Amplitude Distortion of the Pulsed Fields used in Diagnostic Ultrasound," Ultrasound in Med. and Biol., vol. 10 (2), pp. 189-195, 1984.

[2] M.F. Hamilton, V.A. Khoklova, O.V. Rudenko, "Analytical Method for Describing the Paraxial Region of Finite Amplitude Sound Beams," J. Acoust. Soc. Am., vol. 101 (3), pp. 1298-1301, March 1997.

[3] D. Dalecki, E.L. Carstensen, K.J. Parker, D.R. Bacon, "Absorption of Finite Amplitude Focused Ultrasound," J. Acoust. Soc. Am. vol. 89 (5), pp. 2435-2447, May 1991.

[4] T.A. Bigelow, W.D. O'Brien, "Experimental Evaluation of Indicators of Nonlinearity for Use in Ultrasound Transducer Characterizations," Ultrasound in Med. and Biol., vol. 28 (11-12), pp. 1509-1520, 2002.

[5] S.I. Aanonsen, T. Barkve, J.N. Tjøtta, S. Tjøtta, "Distortion and Harmonic Generation in the Nearfield of a Finite Amplitude Sound Beam," J. Acoust. Soc. Am. vol. 75 (3), pp. 749-768, March 1984.

[6] A.C. Baker, V.F. Humphrey, "Distortion and high-frequency generation due to nonlinear propagation of short ultrasonic pulses from a plane circular piston," J. Acoust. Soc. Am., vol. 92 (3), pp. 1699-1705, September 1992.

[7] A.C. Baker, A.M. Berg, A.J. Tjøtta, "The nonlinear pressure field of plane rectangular apertures: Experimental and theoretical results," J. Acoust. Soc. Am., vol. 97 (6) pp. 35103517, June 1995.