INVESTIGATION OF TRANSIENT NONLINEAR PHENOMENA IN ANNULAR THERMOACOUSTIC PRIME-MOVERS

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Abstract

The present paper deals with the transient nonlinear processes responsible for the saturation of acoustic waves in annular thermoacoustic prime-movers. In particular, investigations are focused on the role of nonlinear processes influencing the temperature distribution in the inhomogeously heated parts of the device. Experimental observations of various transient regimes are presented. The results show the possibility for the engine to switch on and off spontaneously and periodically, the possibility for an overshoot of the acoustic wave amplitude before its final stabilization, or the possibility for a double-threshold phenomenon during the amplification process. A numerical approach to study the transient regime is then presented. In the adopted simplified model, the temperature field is coupled to the acoustic field by taking into account the forced convection due to acoustic streaming and the acoustically induced thermal conductivity.

Introduction

Thermoacoustic prime-movers employ for the transformation of the thermal energy into mechanical energy an interaction between an inhomogeneously heated stack of solid plates and resonant gas oscillations. In an annular thermoacoustic prime-mover, the stack is placed in a closed loop resonator, allowing the excitation of traveling acoustic waves. In such a device, the saturation of the thermoacoustic instability is linked not only to classical non linear phenomena such as cascade process of higher harmonic generation and minor losses, but also to nonlinear processes influencing the temperature distribution in the inhomogeneously heated parts of the system such as acoustically enhanced thermal conductivity (equivalent to heat transport induced by gas oscillations [1]) and the excitation of a unidirectional acoustic streaming [2].

In order to understand the independant influence of each one of those nonlinear phenomena, the investigation of the transient regime may provide us useful informations, because the transient development of different nonlinear phenomena might proceed differently (with different time scale). Experimental observations are briefly reported. The obtained results show various regimes, demonstrating the possibility for the engine to turn on and off spontaneously and periodically, the possibility for an overshoot of the acoustic wave amplitude before its final stabilization, or the possibility for a double-threshold process during the amplification regime [3]. A quantitative and exhaustive description of the thermoacoustic amplification process requires a numerical resolution of the equations of motion, but one risks to spend many expensive diagnostics to understand the independant influence of each one of the nonlinear phenomena that control the saturation. We believe that a complementary approach would be to find the simplest description that captures qualitatively the dynamical behaviors which are typically observed in experiments. A simplidied theoretical model is presented, where the one-dimensional heat transfer equation is coupled to the acoustic problem by taking into account the forced convection due to acoustic streaming and the acoustically induced thermal conductivity. Thermoacoustic amplification depends in its turn on the temperature distribution along the stack and the waveguide. The obtained results are qualitatively in good agreement with experimental observations (in particular a periodic switch on-off and the double-threshold phenomena are reproduced).

Experimental results

A schematic diagram of the experimental apparatus is shown in Fig. 1. The torus shaped stainless steel tube of inner diameter $R = 53mm$ and length $L = 2.24m$ is filled with air at atmospheric pressure. The stack (length H_S) is a ceramic porous material with square channels of cross-section $d \times d$ ($d = 0.9mm$). When the temperature gradient along the stack between the cold heat exchanger (located at $x = -H_S$) and the hot heat exchanger (located at $x = 0$) exceeds some critical value, the fluid starts to oscillate at about $150Hz$.

Numerous experimental investigations of the transient regime have been made, with various stack length and stack heating. In particular, we have found that the onset of the thermoacoustic instability could give rise to what we called the double threshold effect [3]. During this transient operation, the initial exponential growth of oscillations (first threshold) is followed by a quasi-stabilization (with wave amplitude slowly growing in time), which before the final stabilization is followed by another exponential growth (second threshold). Here, we present other interesting results, obtained with a 5

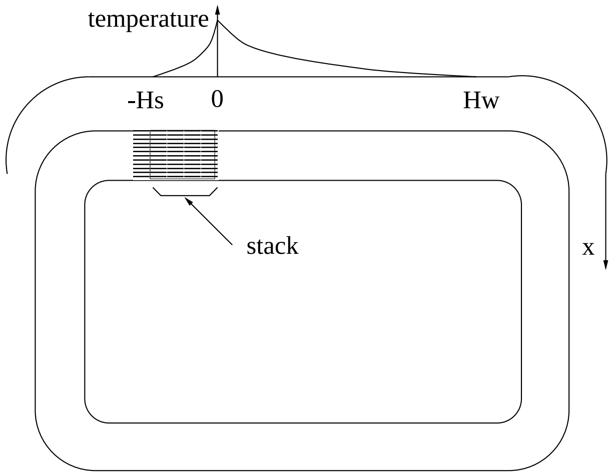

Figure 1: Schematic representation of the annular thermoacoustic prime-mover

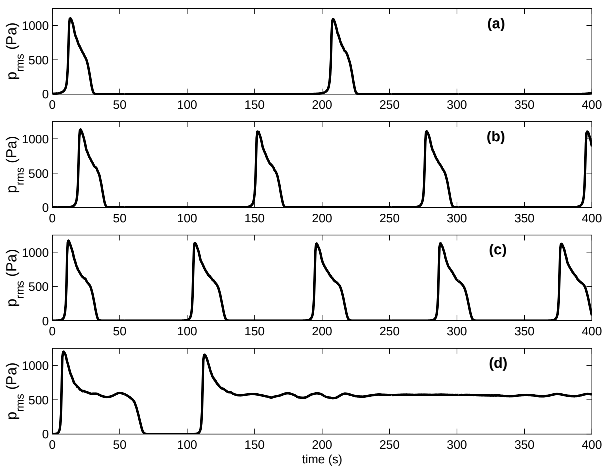

Figure 2: Rms amplitude of acoustic pressure versus time, when the power increment ΔQ above the threshold value Q_0 of input power is gradually increased. The four obtained transient regimes (a),(b),(c),(d) correspond to $\Delta Q = 4W$, $\Delta Q = 8W$, $\Delta Q = 15W$, $\Delta Q = 27W$, respectively

centimeters long stack and with a hot heat exchanger consisting of an appropriately coiled string of Nichrome (i.e. an electrical heat resistance) inserted directly in the resonator at position $x = 0$ and connected to the power supply. For each measurement, the input power Q is initially set just below its critical value corresponding to the onset of the thermoacoustic instability. A small ΔQ increment on Q is then sufficient for the acoustic wave to be generated in the waveguide. In fig. 2, various transient regimes are presented as a function of ΔQ. Several interesting processes are observable. Firstly, the results show the possibility for the device to turn on and off spontaneously and periodically, and the switch on-off period is gradually decreasing when the ΔQ increment is increasing (Fig. 2 (a),(b),(c)). Then, when ΔQ exceeds $27W$, the amplitude of the acoustic wave finally stabilizes to a finite value (Fig. 2 (d)). Note also the presence of an overshoot of the acoustic wave amplitude before its final stabilization (Fig. 2 (d)).

For all the above mentioned experiments, the temperature distribution have been measured using type K thermocouples placed along the stack, and the obtained results (not presented here) show significant transient changes in temperature distribution. Consequently, the nonlinear processes influencing the temperature distribution in the transient regime must be taken into account in the model.

Numerical approach

The device is divided into three parts, i.e. the stack ($-H_S \leq x \leq 0$), the inhomogeneously heated part of the resonator ($0 \leq x \leq H_W$), and the cold part at constant temperature T_C. The interval $-H_S \leq x \leq H_W$ is called the thermoacoustic core. It is made of three compounds (i.e. the stack, the tube walls and the fluid) with associated thermophysical properties (i.e. ceramics, stainless steel, and air at atmospheric pressure, respectively). It is assumed that the thermal coupling between those different elements is so strong that, at any position x along the thermoacoustic core, the temperatures $T_{air}(x)$, $T_{ceramics}(x)$ and $T_{stainless}(x)$ are equal. With this assumption, a simplified monodimensional heat transfer equation can be written in each part of the thermoacoustic core as follows:

$$\forall x \in [-H_S, 0], \partial_t T_-(x,t) + V_-(t)\partial_x T_-(x,t) = \\ (D_- + \delta D_-(t))\, \partial^2_{xx} T_-(x,t) + \frac{T_-(x,t)}{\tau_-}, \tag{1}$$

$$\forall x \in [0, H_W], \partial_t T_+(x,t) + V_+(t)\partial_x T_+(x,t) = \\ D_+ \partial^2_{xx} T_+(x,t) + \frac{T_+(x,t)}{\tau_+}. \tag{2}$$

In Eqs. (1) and (2), the coefficients D_- and D_+ (with subscripts $-$ and $+$ standing for the stack region and the $[0, H_W]$ region respectively) are the global thermal diffusivities of the simplified monodimensional model. They are estimated from thermophysical properties and cross-sectional dimensions of each component (i.e. stack, tube walls, and fluid). The coefficients $\tau_\pm$ are phenomenological parameters which account for exchanges of the device with surroundings. In fact, the transient heat transfer equation is here coupled to the acoustic problem via the acoustically induced thermal diffusivity $\delta D_-(t)$ and the forced convection $V_\pm(t)\partial_x T_-(x,t)$ due to the excitation of a unidirectional acoustic streaming of velocity $V_\pm(t)$. The time dependance (in fact, acoustic pressure-dependance) for streaming velocity and thermal diffusivity will be described later.

Then, the transient evolution of acoustic pressure for a given temperature distribution must be evaluated. This requires to solve the well-known classical linear differential equation of thermoacoustics in the thermoacoustic core. It has been demonstrated in an earlier paper dealing with standing-wave prime-movers [5] that transformation of this equation into an equivalent Volterra integral equation of the second kind leads to an exact solution in the form of an infinit iterative sequence of integral operators. Here, we take advantage of this result, taking into account the specific boundary conditions linked to the annular geometry of the present device. Analytical expressions for the threshold condition and for the corresponding oscillation frequency f are provided, and approximate solutions are calculated by truncating the exact solution to a sufficient order. The structure of the acoustic field can also be calculated in the whole device for an arbitrary temperature field without any restrictions on the stack length. This method, which is not detailed here (see refs. [6], [7]), notably introduces a thermoacoustic amplification coefficient to model the amplification/attenuation of the wave in the thermoacoustic core. Finally, when trying to solve the transient regime of prime-mover operation, the acoustic problem reduces to:

$$\partial_t p_{rms}(t) = \alpha\{T_\pm(x,t)\} p_{rms}(t), \tag{3}$$

where $p_{rms}(t)$ is the root-mean-square amplitude of the acoustic pressure at position $x = 0$ (for instance). In Eq. (3), the thermoacoustic amplification coefficient α depends on the temperature distribution in the whole thermoacoustic core. The sign of α determines whether the acoustic wave is attenuated ($\alpha < 0$) or amplified ($\alpha > 0$) while $\alpha = 0$ corresponds to the threshold condition or to the stationnary regime.

To model the influence of the acoustic field on the temperature field in Eqs. (1)-(2), simple expressions for $V_\pm$ and δD_- are introduced:

$$\delta D_-(t) = \Gamma_D p_{rms}^2(t). \tag{4}$$

$$\partial_t V_\pm(t) + \frac{V_\pm(t)}{\tau_V} = \frac{\Gamma_V}{\tau_V} p_{rms}^2(t). \tag{5}$$

So, the velocity of acoustic streaming and the acoustically induced diffusivity are simply assumed to depend on the square of acoustic pressure, with account of a time delay τ_V of streaming establishment. The parameters Γ_V, Γ_D and τ_V are estimated from experiments [8], theory [4], and numerical calculations [9].

In order to describe the transient regime completely, the following boundary conditions are introduced:

$$T_-(-H_S,t) = T_-(-H_S,0), \tag{6}$$

$$T_+(H_W,t) = T_+(H_W,0), \tag{7}$$

$$T_-(0,t) = T_+(0,t), \tag{8}$$

$$\kappa_-(t)\partial_x T_-(0,t) - \kappa_+\partial_x T_+(0,t) = \Phi(t). \tag{9}$$

In Eq. 9, $\Phi(t)$ is the input power flux at $x = 0$ while $\kappa_-(t)$ and κ_+ stand for thermal conductivities in the stack and $[0, H_W]$ regions, respectively. Finally, the transient regime is computed by solving Eqs. (1),(2) and (3) step by step from the onset of the thermoacoustic instability to the saturation of the wave amplitude (the heat transfer equation being solved using a Cranck-Nicholson finite-difference method [10]).

Results and discussion

In the following, preliminary results of the calculated transient regimes are presented. For all computations, the input power $Q(t)$ is initially set to a fixed value in order that the associated temperature distribution corresponds to the threshold condition. At time $t = 0$, a ΔQ increment on the input power is applied. In Fig. 3, the acoustic pressure amplitude is plotted versus time, for different power increment ΔQ above the threshold. In Fig. 4 and 5, the influence of the parameter Γ_V (characterizing the efficiency of streaming excitation by sound) and of the time delay τ_V of streaming establishment are investigated.

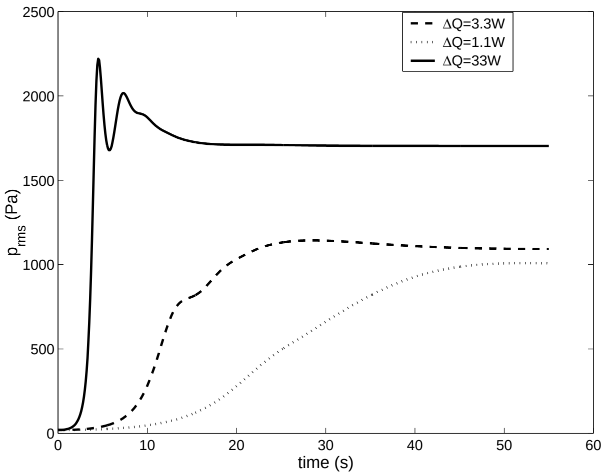

Figure 3: Calculated rms amplitude of acoustic pressure versus time in the transient regime for different power increments ΔQ above the threshold. Other parameters are set to the following values: $\Gamma_V = 2.10^{-8} ms^{-1}$, $\tau_V = 2s$, $\Gamma_D = 7.10^{-7} m^2 s^{-1}$.

At present, is seems premature to draw any quantitative conclusion from the observed results, but it is interesting to notice that the model is able to reproduce qualitatively the experimentally observed processes (overshoot, periodic switch on-off, and double-threshold processes). Furthermore, as illustrated in Fig.

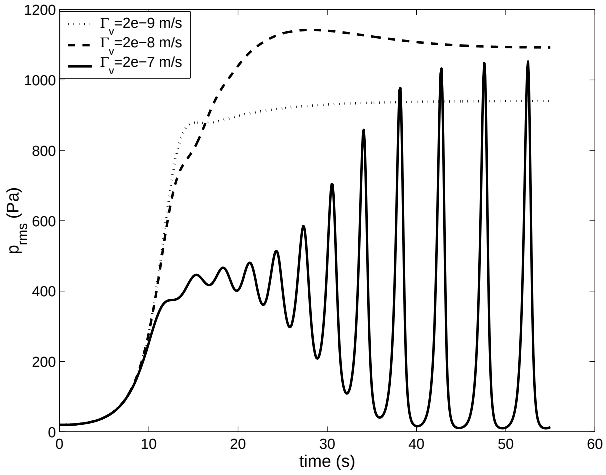

Figure 4: Calculated rms amplitude of acoustic pressure versus time in the transient regime when the parameter Γ_V is varying. Other parameters are set to the following values: $\Delta Q = 3.3W$, $\tau_V = 2s$, $\Gamma_D = 7.10^{-7} m^2 s^{-1}$.

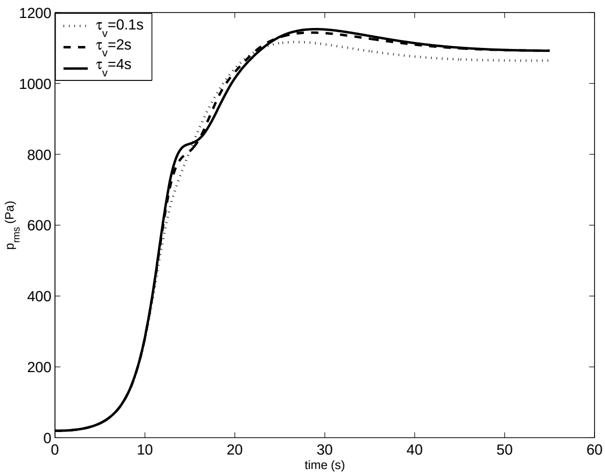

Figure 5: Calculated rms amplitude of acoustic pressure versus time in the transient regime when the parameter τ_V is varying. Other parameters are set to the following values: $\Delta Q = 3.3W$, $\Gamma_V = 2.10^{-8} ms^{-1}$, $\Gamma_D = 7.10^{-7} m^2 s^{-1}$.

5, the account of a time delay of streaming establishment could explain the double-threshold phenomenon. More precisely, we suspect the observed double threshold phenomenon to be a consequence of the streaming induced transient evolution of the temperature distribution, which proceed with a different time scale than the characteristic time of wave amplification [7]. Work is currently in progress in order to relate the parameters controlling transient interactions between acoustic and temperature fields in our simplified theoretical model with the measurable parameters of the experimental thermoacoustic devices.

Acknowledgments

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