CONTROLLING THE EFFECTS OF MODAL INTERACTIONS IN ULTRASONIC CUTTING DEVICES

M. Lucas, A. Cardoni, M.P. Cartmell, F.C.N. Lim
Department of Mechanical Engineering, University of Glasgow, Glasgow, UK
m.lucas@mech.gla.ac.uk

Abstract

Modal interactions, and particularly combination resonances, are characteristic of ultrasonic devices driven at high power in a longitudinal mode. Such behaviour has been modelled previously for parametrically excited simple structures such as beams and bars, and it is possible to demonstrate that ultrasonic devices can exhibit dynamic responses that are qualitatively similar to theoretical models of autoparametric systems. In this study, modal interactions are measured, and the benefits of designing systems with a reduced number of modes are demonstrated.

Introduction

Previous studies of multiple-blade ultrasonic cutting systems have reported and characterised the problems associated with modal interactions [1,2,3]. Combination resonances are a common feature of the dynamic response of systems driven in longitudinal resonance. For ultrasonic devices, multiple tuned-component systems with complex geometries are particularly prone to stress failures due to such modal interactions [1,2]. In order to design systems that can operate at a single driving frequency without significant energy leaks into non-tuned modes, care must be taken to avoid modal frequencies of the system that have special relationships with the driving frequency. This can be achieved if accurate finite element models of the device can be created and subsequent geometry modification strategies can be implemented to effect modal frequency shifts. This design approach is more straightforward for devices where there are few modes below the driving frequency. For devices with a high number of modes, remodelling to effect frequency shifts in a set of modes predicted to interact, is likely just to result in an alternative modal interaction. The most reliable solution is to reduce the number of modes that exist below the driving frequency, so that modal frequency shifting by small geometry modifications can be effective.

Modal Interactions

Modal interactions can occur if special relationships exist between one or more modal frequencies and the excitation frequency. The type of modal interactions that can occur depends on the nature of the excitation and on the system nonlinearity [4,5,6].

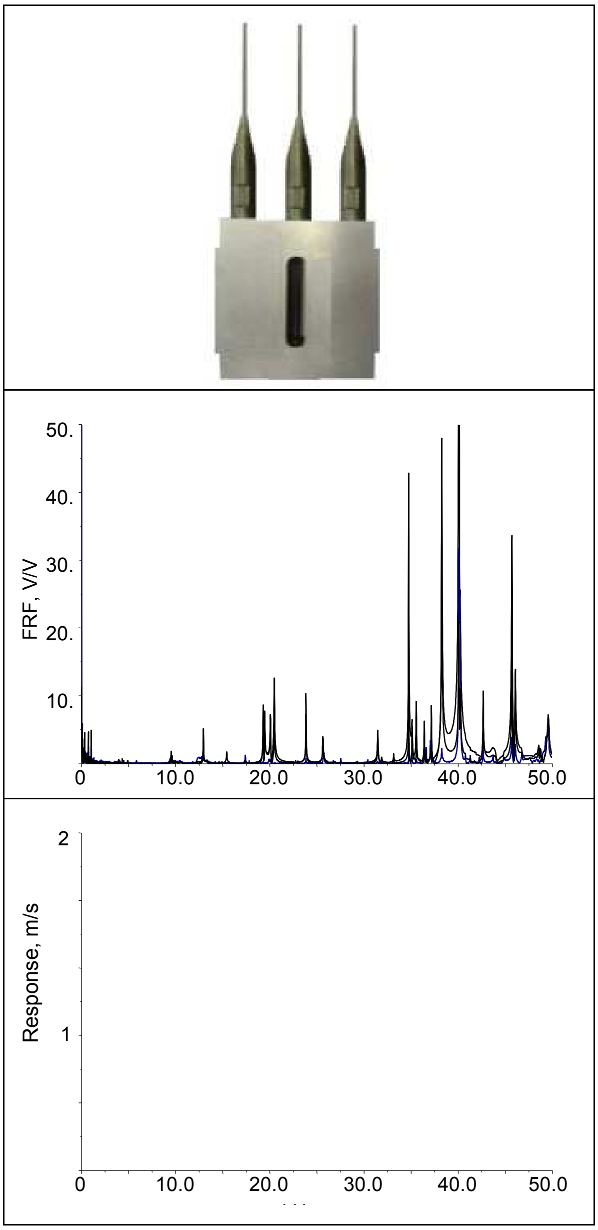
An autoparametric system is one in which the forced response of the primary part of the system, possibly a single forced mode, acts as a parametric excitation on the secondary part, either as a principal parametric resonance involving one mode or a combination resonance involving multiple modes. The net effect is a two-way interaction between the two parts for which overall steady-states are possible. The effects of parametric excitation are such that very large responses may be generated in a plane perpendicular to that of the excitation, provided that certain relationships exist between the excitation frequency and the frequency of the excited internal mode or modes [4,6,7,8]. In practice, for ultrasonic devices, the modal frequency relationships most often measured in systems exhibiting modal interactions are: a modal frequency is approximately equal to half the tuned excitation frequency, called a principal parametric resonance where $\Omega/2 \cong \omega_1$; the sum of two modal frequencies is approximately equal to the tuned excitation frequency, called a combination resonance or three-mode interaction where $\Omega \cong \omega_2 + \omega_3$. Ω is the external excitation frequency (and tuned longitudinal mode resonant frequency of the system), and ω_1, ω_2, ω_3 are three internal modal frequencies [2,3,4,6].

As in the theoretical models, the modal interaction manifests itself as the excitation of large responses in flexural and torsional modes, in a plane perpendicular to the excitation direction, at frequencies lower than the excitation frequency. The responses are easily detected in a measurement of the response spectrum when the device is being driven at the tuned frequency and can be readily heard at their audible frequencies.

Principal parametric resonance

In Figure 1 the response of a three-blade cutting device, driven in longitudinal mode resonance at its tuned wavelength frequency of 34.7 kHz, is shown. The device consists of a piezoelectric transducer providing the external excitation to a single-slotted half-wavelength block horn with three identical half-wavelength high-gain cutting blades attached as shown in Figure 1(a).

The frequency response function (FRF) shown in Figure 1(b), along with an experimental modal analysis of the device, allows all the internal modes to be identified. It is therefore clear from the response spectrum in Figure 1(c), measured when the device is excited at 34.7 kHz, that the externally excited mode couples with an internal mode at 17.35 kHz, at half of the tuned driving frequency, resulting in large responses in an identified flexural mode of the device. The modal interaction modulates the time response and results in a second spike in the frequency response. The measurement is characteristic of a principal parametric resonance.



(aFigure 1 : (a) Cutting device, (b) FRF and (c) frequency response for system driven at 34.7 kHz in second longitudinal mode

Effects of excitation level and frequency on response

As well as measuring the time and frequency response at the driving frequency, the energy leaks into non-tuned modes due to a modal interaction can be detected and illustrated from a slow sine-sweep excitation and response measurement over a small frequency range centred on the driving frequency.

The measurements carried out on the device of Figure 1(a) are characterised by a primary and secondary response, as shown in Figures 2 and 3 respectively. The primary system is the externally excited forced oscillator and the secondary system is parametrically excited by the response of the primary system [4]. In Figure 2, the primary response is the response of the tuned longitudinal mode, excited externally over a frequency range of 300 Hz centred on the longitudinal mode resonant frequency, for an upward (Figure 2(a)) and downward ((Figure 2(b)) frequency sweep, at three different excitation levels. In Figure 3, the secondary response is the response of the coupled internal flexural mode, measured over the same external frequency range, at two different excitation levels.

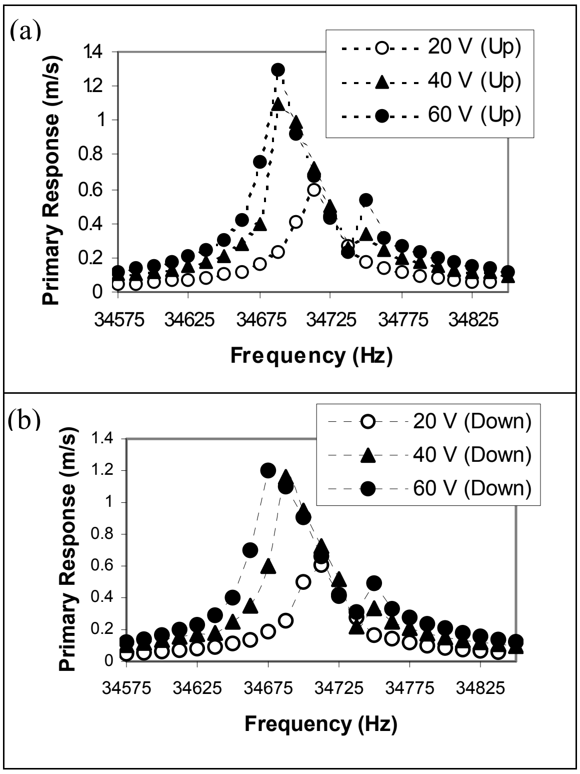


Figure 2 : Measured primary response at three excitation levels; (a) upward frequency sweep, (b) downward frequency sweep

These figures allow the intermodal energy exchange to be studied. It is observed that for the lowest excitation level, the system responds only at the excitation frequency (primary response), and no modal interactions are detected. At the two higher excitation levels, a modal interaction is detected. Considering the upward sweep in Figure 2(a), the primary system response initially increases and there is no secondary response. At 34.69 kHz, very close to the longitudinal mode resonant frequency, there is a large primary response and also a secondary response appears as shown in Figure 3. Both responses then decrease then increase again, until at 34.74 kHz, the secondary response disappears and the primary response decreases. The same responses are recorded for the downward frequency sweep.

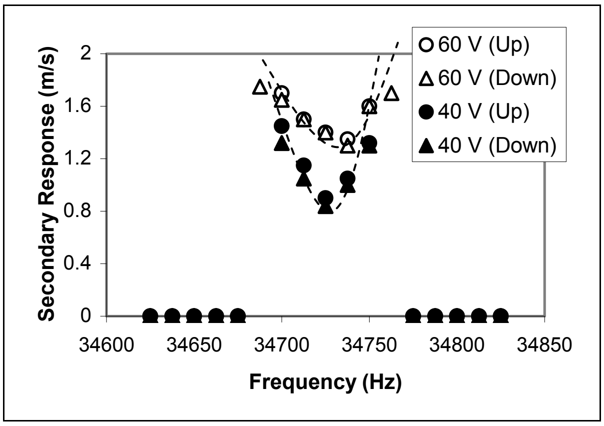


Figure 3 : Measured secondary response at two excitation levels

The measured two-mode coupling exhibits a v-shape in the primary response as illustrated in Figure 2. It can be observed that, once the nonlinear threshold is reached, the primary response is independent of the excitation level in the double response region (v-shape region). By contrast, Figure 3 reveals that the secondary response is dependant on the excitation level.

Figure 4 illustrates the effect that varying the excitation level has on the primary response of the system driven at a constant external excitation frequency. In the measurements illustrated in Figure 4(a) the excitation frequency matches the frequency at which the double response is initially detected (34.69 kHz) during the upward frequency sweep. An excitation frequency corresponding to the frequency of the minimum nonlinear response (at the bottom of the v-shape at 34.73 kHz) is adopted for the measurement plotted in Figure 4(b). In both measurements the primary response increases linearly with excitation level up to the threshold (where the secondary response appears). Further increasing the excitation level, the primary response first decreases and then stays constant. This confirms that the primary response of the system is not governed by the excitation level when the modal interaction is underway [4,6]. From the comparison of Figure 4(a) and 4(b), it is also evident that the drop in the primary response, associated with the internal excitation of the secondary response, is greater when the system is driven at the minimum nonlinear response frequency. This is not a surprising result since the largest amount of energy extracted from the primary mode occurs at this frequency.

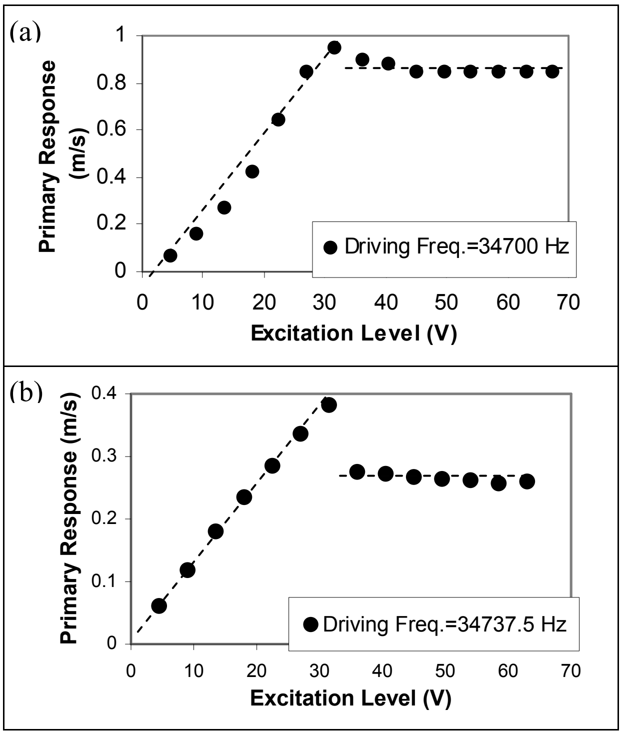


Figure 4 : Primary responses measured as a function of excitation level

Figure 5 shows the secondary system response over the same excitation level range, also for the excitation frequency equal to the minimum nonlinear response frequency. It can be seen that the response of the internally excited mode, above the threshold, increases with excitation level. A response saturation effect, attributable to the inherent nonlinear behaviour of the system, limits the growth at high excitations [3,4,8].

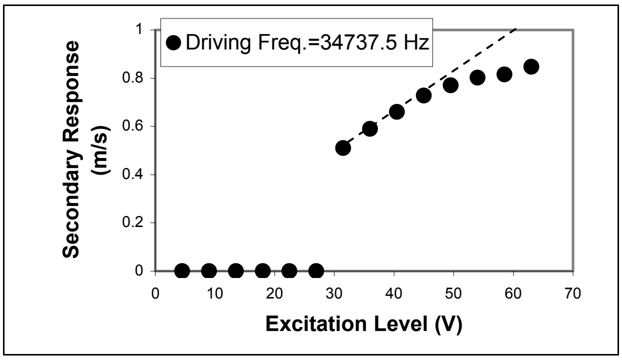


Figure 5 : Secondary response measured as a function of excitation level

Double principal parametric resonance

Theoretical modelling of simple autoparametric systems has predicted that diverse modal interactions can be excited when the external frequency is varied in the vicinity of a modal frequency [4]. In particular it is found that, for certain excitation level thresholds, primary responses are characterised by a number of v-shaped regions each of which indicates a single modal interactions.

In Figure 6, the primary response of the cutting device is measured through a sweep of the excitation frequency over the same range considered in the previous measurements. However, in this case the sweep is performed at a higher excitation level.

The figure shows two v-regions in two distinct frequency bands of the primary response, indicating that the excitation level threshold for two modal couplings is reached. In this case, the first combination resonance, occurring in the lower frequency v-region of the figure, has a higher excitation level threshold, which is not reached, at the lower excitation levels of Figure 2. Hence, this combination is weakly coupled. The second combination corresponds to the same modal coupling detected in the previous measurements at lower excitation levels. This combination is strongly coupled. The threshold and width of the v-region are influenced by damping and the detuning parameter, $\varepsilon\eta$, where $\Omega = 2\omega_1 + \varepsilon\eta$ [4,5]. Both modal interactions feature two internally excited principal parametric resonances coupled with the external mode. The internally excited modes, occurring at 17.3 kHz and 17.35 kHz, are identified in the FRF of the cutting device as shown in Figure 7.

For measurements carried out at even higher excitation levels, an overlap of the two v-shaped regions over a frequency bandwidth will occur. As a result three modal responses, one external and two internal, would be excited in the response spectrum.

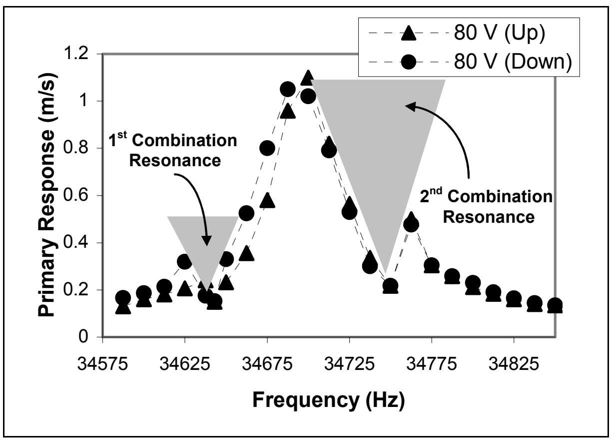


Figure 6 : Primary response for two internal resonances

Since different modal interactions can be excited at different excitation thresholds, it is important in the experimental assessment of an ultrasonic device to check the response up to the excitation level typical of its operating conditions. From the experimental evidence gathered in this and other studies, it would appear that, for high power ultrasonic systems that incorporate multiple tuned components and particularly high-gain components, if one of the frequency relationships which results in a modal interaction exists, the threshold will lie within the operating excitation level range.

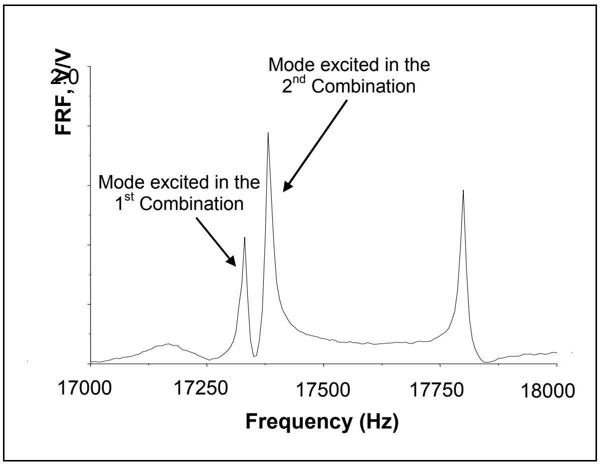


Figure 7 : Measured FRF in 1 kHz frequency range centred at half of the excitation frequency

Combination resonance - three-mode interaction

In Figure 8 the response characteristics of a three-blade cutting device featuring a double-slotted block horn are investigated.

An FRF measurement from a position on one of the outer blades in the 0-50 kHz range is shown in Figure 8(b). The response spectrum measured when the device is driven in the tuned longitudinal mode at 35.5 kHz shows evidence of an internal coupling of the externally excited mode with two flexural modes at 10.8 kHz and 24.5 kHz, as illustrated in Figure 8(c). The measurement is characteristic of a three-mode combination resonance, where a frequency relationship $\Omega \cong \omega_2 + \omega_3$ is satisfied.

Figure 9 shows the primary response of the tuned mode at an 80 V excitation level over a 275 Hz frequency range. The response measurement obtained from a downward frequency sweep exhibits the presence of a v-shaped region corresponding to the three-mode combination resonance. Conversely, when the system is externally excited through an upward frequency sweep it responds only at the excitation frequency (primary response), and no modal interactions are detected. This behaviour is indicative of a weak modal coupling, which may explain the dependence on the sweep direction.

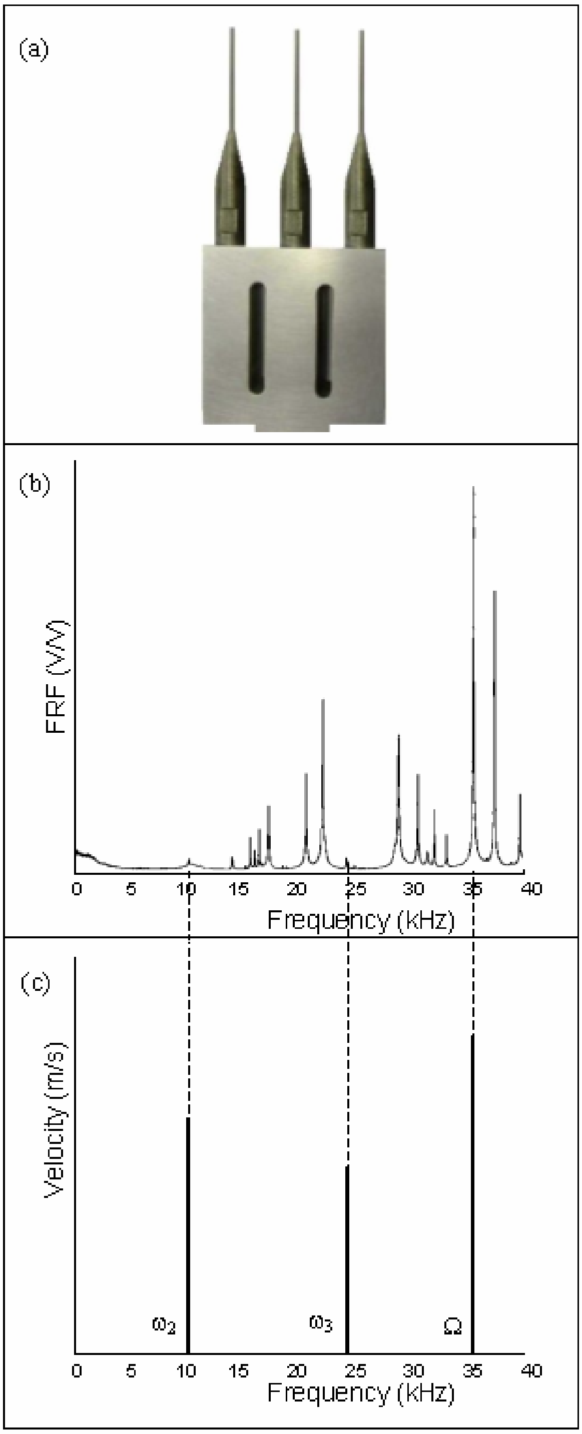


Figure 8 : (a) Cutting device, (b) FRF, (c) frequency response for system driven at 35.5 kHz in tuned longitudinal mode

Impact on high power ultrasonic systems

It is possible to excite any ultrasonic device such that it operates linearly and without modal interactions, if the excitation level is sufficiently low. As it has been shown, each modal interaction has an excitation threshold below which it is not excited or detected. This threshold can be identified from frequency response measurements conducted at different excitation levels. For high power ultrasonic devices, it is difficult to avoid exceeding the threshold for one or more modal interactions. The thresholds may be shifted during operation by the effects of damping from the workpiece material, depending on the mechanism of the material contact, but the thresholds return to their original excitation levels when the system is removed from the material.

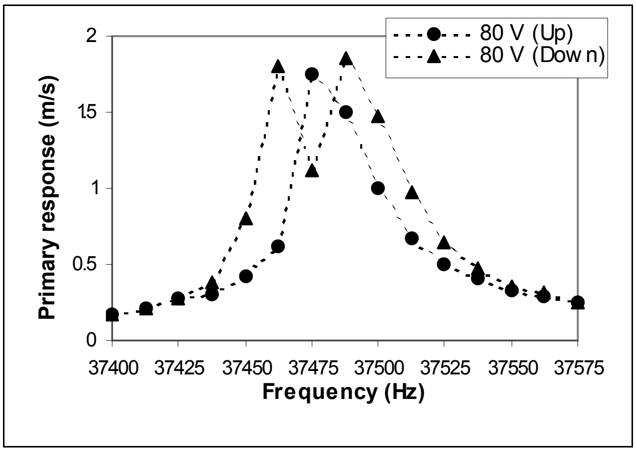


Figure 9 : Measured primary response of a three-mode combination resonance

Reducing the number of modes

Predicting and controlling the dynamic response of high power ultrasonic cutting devices is more easily realised if there are a small number of modes at frequencies below the driving frequency, because modal interactions are a result of energy leaks into lower modes of vibration. If the number of modes is small, there are reduced opportunities to couple the longitudinal resonance with flexural modes and reduced opportunities for the required special relationships between the modal frequencies to exist.

Often, substantial reduction in the number of modes is not readily achievable but usually some reduction can be achieved through geometry modifications. The effect of modifying the block horn is investigated in this study.

An alternative approach, for devices that conventionally have consisted of several serially-coupled half-wavelength components, is to design the system, as far as possible, within a single half-wavelength of the tuned frequency. This can be achieved in two ways; first by driving the original multiple-half-wavelength system at a lower frequency in its first longitudinal mode or, second, by incorporating the essential geometric and dynamic elements within a much shorter length and driving at the original tuned frequency.

Geometry modifications to original cutting device

Half-wavelength block horns are commonly used in ultrasonic systems, both as tools for operations requiring a wide output cross-section, and as intermediate components in devices requiring a wide, anti-nodal, uniform amplitude surface to attach other components.

Block horns are slotted according to a set of design rules in order to compensate for Poisson's effect and to control the output face vibration amplitude uniformity. Slots, however, are responsible for a large number of the modes in the frequency response below the tuned frequency due to the large number of flexural and torsional modes excitable in the columns of the block. For the three-blade cutting device, the block horn geometry that satisfies the slotting design rules is a double-slotted configuration as shown in Figure 10(a).

Two alternative block horn geometries were considered and are shown in Figure 10: (b) a single-slotted block horn and (c) a solid block horn. For both of these alternative configurations, the uniformity of the vibration amplitude at the anti-nodal output face is controlled by modifying the geometry of the side faces, such that a symmetrical block with castellated side faces is designed. A finite element analysis, validated by an experimental modal analysis, was conducted to predict and measure the modes of vibration and the frequency response functions of a three-blade cutting device incorporating each of the three block horns consecutively. The single-slotted block horn reduced the number of modes below the tuned mode by 20% and the solid block horn reduced the number of modes by 40%.

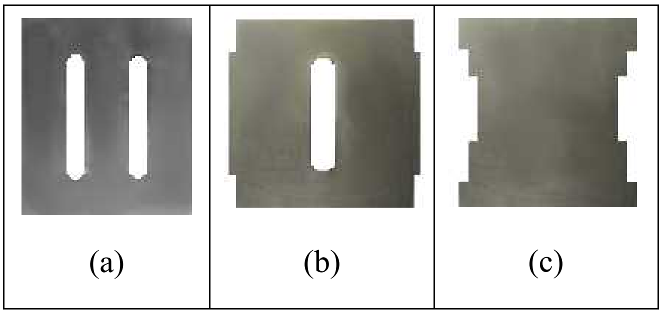


Figure 10 : Alternative block horn geometries

Even with a reduced number of modes, it is still possible to excite modal interactions, as seen for the single-slotted cutting device of Figure 1. However, the internal modes, which are energetically excited by the interaction, can have their modal frequencies adjusted by simply modifying the geometry of the step on the side faces of the block horn. The resulting frequency shifts mean that the special frequency relationship, $\Omega \cong \omega_1$, is no longer satisfied.

Driving at the first longitudinal resonance

To test the validity of driving the three-blade cutting device of Figure 8(a) in its first longitudinal mode, the wavelength device is driven in longitudinal mode resonance at 22 kHz. Although it is not intended to offer a design solution at this new operating frequency, the response measurements are carried out to gain some insight into the effect on the vibration response of restricting the operation of such devices to one half-wavelength of the driving frequency.

For this approach, careful consideration has to be given to the power requirement, modal response, blade gain profile and resulting stress condition, to sustain the required cutting amplitude at the new driving frequency.

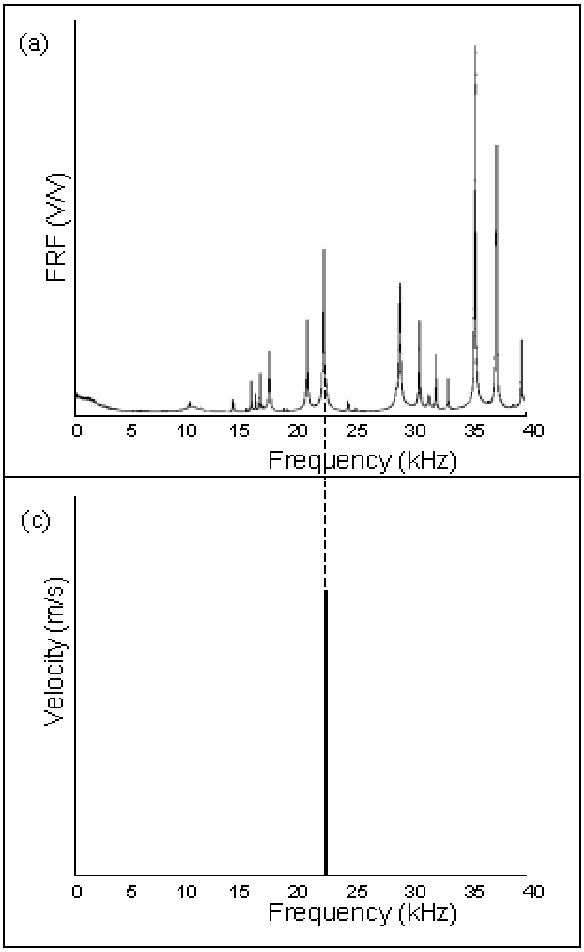


Figure 11 : (a) FRF and (b) frequency response at 22 kHz excitation frequency, for system driven in first longitudinal mode

For this set of exploratory experiments, the ultrasonic transducer worked with low efficiency, but with sufficient power to drive the system such that the same blade tip amplitude as the 35 kHz device is maintained. This required some redesign of the blades to incorporate a higher gain but avoid flexural responses in the blades in longitudinal mode resonance. The system design is carried out by FE modelling based on blade designs reported previously [9].

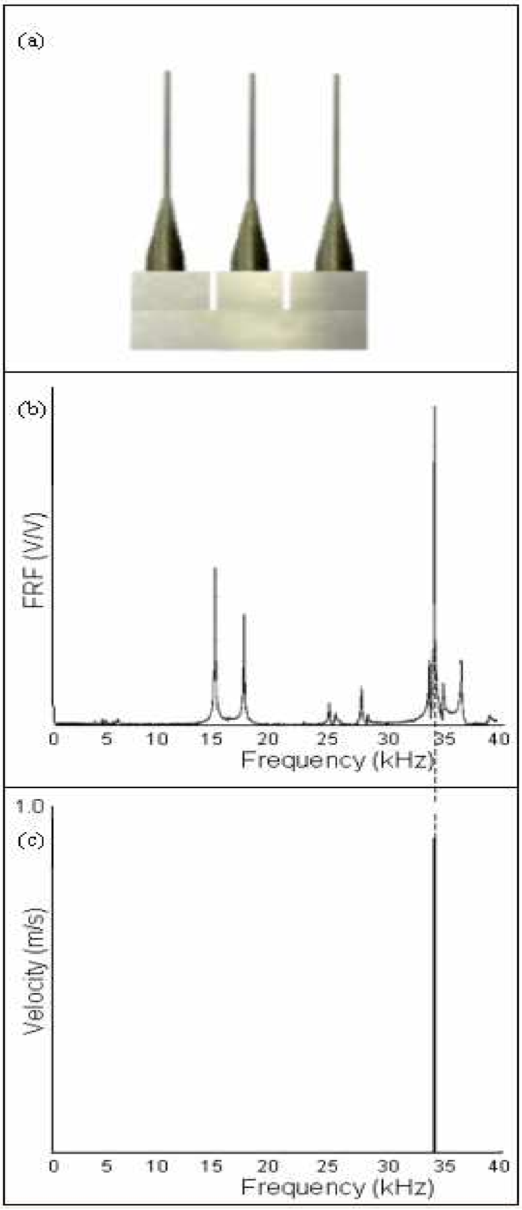


Figure 12 : (a) Half-wavelength cutting device, (b) FRF and (c) response at 34 kHz excitation frequency, for system driven in first longitudinal mode

In Figure 11, the frequency response function exhibits 50 % fewer modes than the FRF associated with the cutting device driven at 35 kHz (cf. Figure 1). The measurement of the response spectrum for this system driven at 22 kHz also clearly shows a single frequency response. No evidence of a modal interaction is found for the range of excitation levels used to drive the system, which is nearly up to the level required to reach the maximum acceptable stress in the blades, and is beyond that required to excite the blade tip cutting amplitude.

The experiments demonstrate that systems can be designed that do not result in modal interactions occurring, and support the strategy of reducing the number of modes below the driving frequency.

Design of a half-wavelength device

For this approach, a new component is designed that contains the block horn and blade components within a half-wavelength of the 34 kHz driving frequency as shown in Figure 12, allowing compactness of the device, while maintaining the required cutting length of the blades and the required cutting amplitude. This device is therefore designed to be driven in resonance in its first longitudinal mode.

Again, the number of modes is significantly reduced and the measured FRF in Figure 12 and the full experimental modal analysis of the device, have shown that the number of modes (in the frequency range up to 34 kHz) is 50 % fewer than for the wave-length device operating at the same nominal driving frequency. The measured response also showed no evidence of inter-modal energy exchanges.

Although the new three-blade half-wavelength cutting device operates successfully without energy leaks due to modal interactions, it is still possible to excite modal coupling in systems exhibiting a small number of modes below the tuned longitudinal-mode frequency. In order to predict the possible occurrence of modal interactions, FRF predictions by FE modelling form part of the design process for new and modified devices. It is possible to search for modal frequencies that satisfy the most common special relationships of a modal interaction, and subsequently adjust geometry parameters in the FE model to predict frequency shifts that will eliminate these critical frequency relationships.

Conclusions

It is possible to characterise the modal interactions in ultrasonic cutting devices by measuring the responses and referring to mathematical models of simple autoparametric systems such as beam-like structures. The two and three-mode interactions, commonly identified in ultrasonic devices, excite responses which are clearly qualitatively similar to such theoretical models.

Energy leaks are characterised for a number of different modal interactions, including principal parametric resonances, double principal parametric resonances and three-mode combination resonances. These are typical responses of high power ultrasonic devices, where energy leaks into modes at frequencies lower than the tuned longitudinal mode frequency, exciting flexural modes of the system.

Modal interactions can be controlled by designing devices with as few modes as possible at frequencies below the driving frequency, within design constraints. Modifications of block horns and design of devices within a half-wavelength of the driving frequency, have proved to be successful strategies. With a lower number of modes, effecting frequency shifts to internal modes by simple geometry modifications can prove successful in eliminating the frequency relationships that lead to modal interactions.

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