PROBING FORCE CHAINS AND INTER-PARTICLE CONTACTS IN GRANULAR MATERIALS BY NONLINEAR SHEAR WAVES

V. Tournat†,*, V. Yu. Zaitsev‡, V. E. Gusev†, V. E. Nazarov‡, P. Béquin† and B. Castagnède†
†Université du Maine, Av. Olivier Messiaen, 72085 Le Mans cedex 09, France
‡Institute of Applied Physics, 46 Uljanova Street, Nizhny Novgorod, 603950, Russia.
*Email: vincent.tournat@univ-lemans.fr

Abstract

Nonlinear acoustic probing of a granular medium by a parametric antenna is reported. Shear waves are used for this purpose for the first time. Nonlinear dilatancy is found, which produces conversion of high-frequency shear waves into low-frequency demodulated longitudinal waves and follows a quadratic or Hertzian power law. A strong anisotropy of nonlinearity for shear waves with different polarizations is observed. This sensitivity of the parametric antenna to the shear pump waves polarization allows to detect the force chains preferential orientation in the uni-axially stressed medium, and to quantify the level of induced anisotropy.

Introduction

Acoustic methods are expected since several years to offer an additional insight in the fundamental properties of granular packings. In particular, due to the high acoustic nonlinearity of the unconsolidated granular materials, the nonlinear effects can be extremely usefull to probe the so-called Hertzian contacts or the force chains network.

The previous experiments [3] indicate that sound propagation is configuration sensitive and that it causes structural relaxation, irreversible and hysteretic phenomena. Recent experiments [4] demonstrated that the acoustic signals emitted by microscopic rearrangements of particles can be used to monitor the dynamics of percolating forced chains in compressed granular media.

The present Communication reports the application of the nonlinear parametric antenna (NPA) [5] to diagnose the granular medium exploiting for the first time self-demodulation of shear high-frequency (HF) acoustic wave bursts. This was motivated by a simple idea that one of distinguishing features of granular packings are inter-grain Hertzian contacts whose nonlinearity is especially high at weak contact pre-loading. As a promising tool with unique possibilities to study such fine material peculiarities, nonlinear wave processes might be used, which have proven to be preferentially sensitive to elastically soft (high-compliant) features of the microinhomogeneous materials [6], [7]. This study followed a series of successful experiments on self-demodulation of HF longitudinal acoustic bursts [8], [9], [10] in sand and artificial granular media. The choice of shear elastic waves was motivated by the expectation that the rectification process in S-waves, which, simultaneously with second harmonic generation, is not efficient in homogeneous isotropic media [11], might be especially sensitive to inhomogeneity and anisotropy of a granular packing and could be useful for studying possible effects of its dilatancy [12], [13] (i.e. the tendency of a medium to expand upon shearing).

First considerations

Concerning granular materials, a challenging problem is to apply nonlinear effects for probing contact loading conditions, in particular, for characterization of the widely discussed (e.g. [1], [4], [14], [15]) weak contacts (“spectators”) and “force chains” transmitting stresses, composed of strong contacts . In favor of the nonlinear approach to these problems one may formulate the following instructive arguments. The Hertz nonlinearity [16] yields, in the simplest case of equally loaded contacts, the material stress-strain relation of the form:

$$\sigma = bn\varepsilon^{3/2}H(\varepsilon) \quad (1)$$

Here factor b depends on elastic moduli of the individual grains, n is the average number of the contacts per one grain and the Heaviside function $H(\varepsilon)$ indicates that only compressed (σ, $\varepsilon > 0$) contacts contribute to the stress in the material. The mean static strain ε_0 of the material determines the linear elastic modulus $d\sigma(\varepsilon_0)/d\varepsilon$ for small amplitude acoustic perturbations with strain $|\widetilde{\varepsilon}| \ll |\varepsilon_0|$, and stress $|\widetilde{\sigma}| \ll \sigma_0 \equiv \sigma(\varepsilon_0)$. In real granular materials, besides the average-loaded contacts, there is a portion of essentially weaker loaded contacts [14], [15], contributing to the resultant $\sigma(\varepsilon)$. Let us suppose for a moment just one additional fraction of weakly loaded contacts. Separating out explicitly the static and oscillatory parts, the stress-strain relation may be rewritten as:

$$\begin{aligned}\sigma_0 + \widetilde{\sigma} \quad = & \\ & bn_0(\varepsilon_0 + \widetilde{\varepsilon})^{3/2}H(\varepsilon_0 + \widetilde{\varepsilon}) + \\ & bn_1(\mu\varepsilon_0 + \widetilde{\varepsilon})^{3/2}H(\mu\varepsilon_0 + \widetilde{\varepsilon})\end{aligned} \quad (2)$$

The mean number n_0 and n_1 of average loaded and weak contacts per grain (estimated using data on wave velocities [17]) may be comparable ($n_1 \sim n_0$). The nondimensional coefficient $|\mu| \ll 1$ characterizes the extent of unloading of the softer fraction. We may allow for $\mu < 0$ in order to describe the initially nonexisting contacts ("holes"), which may be created by acoustic wave with strain $|\widetilde{\varepsilon}| > |\mu|\varepsilon_0$. For initially weakly compressed contacts $0 < \mu \ll 1$ and $|\widetilde{\varepsilon}| \ll |\mu|\varepsilon_0$ the first and higher derivatives of Eq.(2) with respect to $\widetilde{\varepsilon}$ characterize the linear and nonlinear elastic moduli of the material, respectively:

$$\frac{d^m\widetilde{\sigma}}{d\widetilde{\varepsilon}^m}(\varepsilon_0) \sim bn_0(1+\frac{n_1}{n_0}\mu^{3/2-m})\varepsilon_0^{3/2-m} \qquad (3)$$

Thus for the linear modulus ($m = 1$) the relative contribution of the weak contacts is $\sim \mu^{1/2} \ll 1$ and may be negligible. In contrast, for the higher-order, nonlinear moduli ($m \geq 2$) the contribution of the weak fraction is $\sim \mu^{3/2-m} \gg 1$ and thus may strongly dominate.

Experimental results

The observations were made on the features of the demodulation of intensive shear elastic waves in the granular medium (glass beads $2 \pm 0.1mm$ in diameter) placed in a plastic cylindrical container $40cm$-diameter and $50cm$ in height (Fig.1). The vertical loading was achieved by a screw clamp via a rigid plastic cover and was controlled by a force cell (static stress- and strain-ranges were 7-70 kPa and (1-5).10^{-4} respectively). The primary (pump) AM-modulated and burst waves with 80 kHz carrier frequency were excited by S-transducers ($3.5cm$ in diameter). A longitudinal transducer (L) was used for the reception. Orientations and polarizations of the transducers are shown in Fig. 1.

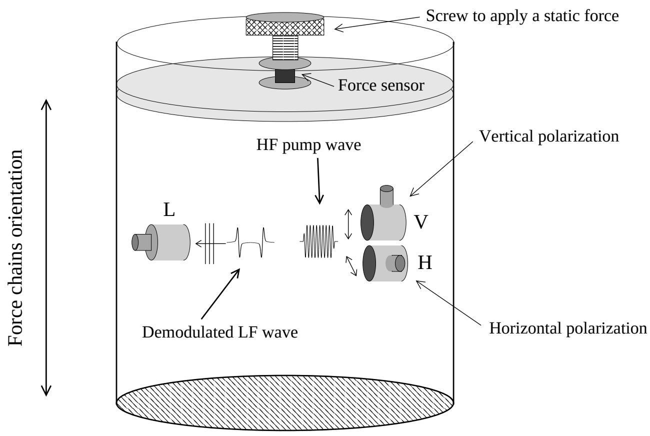

Figure 1: Scheme of the experiment. The propagation direction is orthogonal to the force chains preferential orientation. Propagation distance is $16cm$.

The polarization properties of S-waves provide another interesting possibility to apply the demodulation effect for probing contact anisotropy and the presence

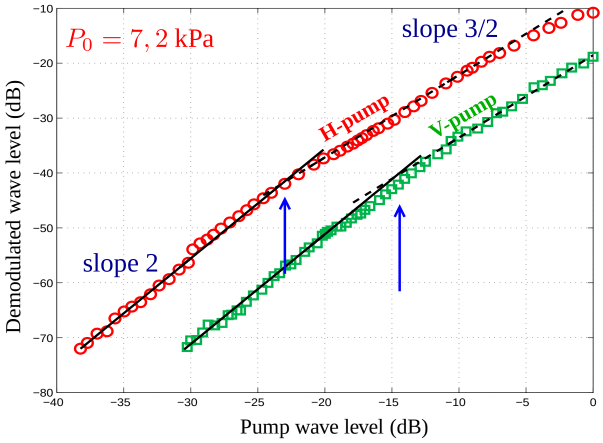

Figure 2: Demodulated signal amplitude dynamics as a function of the pump strain level. The pump wave frequency is $80kHz$ and the demodulated wave frequency is $5kHz$.

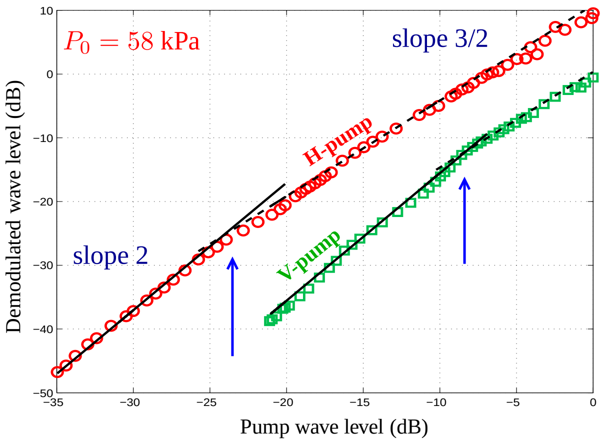

Figure 3: Demodulated signal amplitude dynamics as a function of the pump strain level. The pump wave frequency is $80kHz$ and the demodulated wave frequency is $5kHz$.

of force chains predominantly oriented along the applied static stress direction. As noted above, contact nonlinearity is inversely proportional to the static pre-strain (see Eq.(3)), thus in an anisotropic material different effective nonlinearity should be expected for different S-wave polarizations. Figure 2 presents amplitude dependencies of the demodulated signals from highly identical (this has been verified experimentally) S-pump sources directed horizontally, but having orthogonal vertical (V-pump) and horizontal (H-pump) polarizations. The uni-axial externally applied static stress is $P_0 = 7.2kPa$. The plots firstly indicate that H-polarized pump produced more than $10dB$ higher-amplitude signal. Secondly, transition from a non-clapping regime to a clapping regime (characterized by the transition from a slope 2 to a slope 3/2 in the demodulated signal amplitude dynamics) occured $9dB$

lower in amplitude for H-polarized pump compared to V-polarized pump. Both features consistently indicate several times higher effective nonlinear parameter for the H-polarized pump compared to V-polarized pump waves, which means that horizontal contacts are indeed weaker loaded than vertical ones (roughly an order of magnitude in terms of contact forces).

Table 1: Summary of the presented experimental results in Fig. 2 and in Fig. 3.

Applied static stress	7.2 kPa	58 kPa
Transition 2 $\rightarrow$ 3/2 level for H-polarized pump wave	-23 dB	-23 dB
Transition 2 $\rightarrow$ 3/2 level for V-polarized pump wave	-14 dB	-8 dB
Transition Level difference	9 dB	15 dB
Demodulated signal level difference for -20 dB pump wave level	13 dB	16 dB

In Fig. 3, the same experiment is performed for a higher static stress ($P_0 = 58kPa$). The transition from the slope 2 to the 3/2 slope, associated to the acoustically induced clapping, occurs roughly at the same pump wave level ($-23dB$) for the signal demodulated from horizontally polarized shear pump waves. However, for the signal demodulated from vertically (i.e. along the direction of force chains) polarized shear pump waves, the transition occurs $15dB$ higher in pump wave level. The resultant anisotropy in the effective nonlinear quadratic parameter is then larger for $P_0 = 58kPa$ compared to $P_0 = 7.2kPa$. Most of the applied static stress is supported by the vertically oriented contacts, which creates a network of force chains oriented vertically. The horizontally oriented contacts are not strongly modified in this static stress increase.

This anisotropic nonlinear dilatancy may result, for NPA with HF shear-pump having circular polarization rotating with frequency Ω, in the appearance of the LF demodulated L-wave at even harmonics $2k\Omega$, $k = 1, 2\ldots$

In Tab. 1, the important features of the experimental results are summarized. All the observations and the described interpretations are in agreement. In particular, the level of induced anisotropy can be found through the transition 2 $\rightarrow$ 3/2 level difference and through the demodulated level difference.

Known experimental data and numerical simulations [14] of intergranular forces indicate that distribution $n(\mu)$ of the contacts static loading in disordered granular materials has a rather abrupt decrease above the average loading ($\mu = 1$ in our terms) and has also a significant portion of weakly loaded contacts. If $n_0 \sim n_1$ then the contribution of the soft (weakly loaded) contacts to the nonlinear signal must dominate for sufficiently strong unloading $\mu \leq 0.1 - 0.01$. Such strains correspond to even smaller forces $f/f_0 \leq 0.03 - 0.001$, which are, therefore, far beyond the data region $f/f_0 \geq 0.1$ commonly presented in photo-refraction or carbon-paper experiments (see [14] and references therein). In particular, the occurence of the transition 2 $\rightarrow$ 3/2 at the pump strain levels $-10dB$ or less before the maximum pump strain level $0dB$ is an indication of contacts localization to the low values of μ in the contacts distribution. Considering a flat contacts static strains distribution under $\mu = 1$ and the commonly observed dependence of this distribution for $\mu > 1$ [14], [15], [18], it is possible to predict from the effective medium stress-strain relationship (1) a transition 2 $\rightarrow$ 3/2 at the pump strain level $\sim 0dB$ (this dynamic strain level is, from the estimates, a little less than the mean value of the static strain $\mu = 1$). To obtain such a transition at least $10dB$ lower than the maximum pump level as it is observed in Fig. 2 and 3, there should be more contacts with $\mu < 0.01 - 0.1$.

Conclusions

The results obtained confirm that nonlinear acoustic effects can selectively probe weak contact portions despite a rather high background nonlinearity in granular media. In order to explain the signal magnitude and the clear transition 2 $\rightarrow$ 3/2 in the amplitude dependence of the demodulated wave (found for the first time in a granular material) it is necessary to assume a large amount of weak contacts (over 60-70% of the total). Moreover, this effect requires that the distribution of these weak contacts should contain a significant contact portion strongly enough localized near zero forces. For irregular grain-shapes, as in dry sand, the localization is even stronger, since the quadratic behavior could not be observed at all [8].

The inferences obtained on the weak forces localization agree qualitatively with recent data of inter-grain force simulation [15], indicating an upturn in $n(f)$ at very small forces ($f \leq 0.1f_0$) for packings with friction, and should stimulate further theoretical modeling. The perspectives of the future research might be related with application of the NPA to the evaluation of the slow dynamics (in particular, of such irreversible processes as compaction [18], [19] and contacts heating [6]) and observation of polarization-sensitive nonlinear effects. The optimization of a compact highly-directive parametric emitter and combination of L- and S- pump waves for the diagnostics of granular packings and piles might be considered as a possible practical application.

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