MEASUREMENT OF DRIVING ACOUSTIC PRESSURE IN SBSL 
BY LASER-INTERFEROMETRIC TECHNIQUE

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Abstract
Single Bubble Sonoluminescence (SBSL) is the light emission by a tiny bubble trapped in an acoustic standing wave in a flask filling with liquid. It attaches much interest in physics-acoustics because of its high non-linear characteristic and high concentration of energy. Stable SBSL is sensitive to both the driving acoustic pressure \( P_a \) and the symmetry of standing acoustic wave field in the resonant cell, so \( P_a \) is difficult to be measured real-time by traditional technique. But based on the piezo-optic effect of liquid and Michelson interferometer, the integral of acoustic pressure along the optical path can be detected non-invasively by laser-interferometric technique. This technique makes no disturbance to the bubble and the resonant acoustic field. As long as the relative distribution of acoustic field along the optical path and piezo-optical coefficient of liquid are known, the acoustic pressure of any position in optical path can be obtained. Therefore, the upper and lower thresholds of driving acoustic pressure and the relation between emitting light intensity and driving pressure in stable SBSL are detected by using laser-interferometric technique. And combining with Mie scattering experiment data, another important parameter besides driving pressure in SBSL, the bubble’s ambient radius \( R_0 \) can be fitted.

Introduction
SBSL is a very interesting physical phenomenon. A single bubble of several microns levitates in a standing acoustic wave and emits light. During this process, the acoustic wave of low energy density is transformed to the optic pulse of high energy density by inertial compression. The power density is enhanced of the magnitude of \( 10^{12} \). Some literatures reported that the high pressure of more than \( 10^3 \) ATMs and the high temperature of more than \( 10^4 \) K were generated in the bubble when it collapses violently. \(^{12, 3}\) We also calculated the different pressure increments around the bubble when it was compressed to different radius (Fig. 1).\(^\text{[4]}\) And the possibility of bubble fusion was reported last year.\(^\text{[5]}\)

Stable SBSL is sensitive to the driving acoustic pressure \( P_a \). And it is sensitive to the symmetry of standing acoustic field in the resonant cell too. It is not easy for traditional technique to measure the driving acoustic pressure at the position of the bubble but not to disturb the acoustic field. In order to detect the acoustic pressure non-invasively, a laser-interferometric technique, which is based on piezo-optic effect of liquid and the Michelson interferometer, is introduced.\(^\text{[6]}\) Because this technique makes no disturbance to the acoustic source and field, and it can do absolute and real-time measurement to the acoustic field. As long as the relative distribution of acoustic field and piezo-optical coefficient of liquid are known, the acoustic pressure in the light path can be obtained. Using the measuring laser beam, the optical interference signal and the Mie scattering signal were detected at the same time. From them, the driving acoustic pressure and the bubble’s ambient radius were obtained.
Formulation

The principle of laser-interferometic technique for measuring the acoustic pressure distribution in the spherical resonant cavity is shown in Fig. 2. The interference fringes passing through the PMT reflect the change of the optical pathlength difference. In SBSL, the driving acoustic pressure changes the liquid’s density in the flask and then the liquid’s refractive index and the optical pathlength difference. So the number of the moving interference fringes through the PMT reflects the pressure.

In our experiment, the spherical flask was driven at its fundamental frequency, so the fundamental harmonic acoustic field can be expressed as:

\[ p(r) = A_0 j_0(k_0 r), \]

where \( A_0 \) is the amplitude of the acoustic wave, \( k_0 \) is the fundamental wave vector and \( j_0(x) \) is the zero-order spheric Bessel function.

So the integral \( M \) of driving acoustic pressure \( p \) along the light path is

\[ M = 2 \int_0^\infty p(r) dr = \frac{2aA_0}{\pi} \int_0^\pi j_0(x) dx, \]

where \( a \) is the radius of the spherical liquid (the glass thickness of the flask is reckoned in it).

The relationship between the liquid’s density \( \rho \) and the liquid’s refractive index \( n \) can be expressed by piezo-optic coefficient \( \alpha_\rho \) according to a formula due to Lorentz (1880),

\[ \alpha_\rho = \rho \frac{dn}{d\rho} = \frac{(n^2-1)(n^2+2)}{6n}. \]

For water, when the optical wavelength \( \lambda \) is 632.8 nm, \( \alpha_\rho = 0.324 \) when its temperature \( T = 288 \) K.\(^7\)

The variation of \( \rho \) is associated with the acoustic pressure \( p \): \( p = c^2 \Delta \rho \) , where \( c \) is the acoustic velocity in liquid. So,

\[ \Delta n = \alpha_\rho \frac{\Delta \rho}{\rho} = \frac{\alpha_\rho p}{pc^2}. \]

The optical pathlength difference caused by the acoustic pressure is

\[ \Delta \psi = 2 \int_0^l (\Delta n) dl = \frac{4\alpha_\rho}{pc^2} \int_0^\pi p(r) dr. \]

When \( \Delta \psi \) changes \( N \lambda \), \( 2N \) interference fringes are moved in the half circle of the acoustic signal \( T/2 \).\(^7\) The relationship between the integral \( M \) of acoustic pressure \( p \) along the light path and the moved interference fringes \( 2N \) can be obtained

\[ M = 2 \int_0^\infty p(r) dr = 2 \cdot N \lambda \cdot \frac{\rho c^2}{4\alpha_\rho}. \]

Where, \( \lambda \) is the optic wavelength.

So the amplitude of acoustic wave \( A_0 \) is

\[ A_0 = \frac{2N}{a} \cdot \frac{\rho c^2 \lambda}{4\alpha_\rho} \cdot \frac{\pi}{2T}, \]

where \( I = \int_0^\pi j_0(x) dx \). The last two items in the right of this equation are constants. Thus, the driving acoustic pressure is

\[ p(r) = \frac{2N}{a} \cdot 911 \cdot j_0(k_0 r). \]

So, the driving acoustic pressure is direct proportional to the number of the interference fringes and inverse proportional to the radius of the resonant cell. Therefore, if \( 2N \) is measured, the pressure at any position in the acoustic field can be obtained.

Results

The experiment apparatus was shown in figure 2. Before the splitter mirror, the He-Ne laser beam passed through a beam collimator (NRC 675). Then with a beam radius much larger than the bubble’s radius, it was trained on the bubble which located in the center of the flask and scattered by it. The output signals of PMT 1 and PMT 2 (HAMAMATSU CR114) were recorded by a digital oscilloscope (Tektronix TDS3032). The SBSL intensity was detected by a high-sensitivity and wide-range photoradiometer (PRM, SDS-II).

The interference fringes at some driving acoustic pressure when the bubble was emitting light were
shown in figure 3. The experimental frequency was around 21 kHz and the temperature of water was 285 K. The output of function generator (HP 3325A) was 185 mV and the gain of the power amplifier (B&K 2713) was 46.0 dB. The bubble was emitting light. The interference fringes $2N$ equaled 4.8, so the center pressure of the flask was 1.17 ATMs. The corresponding Mie scattering result was shown in figure 4. In this figure we can see that the scattering light and the SL light were received at the same time and the SL light occurred close to the bubble’s minimum radius at time scale.

Figure 3 Interference fringes when the bubble emitted light. $T = 285\, \text{K}, f = 21.14\, \text{kHz}, 2N = 4.8$, so the driving acoustic pressure at the position of the bubble $P_d$, i.e. at the center of the flask, was 1.17 ATMs.

Figure 4 Mie scattering and SL signals at $P_d = 1.17$ ATMs, $T = 285\, \text{K}, f = 21.14\, \text{kHz}$

The interference fringes at the upper and lower driving acoustic pressure thresholds were shown in figure 5a and 5b. The outputs of the function generator were 111 mV and 94 mV respectively, and the gain of the power amplifier was 48.0 dB for the both. The numbers of the interference fringes in half an acoustic cycle were 5.1 and 5.8 respectively, and the corresponding driving acoustic pressures in the flask center were 1.41 ATMs and 1.24 ATMs. Here the driving acoustic pressure region of stable SBSL increased a little compared with Fig 3 because the water temperature increased a little. Figure 6 showed the relation between the SBSL intensity and the driving acoustic pressure under the same condition. Between the lower and the upper thresholds, emitting

Figure 5a Interference fringes at the upper driving acoustic pressure threshold. $T = 288\, \text{K}, f = 21.08\, \text{kHz}, 2N = 5.8$, so $P_d = 1.41$ ATMs.

Figure 5b Interference fringes at the lower driving acoustic pressure threshold. $T = 288\, \text{K}, f = 21.08\, \text{kHz}, 2N = 5.1$, so $P_d = 1.24$ ATMs.

Figure 6 Relation between the SBSL intensity and the driving acoustic pressure. The upper and the lower threshold were shown in Fig. 5a and 5b.
light intensity was approximatively in direct proportion to driving acoustic pressure.

Mie scattering result was shown in figure 7. The driving acoustic pressure was rather low, so the bubble did not emit light yet. From the corresponding interference fringes chart, the driving acoustic pressure was 0.90 ATM. Combining Mie scattering data with the theoretical curve from Rayleigh-Plesset Equation, another important parameter, the bubble’s ambient radius could be fitted (see Fig.8). After the modification of phase, background and longitudinal radio (through expansion radio) and the correlation with theoretical curve, we obtained the ambient radius was 6.2 µm when driving pressure is 0.90 ATM.

Conclusion

By laser interferometric technique, it was measured that the driving acoustic pressure of SBSL and its upper and lower thresholds non-invasively. The relation between the emitting light intensity and the driving acoustic pressure in stable SBSL was detected too. And combining with Mie scattering experimental data, another important parameter, the bubble’s ambient radius was fitted.

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Reference


Supplement

Figure 9 is a photo of the resonant flask used in our experiment. There were four transducers which are parallel connected at the both sides of it. A tiny bubble located in the center of the flask. And a laser beam with a beam radius larger than the radius of the bubble was trained on the bubble and scattered by it.

Figure 7 Mie scattering result, Non-SL. $T = 285$K, $f = 21.14$kHz, $2N = 3.7$, so $P_a = 0.90$ ATMs.

- $T = 285$K, $f = 21.14$kHz, $2N = 3.7$, so $P_a = 0.90$ ATMs.

Figure 8 Fitted $R(t)$ curve with Mie scattering data. The black and dotted line was the theoretical curve from RP equation, and the red and solid was the fitted curve. From Fig. 6, $P_a = 0.90$ ATM, so the bubble’s ambient radius $R_0 = 6.2$ µm.