RECONSTRUCTING ULTRASOUND TRANSDUCER SOURCES USING A REDUCED NUMBER OF ACOUSTIC FIELD MEASUREMENTS

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Abstract

The Angular Spectrum of Plane Waves (ASPW) method is commonly used to determine the surface displacement/peak velocity distribution of piston-like transducers from measurements of the radiated pressure field distribution. This requires, typically, plane scans of 100×100 points and a corresponding computational effort.

The finite element approach is an alternative method that can predict source distributions and radiated fields to an acceptable accuracy from relatively few measurements. The radiating surface is divided into a number of elementary areas over each of which the displacement/peak velocity is assumed constant. The acoustic field at any point is approximated as the superposition of the fields due to the individual surface elements. Carrying out measurements at N different points, at least equal to the number of elements, enables a set of N linear equations between the unknown element values and the measured field values to be found. The (approximate) surface distribution is recovered by inverting these simultaneous equations.

By assuming that the source distribution is a function of the radial co-ordinate only, surface velocity distributions of plane, circular, piston-like transducers have been reconstructed from a limited number of pressure measurements (typically < 10). The present contribution also investigates non-uniform source distributions. This necessarily increases the number of elements required for a satisfactory source reconstruction. The significant increase in the number of measurements required means that ill-conditioning of the equations is a significant issue.

Introduction

It is well known that the surface velocity distribution of a plane piston-like transducer can be determined indirectly from measurements of the radiated pressure field distribution over a plane parallel to the transducer surface by using the Angular Spectrum of Plane Waves (ASPW) method.

This approach often requires a large number of time-consuming precision measurements and a corresponding computational effort. Scans of 100×100 points are typical for transducers operating in the Megahertz region. Thus there is an incentive to investigate alternative approaches that can predict both source distributions and radiated fields to an acceptable accuracy but requiring only relatively few measurements.

THEORY

General

The acoustic field produced by a bounded vibrating surface S can be found from the Rayleigh integral:

$$p(\mathbf{r},t) = \frac{i\omega\rho}{2\pi r}\int_S \exp(-ikr)\mathbf{U}(\mathbf{r}_0)dS \qquad (1)$$

The surface S is subdivided into N regions and the strength of the vibration over each region is assumed to be constant. The acoustic field can then be expressed as

$$p(\mathbf{r},t) \cong \frac{i\omega\rho}{2\pi r}\sum_{i=1}^{n}\int_{S_i} \exp(-ikr)\mathbf{U_i}(\mathbf{r}_0)dS_i \qquad (2)$$

If measurements are made at M different field points, then (2) yields a set of M simultaneous linear equations which can be represented in matrix form as

$$[\mathbf{P}_j] = [\mathbf{Q}_{ji}][\mathbf{U}_i] \qquad (3)$$

The formal solution to the system (3) is given by

$$[\mathbf{U}_i] = [\mathbf{Q}_{ji}]^{-1}[\mathbf{P}_j] \qquad (4)$$

and may be solved provided M≥N.

Transverse reconstruction

The initial work [3] used computer simulations based on the circular plane piston to establish the feasibility of the principle. The radiating surface was divided into N regions using N-1 concentric circles.

The field data is taken on a radial line parallel to the transducer surface plane. The axis field expression required for the N×N coefficient matrix was calculated from (1), using the form of Rayleigh Integral due to Fung, Cobbold and Bascom [4].

$$p(x,z) = \rho c U_0 ka[\int_0^{\pi/2} J_1(ka\sin\theta)J_0(kx\sin\theta)\exp(jkz\cos\theta)d\theta$$
$$-\int_0^{\infty} J_1(ka\cosh\beta)J_0(kx\cosh\beta)\exp(-kz\sinh\beta)d\beta] \qquad (7)$$

The investigations included both computer simulations and experimentally derived data. These were carried out for a range of source-measurement plane distances in order to test the range of applicability of the reconstruction algorithm. It was found that the reconstruction quality is relatively insensitive to the measurement distance.

Accuracy of Reconstructions

The goodness of a simulated reconstruction depends on two factors. These are:

(1) The nature of the approximating function
(2) The numerical accuracy of the solution to the system of inverse equations.

This latter depends on the conditioning of the system of equations. In addition, reconstructions from experimental data will incur further errors due to the inevitable measurement uncertainties (noise). Simulated reconstructions of uniform sources demonstrate directly the effect of the equation system's conditioning. This is illustrated by the 8-point reconstructions shown in Figure 2. The maximum error in the magnitude is less than 2%, while the phase error is only a fraction of a milliradian. This gives some confidence that good reconstructions of piston-like sources should be possible. This was checked by simulating a piston-like source with a linear velocity taper to the transducer boundary. The result of one such reconstruction is shown in Figure 3. The reconstructions give a reasonable representation of the original magnitude and phase distributions

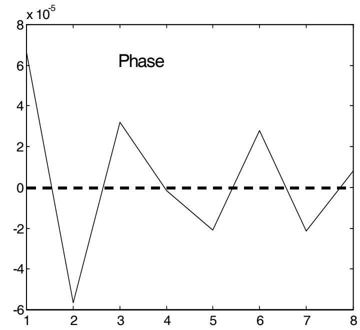

FIGURE 2. 8-point reconstruction of the amplitude and phase distributions for a uniform source.

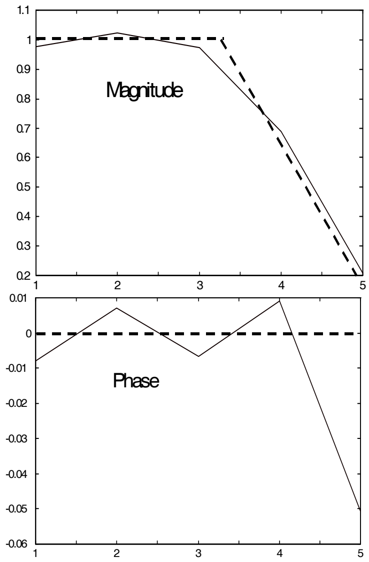

FIGURE 3. 5-point reconstruction (solid lines) of a piston-like source with a linear taper (dashed lines). a=15mm, frequency =200kHz, z =30mm, z_n = 1.

Reconstructions from measured data

A series of transverse field measurements were carried out in a PC-controlled scanning tank.

The source was a nominal 500kHz, 30mm dia. plane transducer (Dupont) and the field probe was a Precision Acoustics 0.5mm dia. PVDF Needle hydrophone and HPM05/2preamplifier combination. Various frequencies in the range 200-500kHz were used in order to produce different source distributions. Two such reconstructions are shown in Figures 4 and 5.

Figure 4 shows a 6-point reconstruction of the source when driven at 400kHz A cubic spline interpolation through the reconstructed values is used here for clarity. Figure 5 compares the results of a 25×25-point ASPW source reconstruction with an 8-point transverse (radial) reconstruction. The operating frequency was 200kHz. Although the 8-point reconstruction broadly captures the features of the source distribution, it is evidently far from perfect. The main reason for this was revealed by the full 2-D ASPW reconstruction, which shows that the surface vibration is more complicated and depends both on the radial and azimuthal co-ordinates.

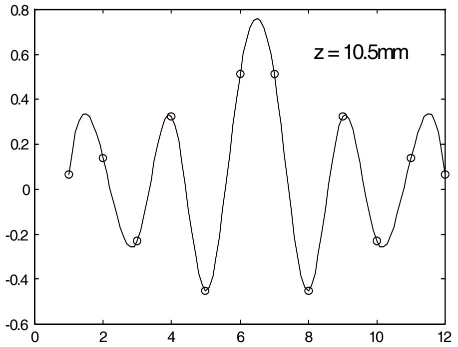

Figure 4. A 6-point radial reconstruction from data measured along a transverse line at distance z =10.5mm from the transducer's face. The operating frequency was 400kHz. The curve is a cubic spline interpolation through the measured points.

Ill-conditioning problems proved to be quite mild for these reconstructions due to the coarse discretisation employed here.

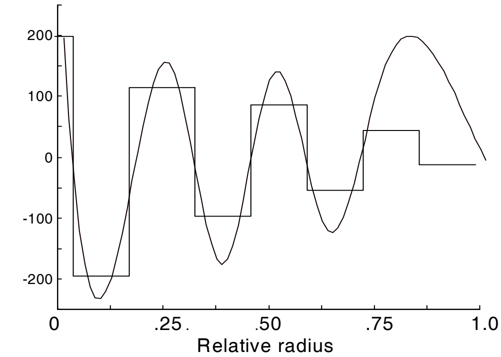

FIGURE 5. Comparison of source reconstructions for a 30mm dia. transducer, driven at 200kHz. The curve is a radial section of the back projection. The line graph is the 8 point radial reconstruction.

Two–dimensional reconstructions

This result shown in Fig.5 provides the motive for extending the investigations to non-symmetric source vibrations. In contrast to the essentially 1-D reconstructions, the number of source elements required to give an acceptable map has to be increased. This in turn suggests that ill-conditioning of the transfer matrix **Q** will now be a significant issue.

Both square and circular sources were investigated and their vibrations were approximated using 25 surface elements, requiring measurements at 25 field points.

The field expression required for the 25×25 term **Q** matrix was calculated from the fully discretised form of Rayleigh Integral, appropriate to each co-ordinate system. These take the form:

$$p(r) \approx \sum_{u=1}^{m}\sum_{v=1}^{n} \Delta S_{mn} \frac{e^{-ikr_{uv}}}{r_{uv}} \qquad (8)$$

The radiators are thus approximated by discrete point sources whose strengths are proportional to the surface element areas ΔS_{mn}. The transfer matrices **Q** in each investigation were extremely ill conditioned, but applying the Tikhonov regularization process [5] can yield acceptable approximate solutions.

As an illustration, Figure 6 compares the unregularized and a regularized 5x5-point reconstruction of a square source.

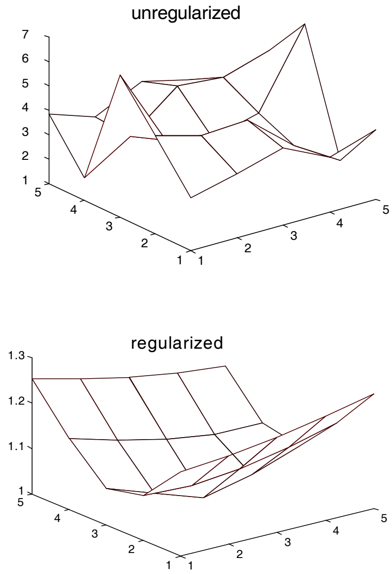

Figure 6. Comparison of the unregularized and regularised 5x5 point reconstructions for a 2λ side square source.

The regularized reconstruction deviates by ±10%, which precludes the imaging of moderate deviations from the ideal source. The reason for the relatively poor reconstruction appears to be the use of a point source approximation, rather than a uniform area elementary source.

Conclusions

The current investigations have shown that an acceptable reconstruction of a transducer's surface vibration can be achieved from a limited number of measurements when circular symmetry can be assumed. They indicate that good reconstructions can be obtained in the range $0.05\ a^2/\lambda < z < a^2/\lambda$. from the source. Other experiments (not illustrated here) have shown that the predicted radiation patterns for the few-point source reconstructions compare closely to the true source radiation patterns.
When extending these investigations to non-symmetrical distributions, it was anticipated that ill-conditioning of the system of equations (4) might limit the usefulness of the method, and this is indeed the case.

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