PARTICLE POSITIONING BY A TWO- OR THREE-DIMENSIONAL ULTRASOUND FIELD EXCITED BY SURFACE WAVES

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Abstract

The controlled positioning of small particles is still a challenge in micro technology. Acoustic forces can be used to manipulate single or few particles in a fluid. These forces act on particles in a sound field. However, existing systems are often very complicated and can only be used in special environments.

We developed a method to position and move particles in a fluid by acoustic forces. These forces arise in an ultrasound field that is excited by surface waves of a solid: A vibrating plate emits sound that is reflected at the plane surface of another body. Depending on the surface movement of the plate (one- or two-dimensional) a two- or three-dimensional standing sound field is built up between these two bodies. With this sound field it is possible to position small particles in one or two dimensions and to hold them in an equilibrium position in the vertical dimension. By changing the sound field the particles can be moved.

In experiments we use glass spheres of a diameter between 10 microns and 100 microns as particles. With a one-dimensional vibration of the plate a two-dimensional sound field is built up and the particles are concentrated in lines. The particles can be in contact with the mentioned surface or they can levitate in the fluid.

The technical setup is presented and the excitation of the vibrations is explained. A theoretical calculation of the force field that acts on the particles will be introduced. In addition several methods to use this technique to concentrate particles at predetermined positions and to move them are shown. Experimental results are presented and compared with the theoretical model.

Introduction

State of the Art, Application of Ultrasonic Forces

The most important applications of ultrasonic forces are the separation of particles from a fluid (ultrasonic filtering) and the manipulation of particles.

The general principle of ultrasonic filtering is explained in detail in [1]. Often a plane standing wave is established in a fluid. Particles are concentrated at certain locations in this wave. From these locations the particles can easily be removed.

There are many approaches for the manipulation of few or single particles. [2] introduces acoustical tweezers: two focused ultrasonic beams emitted by spherical transducers trap the particles. An almost arbitrary sound field can be generated with a plurality of transducers like mentioned in [3]. With this special sound field the particles can than be manipulated.

Overview of this Work

To position small particles a sound field is excited in a fluid by a vibrating plate and reflected by a rigid surface. Fig. 1 shows this setup. The glass plate is

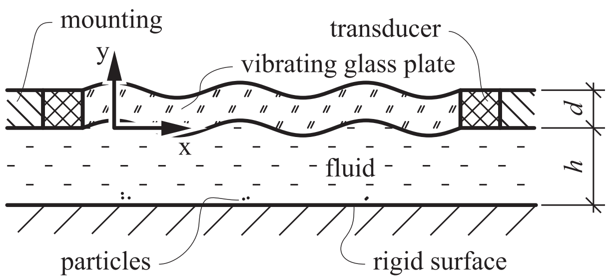

Figure 1: Schematic view of the setup.

excited to structural vibration by one or two transducers. As a result of the surface movement of this plate a sound wave is emitted into the fluid. This wave is reflected by a rigid surface so that a stationary sound field appears in the fluid. Particles that are suspended in the fluid will be concentrated according to the sound field at certain positions.

In this work first the forces that act on small particles in a sound field are addressed. Then an analytical description of the sound field is given. To know the wavelength of the plate vibration the dispersion relation of the plate with fluid layer will be deduced. After that several principles to move particles are explained and finally experimental results will be shown.

Forces on Particles in the Sound Field

There always acts a force on a particle when it is placed in a sound field. If the sound field is strong and the particles are small these forces are large enough to manipulate the particles. The formulas for the time averaged force $\langle F \rangle$ in an arbitrary sound field are very complicated. A simplification for a plane standing wave and a compressible sphere was given in [4]:

$$\langle F \rangle = \pi \rho_F \Phi^2 (k_F r_S)^3 F_Y \sin(2 k_F h) , \quad (1)$$

where k_F is the wave number of the sound wave in the fluid, r_S the radius of the sphere, h is the position of the sphere in the axis of propagation of the sound wave and Φ the amplitude of the velocity potential. F_Y is the density-compressibility factor with

$$F_Y = \frac{1}{3}\left(1 - \frac{\rho_F c_F^2}{\rho_S c_S^2}\right) + \frac{\rho_S/\rho_F - 1}{2\rho_S/\rho_F + 1}, \qquad (2)$$

where ρ_F and ρ_S are the densities of the fluid and the sphere and c_F and c_S are the speeds of sound, respectively.

The opportunity to calculate the force easily was provided by [5]. The respective force potential U is

$$U = 2\pi r_S^3 \rho_F\left(\frac{1}{3}\frac{\langle p^2\rangle}{\rho_F^2 c_F^2} f_1 - \frac{1}{2}\langle q^2\rangle f_2\right). \qquad (3)$$

$\langle p^2\rangle$ and $\langle q^2\rangle$ are the mean square fluctuations of pressure and velocity of the incident wave at the point where the particle is located, $f_1 = 1 - \rho_F c_F^2/(\rho_S c_S^2)$ and $f_2 = 2(\rho_S - \rho_F)/(2\rho_S + \rho_F)$. The force is given by the negative gradient of the force potential:

$$\langle \vec{F}\rangle = -\nabla U. \qquad (4)$$

Theory

The Sound Field in the Fluid Gap

Sound waves are emitted by the vibrating glass plate and reflected by a rigid surface (Fig. 1). The sound field in the fluid will be described by the velocity potential ϕ_F and the velocity field in the fluid gap by $\vec{v} = -\nabla\phi$. The surface movement of the vibrating plate at $y = 0$ is sinusoidal and its displacement in y-direction is given by $u_P = u_{P0}\cos(xk_P)e^{\omega it}$, where u_{P0} is the amplitude, ω the frequency and $k_P = 2\pi/\lambda_P$ the wave number. At the rigid surface there is no displacement of the fluid in y-direction: $v_y(y = -h) = 0$. The emission and reflection of sound

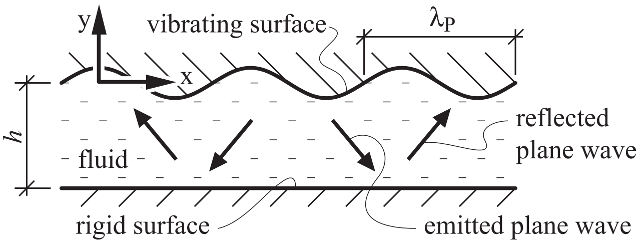

Figure 2: The sound field in the fluid gap.

waves leads to a generation and superposition of four plane waves in the fluid gap as shown in Fig. 2. The velocity potential ϕ_F is then given by

$$\phi_F = \frac{\omega u_{P0}}{k_{Fy}}\frac{e^{2hik_{Fy}}}{1 - e^{2ihk_{Fy}}} \times$$
$$(e^{iyk_{Fy}} + e^{-ik_{Fy}(y+2h)})\cos(xk_{Fx})e^{i\omega t}. \qquad (5)$$

$k_{Fx} = k_P$ and k_{Fy} are the wave numbers in the fluid in x- and y-direction, with $\sqrt{k_{Fx}^2 + k_{Fy}^2} = k_F = \omega/c_F$. If the wavelength of the plate vibration λ_P is smaller then the wavelength of sound in the fluid there is no sound emission.

When the surface movement is two-dimensional and assumed to be $u_P = u_{P0}\cos(xk_{Fx})\cos(zk_{Fz})e^{\omega it}$ then the velocity potential ϕ_F has the form

$$\phi_F = \frac{\omega u_{P0}}{k_{Fy}}\frac{e^{2hik_{Fy}}}{1 - e^{2ihk_{Fy}}} \times$$
$$(e^{iyk_{Fy}} + e^{-ik_{Fy}(y+2h)})\cos(xk_{Fx})\cos(zk_{Fz})e^{i\omega t}. \qquad (6)$$

The Force Field in the Fluid Gap

The force potential in the fluid gap is given by Eq. (3) with the velocity and pressure calculated from Eqs. (5) or (6). For a two-dimensional vibration of the glass plate, that leads to a sound field like Eq. (6), the force potential is

$$\begin{aligned} U = U_0(&\tfrac{1}{3}f_1 k_F^2\cos^2(xk_{Fx})\cos^2((h+y)k_{Fy})\cos^2(zk_{Fz}) \\ &-\tfrac{1}{2}f_2\{k_{Fx}^2\sin^2(xk_{Fx})\cos^2((h+y)k_{Fy})\cos^2(zk_{Fz}) \\ &+k_{Fy}^2\cos^2(xk_{Fx})\sin^2((h+y)k_{Fy})\cos^2(zk_{Fz}) \\ &+k_{Fz}^2\sin^2(zk_{Fz})\cos^2((h+y)k_{Fy})\cos^2(xk_{Fx})\}), \quad (7)\end{aligned}$$

where U_0 is a factor independent of space and time. The locations where the particles are collected are given by the minimum of this force potential. The points (x, y, z) where U has an extremum are at

$$xk_{Fx}\ ,\ (y+h)k_{Fy}\ ,\ zk_{Fz} = \pi m/2 \quad (m \in \mathbb{Z}). \qquad (8)$$

Whether one of these points is a minimum, a maximum or a saddle point depends on the factors f_1 and f_2 and consequently on the material properties of the particle and the fluid.

The force field in the fluid gap can be calculated with Eq. (4). Fig. 3 shows the force field on the reflecting surface at $y = -h$ for glass spheres in water.

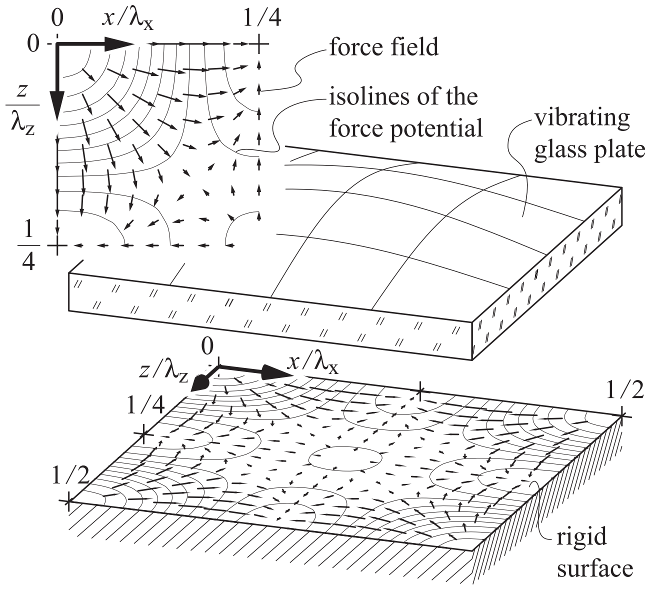

Figure 3: Force field on the reflecting surface.

Dispersion relation of the Plate and the Fluid Layer

The dispersion relation of the plate in contact with the fluid layer should be determined to know the relationship between the excitation frequency and the wavelength. Waves in an unloaded plate are described by Lamb waves. However, in our case the plate is on one side in contact with the fluid. In [6] a plate with fluid loading is considered, where the fluid has a free surface.

In our case the fluid is on one side in contact with the vibrating plate and on the other side with a rigid surface (Fig. 1). For the calculation of the dispersion relation it is assumed that the plate, the fluid and the rigid surface spread infinitely in x-direction. The displacement potential of the sound field in the fluid θ_F is similar to the velocity potential like in Eq. (5) but has an other amplitude Θ_F with other dimensions. The movement of the plate is described by the superposition of the displacement potentials of the longitudinal wave θ_P and of the transversal wave ψ_P. With the respective amplitudes Θ_s, Θ_a, Ψ_s and Ψ_a the equations are:

$$\left.\begin{aligned}\theta_F &= \frac{\Theta_F}{e^{-2hik_{Fy}}-1}\left(e^{iyk_{Fy}}+e^{-i(y+2h)k_{Fy}}\right)e^{i(\omega t - xk_P)}\\ \theta_P &= \left(\Theta_s\cosh(qy)+\Theta_a\sinh(qy)\right)e^{i(\omega t - xk_P)}\\ \psi_P &= \left(\Psi_a\cosh(sy)+\Psi_s\sinh(sy)\right)e^{i(\omega t - xk_P)}\end{aligned}\right\}\quad(9)$$

$q = \sqrt{k_P^2 - k_l^2}$ and $s = \sqrt{k_P^2 - k_t^2}$, where k_l and k_t are the wave numbers of the longitudinal and transversal speed of sound.

Further on, the system is determined by the following boundary conditions. At the upper surface of the plate at $y = d$: no normal stress $\sigma_{yy} = 0$ and no shear stress $\sigma_{xy} = 0$. At the interface of the plate and the fluid at $y = 0$: no shear stress in the plate $\sigma_{xy} = 0$, the normal stress in the plate and the negative fluid pressure are equal $\sigma_{yy} = -\rho_F\theta_{F},_{tt}$ and the surface movement is equal to fluid movement $\theta_{P},_{y} + \psi_{P},_{x} = \theta_{F},_{x}$.

The set of equations (9) and the boundary conditions lead to the characteristic equation:

$$\begin{aligned}0 = {} & [16k_P^4q^2s^2 + (k_P^2+s^2)^4 - 4qs(k_P^2+s^2)^2k_P^2 \times \\ & (\tanh(\tfrac{1}{2}ds)\coth(\tfrac{1}{2}ds) + \tanh(\tfrac{1}{2}ds)\coth(\tfrac{1}{2}ds))] \\ & + \{(\rho_F/\rho_P)(q/k_{Fy})\cos(hk_{Fy})k_t^2(k_P^2-s^2) \times \\ & (\coth(dq)(k_P^2+s^2)^2 - 4qs\coth(ds)k_P^2)\}. \quad (10)\end{aligned}$$

This equation consists of two parts. The first part (in square brackets) is the product of the characteristic equations of the symmetric and antisymmetric Lamb waves. The second part (in curly brackets) represents the influence of the fluid and leads to a coupling of the symmetric and antisymmetric waves in the plate. It is clear that there is no coupling when there is no fluid, which means that $\rho_F/\rho_P = 0$.

Displacement of Particles

In this section some principles to move particles will be introduced. The initial state is shown in Fig. 4.

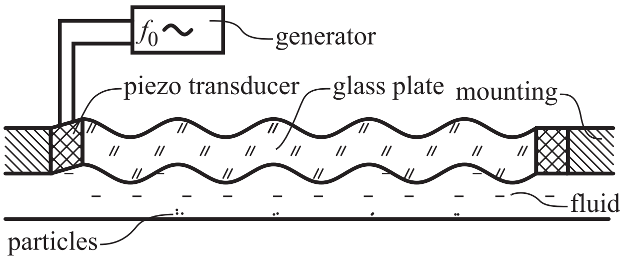

Figure 4: Initial State for displacement.

The particles are concentrated according to the sound field excited by the vibrating glass plate. An easy method to displace the particles can be seen in Fig. 5

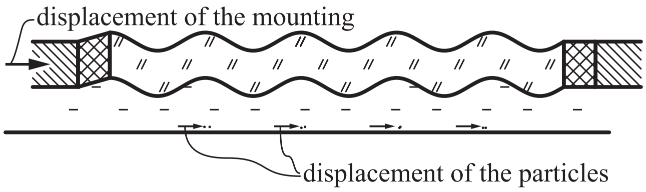

Figure 5: Displacement by moving the mounting.

where the full device is moved relative to the surface on which the particles are placed. The particles will follow that movement. A further method is to excite

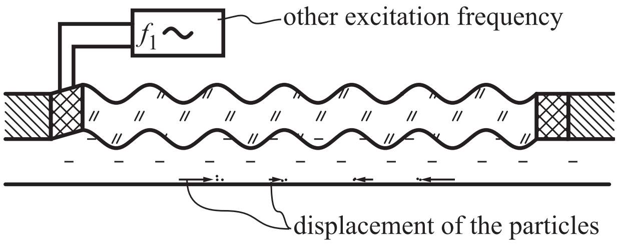

Figure 6: Displacement by changing the frequency.

the transducer with another e.g. higher frequency like in Fig. 6. In this way the particles can be moved without changing the position of the mounting. The draw-

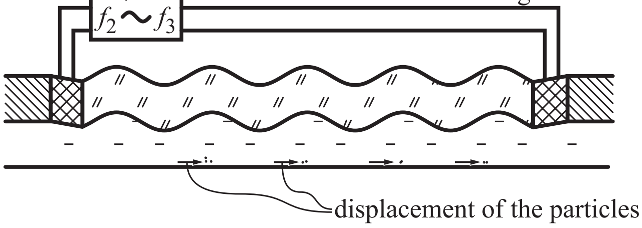

Figure 7: Displacement by modulating the vibration.

back is, that the particles can not be moved continuously, as the different vibration modes of the plate differ discretely. A third possibility lies in the modulation of the plate vibration (Fig. 7). This can be done for example by running two transducers, each on one end of the vibrating plate. The frequencies f_2 and f_3 may differ in amplitude, frequency or phase. It is than possible to move the particles continuously without moving the mounting.

Experiments

Experimental Setup

To be able to optically control the manipulated particles the vibrating plate is made of glass. On the edges of the glass plate piezo transducers are attached to excite the vibration. The glass plate and the transducers are held by a clamping. This device is placed

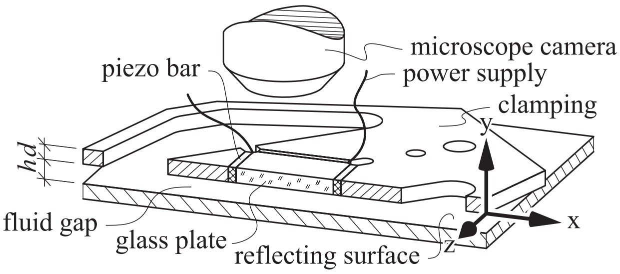

Figure 8: Experimental setup.

over a rigid surface. The particles are observed through the glass plate by a microscope camera. Fig. 8 shows a sketch of the setup. The clamping is connected to a translation stage so that it can be positioned in the x/y-plane.

Experimental Results

In this section the results for a one- and two-dimensional positioning of particles shall be introduced. In Fig. 9 the one-dimensional positioning can be seen. The left one shows glass spheres of a diameter between 10 am 100 µm which are arbitrarily dispersed on a flat surface. On the right picture the glass plate is excited to vibrations and a sound field is build up. This sound field concentrates the particles in lines.

The two dimensional case is displayed in Fig. 10. The minimum of the force potential is not any longer line-shaped like in the one-dimensional case, but the minimum lies at discrete points. In these experiments a

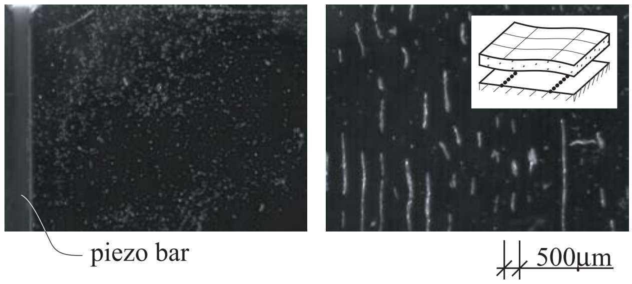

Figure 9: One-dimensional positioning of particles.

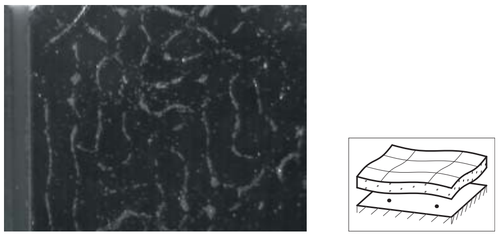

Figure 10: Two-dimensional positioning of particles.

large number of particles was used, so that in each minimum of the force potential many particles are collected.

Transaction, Proceedings or Journal Articles, Patents:

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