

## PHASE SENSITIVITY OF LOW FREQUENCY ULTRASOUND WAVE EMITTED BY NONLINEAR INTERACTION OF PHASE CONJUGATE ACOUSTIC BEAMS

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### Abstract

Phase conjugation of ultrasound waves is used for automatic overlap of nonlinearly interacting acoustic beams scattered by an object immersed in a liquid. Low combination frequency emission (LFE) by means of interaction of phase conjugate beams of frequencies 10MHz and 11MHz is studied experimentally and theoretically. While variation of the object position for interaction of co-directional waves leads to a normal phase shift of the emitted wave proportional to its frequency  $\Omega_- = \Delta\omega = \omega_1 - \omega_2$ , the anomalous phase shift proportional to  $\Omega_+ \gg \Delta\omega$  ( $\Omega_+ \cong 2\omega_{1,2}$ ) is obtained for interaction of contra-propagating waves. Application of anomalous phase shift for "super resolution" in object positioning by means of nonlinear low frequency emission is discussed.

### Introduction

Development of the supercritical parametric technique of acoustic wave phase conjugation (WPC), notable for giant amplification of phase conjugate wave (PCW) [1], stimulated the beginning of investigations in the field of nonlinear wave front reversal acoustics.

Interaction of waves with different frequencies  $\omega_1$  and  $\omega_2$  in a nonlinear acoustic medium leads to generation of sound waves of combination frequencies. Recently it was proposed to use nonlinear low frequency emission for local ultrasonic diagnostics of biological tissues [2] and for vibro-acoustic technique [3]. Utilization of focused acoustic beams increases as usual the efficiency of nonlinear interaction and improves localization of the interaction area.

Application of supercritical WPC for nonlinear low frequency ultrasound emission provides the new options such as auto-confocal overlap of the interacting beams and auto-targeting of the PCW beams onto scattering objects.

The experiments were carried out on focused acoustic beams scattered by a local object immersed in a liquid. The first results on low frequency emission (LFE) by means of nonlinear interaction of phase conjugate waves were reported in ref.[4,5]. Comparable LFE efficiency for resonance interaction of co-directional waves and non-resonance interaction of contrapropagating waves were observed. In this

paper, the influence of the size of scatterer on ratio of LFE efficiency in this two cases is studied experimentally. Application of LFE for imaging of phase profile of the scattering object is demonstrated. The mechanism of LFE for non-resonance interaction of contra-propagating phase conjugate wave is discussed. Anomalous phase sensitivity of LFE signal to the scatterer position is explained theoretically.

### Experimental Technique and Results

An incident ultrasonic wave with 10MHz frequency and 15 $\mu$ s duration was emitted in a water tank by a piezoelectric transducer of 3 cm focal distance and 10mm aperture, focused on, and scattered by, the object. The MAPC (Magneto-Acoustic Phase Conjugator) axis was turned relatively to the axis of the transducer with an angle about  $\pi/2$ . MAPC was excited parametrically by 20 MHz electromagnetic pumping in order to provide the phase conjugation of the scattered wave.

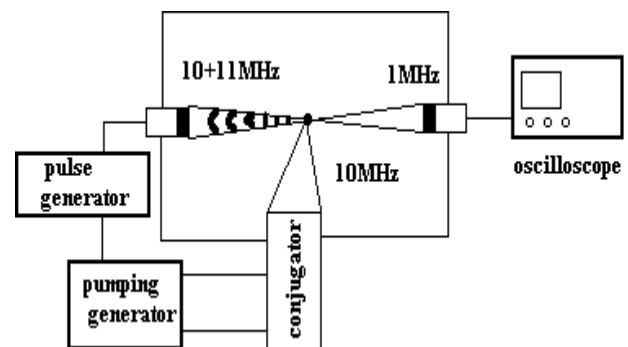


Figure 1. Experimental setup

The conjugate wave of same frequency came back, reflected by the object in the direction of the emitting transducer and interacted with a second incident wave of 11 MHz emitted with a proper delay by the transducer towards the focus. The interacting beams overlap automatically due to wave front reversal. Maximum of the LFE signal was observed when the 1 MHz receiver was placed behind the object on the axis of the transducer.

There were two kinds of nonlinear interaction observed in the described experiments. In the first one, the second wave with the frequency 11MHz was emitted by the same transducer in order to interact with the phase conjugate wave at the moment of its reflection by the object. In this case the two

interacting waves were propagating in the opposite direction. In the second case the 11MHz wave was emitted with a time delay providing interaction with the phase conjugate wave after its reflection on the surface of the transducer. In this case the two waves were propagated in the same direction.

While variation of the object position for resonance interaction of co-directional waves did not affect the phase of the emitted low frequency wave (1 MHz), an anomalous phase shift proportional to the sum of frequencies (21 MHz) was obtained for non-resonance interaction of contra-propagating waves. The efficiencies of interactions for co-directional and contra-propagating waves were comparable. The dependence of the ratio of efficiencies in this two kinds of nonlinear interactions versus the size of the object was revealed in these experiments. For detailed study of this dependence, a special object with variable diameter was fabricated from a glass conical tube. The diameter of the conical section of the tube varied from 300 $\mu\text{m}$  to 100 $\mu\text{m}$  over a 1mm length along tube axis.

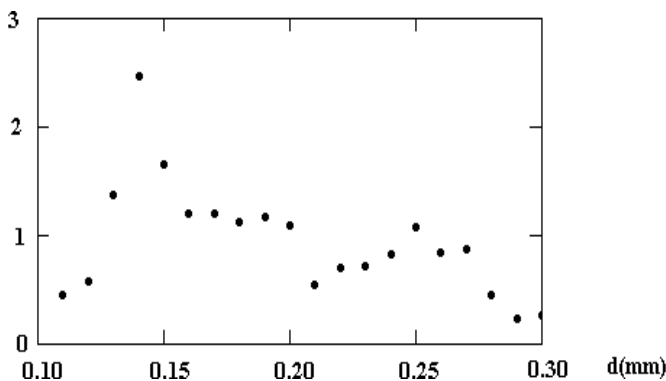
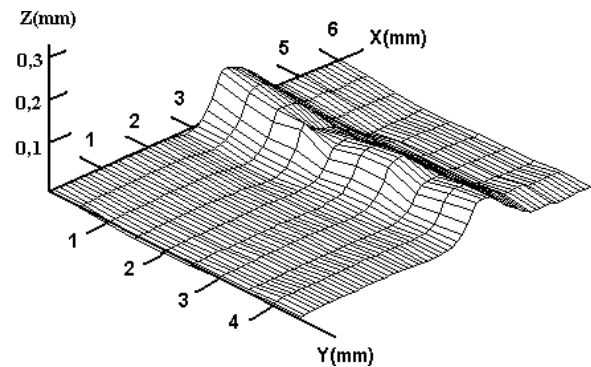


Figure2. Ratio of LFE efficiencies for contra-propagating and co-directional waves

The obtained results show that the efficiency of nonlinear interaction for co-directional waves was higher than for contra-propagating waves if size of the object was bigger than 200 $\mu\text{m}$  (Fig2.). If the size of the object is smaller than 200 $\mu\text{m}$ , the efficiency of interaction of contra-propagating waves predominates. The ratio of efficiencies of the two processes achieves maximum when the size of the object is close to 150 $\mu\text{m}$ , which corresponds to the wavelength of ultrasound wave in water at the frequency 10MHz. In this case the small size of the object provides strongly localized interaction of the scattered field with incident 11MHz wave and leads to the growth of the efficiency of interaction of the contra-propagating waves. Decrease in the efficiency of the nonlinear interaction for contra-propagating waves when the size of the scattered object is lower than 150 $\mu\text{m}$  is

caused by the decrease of scattering efficiency for the incident wave.

In the previous experiments [4,5] an anomalous phase shift proportional to the sum of frequencies was obtained for non-resonance interaction of the contra-propagating waves. Here, this effect was used to provide the image of the phase profiles of the objects. In this case the object was a strip made from thin aluminum foil bonded on 6mm x 5mm glass plate and 100 $\mu\text{m}$  of thickness.



a



b

Figure3. a - phase profile of the object; b - brightness image of the phase profile of the object

A glass plate of such thickness is transparent for 1MHz ultrasound and almost opaque for 10MHz ultrasound. The aluminum strip was 100 $\mu\text{m}$  thick and 1mm wide.

The results of experiments are shown on Fig.3. In Fig.3a z-coordinate for the space profile of the object is calculated from the expression  $z = \Delta\phi/k_+$  where  $k_+ = (\omega_1 + \omega_2)/c$ . In Fig.3b the brightness image of the phase profile of the object is shown. Irregularity of intensity of the obtained image is explained by non perfect gluing of aluminum strip and glass substrate. The phase image of this sample obtained by LFE detection in non-resonance interaction of contra-propagating waves demonstrates high phase

resolution, which corresponds to the sum of frequencies of interacted ultrasonic waves.

**Discussion**

To explain the main observed features of the LFE phenomenon lets consider interaction of two confocal beams propagating in the area between transducer and its focal plane ( $0 < z < d$ ). LFE is described at the second order of the perturbation theory by the general formula:

$$\rho^{(2)}(r_0, \theta, t) = \int_0^d \int_0^{2\pi} \int_0^\infty \frac{Q(r, z, t - R/c_0)}{R} r dr d\phi dz, \quad (1)$$

where

$$R = r_0 + \frac{r^2}{2r_0} - r \sin \theta \cos(\alpha - \phi) - z \cos \theta$$

here  $(r_0, \theta, \alpha)$  are coordinates of the observation point,  $Q(r, z, t)$  is the nonlinear emission source function equal to:

$$Q = -\frac{1}{4\pi\rho_0} [(\gamma-1)\pm 2](k_1 \mp k_2)^2 \rho_1 \rho_2^* \exp [i\Omega_2 t - i(k_1 \mp k_2)z] + c.c., \quad (2)$$

here  $\omega_{1,2}$  are the frequencies of the two interacting waves,  $\Omega_2 = \omega_1 - \omega_2$ ,  $k_{1,2}$  are the wave numbers,  $\gamma$  is the nonlinear parameter and  $\rho_0$  is the equilibrium density of the liquid. The upper and lower signs correspond to co- and contra-propagation of the interacting waves respectively,  $\rho_1 = \rho_1(t-z/c)$ . For co-directional propagation  $\rho_2$  is equal to  $\rho_2 = \rho_2(t-z/c)$  and for contra-propagation is equal to  $\rho_2 = \rho_2[t+(z-d)/c]$ . In the latter (non-resonance) case the amplitude of the nonlinear source  $Q$  is much higher than for the resonance case due to  $(k_1+k_2)^2 \gg (k_1-k_2)^2$ . This feature explains comparable efficiency of the resonance and non-resonance LFE observed in our experiment. The result of integration of the equation (1) for contra-propagating waves is equal to:

$$\rho^{(2)}(r_0, \theta, t) = \frac{(\gamma-3)(k_s a)^2 \exp[i(\Omega_2 t - k_d r_0)]}{16\rho_0^2 r_0} \times \frac{\exp[i(k_s - k_d \cos\theta)d]}{(k_s - k_d \cos\theta)} \times \rho_1(t - \frac{r_0}{c_0}) \left\{ g(\xi) \otimes \rho_2(t - \frac{r_0-d}{c_0}) \right\} \quad (3)$$

where  $a$  is the transducer aperture,  $k_d = k_1 - k_2$  and  $k_s = k_1 + k_2$ . The function  $g(\xi)$  reflecting effect of focusing of the beams is equal to:

$$g(\xi) = e^{j\xi} / \left( 1 + j \left( \frac{a}{2d} \right)^2 \xi \right). \quad (4)$$

For our experiments, when  $(a/2d)^2 = 0.007$ , the numerical analysis of the function  $g(\xi)$  show that the main contribution to the convolution integral (4) is given by the area near  $\xi=0$ . As a result the shape of time envelope of the convolution function reproduces the envelop of the wave  $\rho_2(t)$ . It means that the amplitude of LFE signal is proportional to the product of the envelopes of the interacting waves in accordance with the experimental observations [4,5].

The phase factor  $\varphi = (k_s - k_d \cos\theta)d$  provides a high sensitivity of LFE phase to the object position coincident with  $d$  in our calculations. For direction  $\theta=0$  the phase factor is equal to  $\varphi = 2k_d d = 2(\omega_2/c)d$ . This result is in a good quantitative agreement with our measurements. In spite of low frequency of the detected wave  $\Omega_- = \omega_1 - \omega_2$  the phase shift is proportional to high frequency  $2\omega_2$  that explains anomalous sensitivity of phase to the object position. Scattering of the phase conjugate wave on the small object of size comparable with the wave length increases the ratio of non-resonance to resonance LFE efficiencies due to better localization of the nonlinear interaction area.

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