FORMATION AND TRANSPORTATION OF HEAP IN AN INCLINED AND VERTICALLY VIBRATED CONTAINER

Guoqing Miao*, **Kai Huang, Yi Yun, and Rongjue Wei**
State Key Laboratory of Modern Acoustics and Institute of Acoustics, Nanjing University, Nanjing, P. R. China
*Email: miaogq@nju.edu.cn

Abstract

An experimental study is performed on formation and transportation of heap in granular materials in an inclined and vertically vibrated container. A relationship is presented of how the velocity of heap changing with driving acceleration and frequency. The shape of the heap bottom is measured by detecting the colliding phase of the heap bottom with the floor of container. An analogous experiment is performed with a heap-shaped Plexiglas block. Pressure gradient of ambient gas plays a crucial role in generating and maintaining a heap. Ratchet effect causes transportation of heap.

It is well known that under vibration many processes, e. g. segregation, convection, heaping, etc. [1], which govern the physics of granular materials, are quite unusual, so that the properties of such materials are not well understood. For example, heaping is one of long-standing problem since Faraday. Several physical mechanisms have been identified as possible causes of heaping: Friction between the walls and particles [2], analog of acoustic streaming if the shaking is nonuniform [3], and gas pressure effect [4], etc. [5]

Recently we have observed experimentally the formation of a heap and its transportation from the lower to the higher end in an inclined and vertically vibrated container. The experiments are conducted in a Plexiglas rectangular container [370 mm (length) $\times$25 mm (width) $\times$80 mm (height)]. We use two kinds of quartz sands: Spheres of diameter 0.15-0.20 mm and grains of irregular shape or coarse surface of diameter 0.3-0.5 mm. The container is inclined with the inclination α from 2.0° to 5.5° by putting a pad underneath the container (shown in Fig.1). The vibration exciter (Brüel and Kjær 4809) is driven by a sinusoidal signal, and controlled by the vibration exciter control type 1050. Driving frequency f and dimensionless acceleration amplitude $\Gamma = 4\pi^2 f^2 A/g$ (where A is driving amplitude and g the gravitational acceleration) are used as two control parameters.

Experiments show that even in the horizontal container the horizontal acceleration can cause movement of the heap. The horizontal acceleration of our exciter is 3.5% of the vertical acceleration. To eliminate the influence of the horizontal component of the driving acceleration on the movement of the heap along the length direction of the container, we adjust the direction of the container orthogonal to the horizontal component of acceleration of the exciter. The range of Γ we use is from 1.5 to 2.5, which is a good range for heap formation. At first, about 80 ml of sands are uniformly put into the lower part of the vessel. Then as Γ increased to and beyond some critical acceleration Γ_c, heap will form, in the meanwhile it moves upward the container. Fig.1(a) and (b) are the typical photos of heaps. In both of them the back surface is longer than the front one, while the slopes of both surface relative to horizontal are the same. The differences between two heaps are: The dynamical angle of repose of heap formed by coarse sands is larger than that formed by spherical sands; The heap formed by coarse sands completely separated from the end walls, i. e. this heap has nothing to do with the end walls completely, while the heap formed by spherical sands has a long and very thin tail behind it, and connected to the back end wall.

The heaps move upward the container with a nearly uniform velocity. We measure the velocity for four different frequencies. The result is shown in Fig.2. One can see that the velocities of both heaps increase with the driving acceleration for all of frequencies, but decrease as driving frequency increases for every value of acceleration (when $2.1 < \Gamma < 2.5$, the velocities of heap formed by spherical sands at $f = 13Hz$ is larger than that at $f = 11Hz$, this may be resulted from measurement error). The velocity of heap formed by coarse sands is greater than that of heap formed by spherical sands. For example, at most situations of $\Gamma < 1.7$ the velocity of heap formed by spherical particles is below 0.05 cm/s, while that of heap formed by coarse particles is all larger than that value. And at 11 Hz the velocities of heap formed by coarse sands are much larger than that of heap formed by spherical sands.

We measure the shape of the heap's bottom by detecting the colliding phase of different parts of the heap's bottom with the container. A pressure transducer is mounted in the middle of the floor of container, and an oscilloscope (Agilent 54810A) is used to acquire pressure signals. In our driving parameter range, heap collides with the floor of the container once a cycle [6]. Then as heap moves upward across the transducer, the collision phases of the specified points at the heap's bottom with the transducer relative to that of the reference signal (exciting signal) determine the shape of the bottom of the heap. Fig.1(c)(d) are the shapes of the heap's bottom drawing according to the phase difference [note that the divisions of H axes are $10^{-4} m/\mathrm{div}$ in (c) and

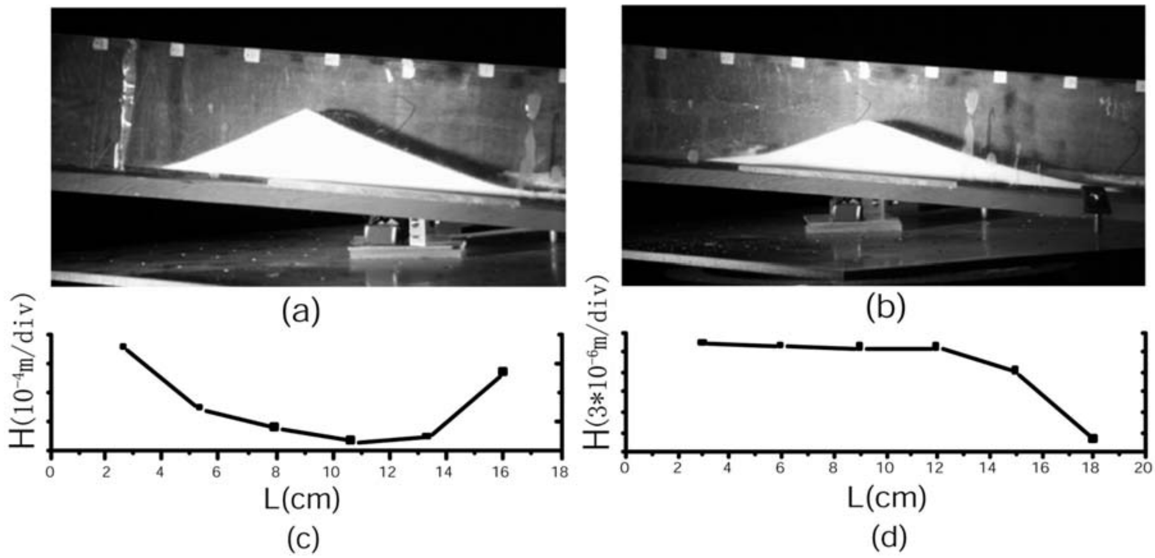

Figure 1: Heap formed by (a) coarse sands and (b) spherical sands. (c) The shape of heap bottom in (a) (amplified in vertical direction) according to the colliding phase of bottom with the floor of container. (d) The shape of heap bottom in (b). L indicates location of the heap bottom.

.

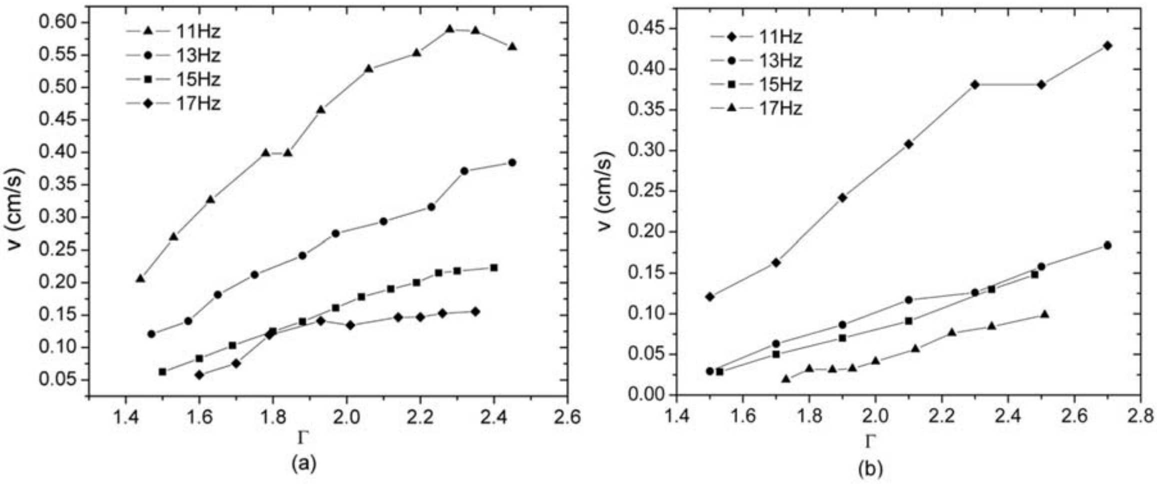

Figure 2: The heap transport velocity v as a function of dimensionless acceleration Γ for different frequencies. (a) is the result of coarse sands. (b) is for spherical sands. Inclination angles of both experiments are $\alpha = 2.6°$.

$3 \times 10^{-6} m/\mathrm{div}$ in (d)]. We can see that the bottom of heap formed by coarse sands is a convex, while that by spherical sands is concave.

If we pump the air out of the container, as pressure is decreased, heap reaches a flat state gradually; If there is no heap initially, never can any heap generate. This shows that ambient gas plays a important role in generating and maintaining a heap.

Obviously, the transport of the heap is a collective behavior of the granular materials. To examine this idea we put a Plexiglas block of the same in shape and dimension as the sand heap on the same vibrated inclined plane, a similar transport of the block upward the inclined container is observed. So we consider that the transport of the heap is similar to that of a solid block. But why does it move, or what is the mechanism of the transport? We use a high speed camera (Redlade Motion Scope PCI 2000SC) to record the movement of the block as it moves upward the container with record rate of 250 fps (frames per second), then play back slowly (e. g. 25 fps). In this way, we can see the detail of the movement of the block. In each cycle, when $\Gamma \cos 2\pi f t < -g$, block separates with container, and flies until collision with the container. We mark the mass center of the block with a black point, and observed a forward ballistic trajectory of the center of mass. Upon colliding with floor, block moves together with the container in vertical direction until next separation with container. The friction force between the block and the floor of container is large enough so that sliding down is small, and there is a net nonzero displacement upward the container. Therefore the block moves one step upwards the container in each cycle. Next, to examine the effect of air, we evacuate the air from the container and observe no transport at all. This shows that air plays a critical role for the movement of the block upward the container. In the period of free flight, the pressure in the gap is less than that at the outside. The total force resulted from the pressure difference between the gap and the outside perpendicular to the bottom surface of the block and, together with gravitation force, forces the block to move along a forward ballistic orbit. The analysis above shows that the transport of block upward container is a typical ratchet effect, which is caused by the pressure difference between gap and outside of block and the friction force between block and floor of the container.

For the granular bed, if for any reason, some small initial heap formed. In the period of free flight, partial vacuum in the gap leads to a pressure gradient (or force) in the interior of the heap. The horizontal component of the force (we call it as a cohesion force) enhances the heap. The total force is perpendicular to the bottom of the heap and, together with gravitational force, forces the heap as a whole to move upward the container. Like block, the transport of heap upward container is also a typical ratchet effect, which is caused by the pressure gradient and the friction force between heap and floor of the container.

Now we investigate further the evolution of the state of heap. During the period of free flight, we consider the granular bed as a weightless fluid [this is suitable especially to the case in Fig.1(a). For Fig.1(b), because it has a tail connected to back end wall, we elucidate it in the following]. In a reference frame moving with an acceleration due to the total force of pressure gradient and gravitational force, heap still encounters a cohesion force, i. e. the force exerted on the part below front surface points right, while that below the back surface points left. We describe this weightless fluid with two-dimensional Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \bigtriangledown)\mathbf{u} = -\bigtriangledown \frac{p}{\rho} + \nu \bigtriangledown^2 \mathbf{u} - \mathbf{a}. \qquad (1)$$

and continuity equation

$$\bigtriangledown \cdot \mathbf{u} = 0. \qquad (2)$$

where $\mathbf{u}$ is the velocity of the fluid, p the pressure fluctuation, ρ the mean mass density of the granular bed (which is assumed as constant), and $\mathbf{a}$ the acceleration caused by the horizontal component of the pressure gradient. ν is named kinematic viscosity coefficient. The initial shape of the weightless fluid is taken as that of the heap in Fig.1. All surface are free. Fig.3 is the velocity distribution of the fluid obtained from the numerical solution of the equations (1) and (2). It is shown that heap is

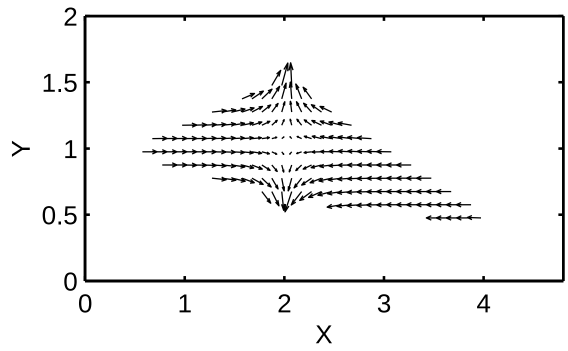

Figure 3: The velocity distribution of the weightless fluid in a reference frame moving with an acceleration due to the total force of pressure gradient and gravitational force [obtained from the numerical solution of the equations (1) and (2)].

"compressed" in horizontal direction while elongated in vertical direction. The bottom of the heap is convex [this is just the case of Fig.1(a)]. During the period of bed-floor collision, the center part of the bottom touch floor first, then the other parts touch floor consecutively from center to outer. This results in a further enhancement of the heap. Then a new cycle begins. In this way heap becomes bigger and bigger, and its slope becomes larger and larger. When the slope angle reaches and exceeds dynamic angle of repose, the avalanche occurs in the period of heap-floor collision. So when the heap reaches a steady state, in the laboratory reference frame, one can see a steady convection flow: In the center part, grains move upward, while at the surface grains move rapidly downward. In Fig.1(b), the friction force of the back end wall of the container exerted on the heap's tail makes the bottom of the heap concave as in ref. [7].

Now we describe in detail the results in Fig. 2 in terms of ratchet effect. We denote β be the ratio of free-flight time and the period T of excitation, then in each cycle in the period of free flight, heap flies upward the container. The distance of free flight is $s_1 = \frac{1}{2}a_1\beta^2T^2\cos\alpha$, where a_1 is the acceleration due to the horizontal component of total force caused by pressure gradient. If we denote total force with F, then $a_1 \sim F\sin\alpha$, and $s_1 \sim \frac{1}{4}F\beta^2T^2\sin 2\alpha$. In period of bed-floor collision, heap slides downward the container. The distance of sliding is $s_2 = \frac{1}{2}a_2(1-\beta)^2T^2$, where a_2 is the acceleration downward the container, which is determined by the friction between heap and floor, inclination α of the container and gravitation g through $a_2 = g\sin\alpha - \mu g\cos\alpha$, where μ is the friction coefficient between heap and floor. The total (or net) displacement of heap upward the container in each cycle is $s = s_1 - s_2$, timing driving frequency f gives the velocity v of heap upward the container, i. e. $v = sf$. The ratio β increases with driving amplitude $A = \Gamma g/4\pi^2 f^2$ [6], i. e. v increases as Γ, but decreases as f increases. The larger the μ, the smaller the s_2, and the lager the v. Because the friction between coarse sands and floor of the container is larger than that between spherical sands and floor of the container, the transport velocity of heap formed by coarse sands is larger than that of heap formed by spherical sands.

This model also suitable for the heap formed in a horizontal container, where the horizontal force plays the same role (enhance the heap) as that in a inclined one. The total force caused by pressure gradient is parallel to gravitational force, no force forces the heap to move in horizontal direction.

Our conclusion is: Pressure gradient, which makes cohesionless granular materials cohesive, plays a crucial role in generating and maintaining a heap in vibrating granular materials; Pressure gradient plus friction force between heap and floor of container, which lead to a ratchet effect, are unique cause for transportation of heap upward an inclined container.

This work was supported by the special Funds for Major State Basic Research Projects and National Natural Science Foundation of China through Grant No. 10074032.

References

[1] H. M. Jaeger, S. R. Nagel and R. P. Behringer, ”The physics of granular materials”, Physics Today, vol. 49 (4), pp. 32-38, 1996.

[2] E. Clément, J. Duran, and J. Rajchenbach, ”Experimental study of heaping in a two-dimensional “sandpile””, Phys. Rev. Lett., vol. 69, pp. 1189-1192, 1992.

[3] S. B. Savage, ”Streaming motions in a bed of vibrational fluidized dry granular material”, J. Fluid Mech., vol. 194, pp. 457-478, 1988.

[4] H. K. Pak, E. Van Doorn, and R. P. Behringer, ”Effects of ambient gases on granular materials under vertical vibration”, Phys. Rev. Lett., vol. 74, pp. 4643-4646, 1995.

[5] Weizhong Chen, Rongjue Wei, Benren Wang, ”Formation mechanism of the soliton-shape heap and convection in granular materials under vibration”, Phys. Lett. A, vol. 228, pp. 321-328, 1997.

[6] Guoqing Miao, Lei Sui, and Rongjue Wei, ”Dissipative properties and scaling law for a layer of granular material on a vibrating plate”, Phys. Rev. E, 63, pp. 031304-1-031304-3, 2001.

[7] Banko Thomas and arthur M. Squires, ”Support for Faraday’s view of circulation in a fine-powder Chladni heap”, Phys. Rev. Lett., 81, pp. 574-577, 1998.