ACOUSTIC WAVE - CRACK INTERACTION: MECHANISMS OF NONLINEAR ELASTIC AND INELASIC DYNAMICS AT DIFFERENT TIME-SCALES

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Abstract

In the present communication the complementary elastic and inelastic effects in acoustic wave-crack interactions are discussed from a unified point of view. Experimental results obtained for samples containing individual cracks are presented. The data provide instructive examples of the important role of crack-induced variations in the sample elasticity as well as in elastic-wave dissipation, both of them being pronouncedly sensitive even to low acoustic strains ($e \le 10^{-6}..10^{-5}$). Difficulties in conventionally used interpretations of such observations are pointed out. It is argued that just a few rather general geometrical crack features do essentially determine the character of these phenomena including both "instantaneous" and slow dynamics effects. In particular, a thermo-elastic mechanism allowing for a unified and consistent interpretation of the observed variety of the effects is considered. Possible diagnostic applications of the microstructure-induced nonlinear effects are pointed out.

Introduction: main experimental facts and conventional interpretations.

There is general consensus that cracks do significantly affect acoustic properties of solids, especially their nonlinearity. In this aspect, one may mention the following important distinctions of crack containing materials from perfect crystals and homogeneous amorphous solids:

- Highly-increased level of elastic nonlinearity
- « Non-classical » character of the nonlinearity (non-monotonous amplitude dependences, fractional powers, etc.)
- Increased dissipation for elastic waves.
- Pronounced pressure-dependence of the dissipation.
- Slow dynamics effects (memory of the precedent acoustic activation, log-time behavior…)

When the density of the cracks is significant the linear elastic moduli of the material can be affected (reduced) quite noticeably, however, at small crack density the aforementioned nonlinear elastic and dissipative manifestations can be very pronounced, whereas the complementary variations in the linear elastic moduli still remain hardly noticeable. In general it is quite clear that all of the mentioned manifestations are more or less directly related to high compliance of the crack-like defects. However, conventional interpretations often leave open essential problems which are briefly outlined below.

Elasticity: high compliance of cracks and their sensitivity to very low strains.

One often argues that, for the elastic stress, cracks may be considered as nonlinear "diode-type" inclusions in the near-perfectly linear elastic matrix. Indeed, under tensile stress, crack interfaces can be relatively easily separated. In contrast, under compressing stress the crack closes, so that the material response to the compression is practically the same as for an intact solid. Such a representation, although sometimes useful, is rather over-simplified. In particular, an open question remains how average elastic strains as low as 10^{-6} and even less may noticeably affect the material properties, which implies essential change in the state of the cracks. Various crack models consistently yield the "rule of thumb"[1] according to which a crack may be completely closed by an average strain roughly equal to crack's aspect ratio *d/L*, where *d* and *L* are characteristic crack opening and diameter, respectively. Typical aspect ratios for cracks (e.g. in rocks) are 10^{-3} -10^{-4}. In the pressure-dependence of the elastic wave velocities in rocks this fact results in initially pronounced increase of the wave velocities under increasing static pressure, which indicates gradual closure of the cracks. Then, at average static strain in the material ~ 10^{-3}, the rate of the velocity variation strongly decreases. At such strains the elastic moduli practically reach saturated values indicating that most of the cracks are already closed (see e.g. [2]). Application of a relatively intensive oscillating strains also causes period-averaged variation in the material elasticity, which may be conveniently monitored via observation of shift of the resonance frequency for another small-amplitude probe wave (see e.g. [3], where the data are reported for a sandstone very similar to samples discussed in [2]). Importantly that quite a noticeable variation in the elastic moduli is observed in such experiments for low strains $e \sim 10^{-6}..10^{-5}$. From the point of view of such conventional models as elliptical cracks or penny-like cuts this should be attributed to closure of cracks with aspect ratios $\sim 10^{-6}$. However, for typical crack sizes of millimeter- and submillimeter-scale, such small aspect ratios imply essentially sub-atomic separation of crack interfaces, whereas the minimal separation is physically limited to the atomic scale. This apparent paradox indicates that the aforementioned popular crack models are not sufficient in this case.

Dissipation: adhesional/frictional hysteresis and "sub-atomic friction".

Concerning the problem of dissipation in crack-containing solids, models assuming the dominant role of frictional/adhesional hysteresis at crack interfaces have been developed over 40 years, especially in application to geophysics (see, for example [4,5]). Such models had evident positive features since they yielded reasonable estimates of the dissipation in rocks using friction coefficients typical for macroscopic experiments and assuming verisimilar crack densities. Furthermore, the frictional mechanism suggested a way to account for nearly-constant Q-factor typical for many rocks in a wide frequency band. Amplitude-dependent attenuation could be also incorporated in such models. However, in the framework of such models the problem of explanation of low-amplitude attenuation remains open. The following instructive citation from [4] indicates that this difficulty was understood quite long ago: *"...For the range of strains used in our experiments an upper limit of interface displacement ranges from* 10^{-12} *cm in the low strain amplitude experiments... The displacements are so small that the friction characteristics of the interfaces should be quite different from what would be observed in a macroscopic friction experiment...".* Recent data of nano-tribology based on AFM-technique have directly demonstrated that frictional slip is essentially threshold-type phenomenon and inherently implies minimal atomic-scale displacements at the contact interface (see e.g. [6]). Nevertheless, often the estimates based of the assumption of frictional/adhesional dissipation still are applied to small-amplitude range with far subatomic displacements at the crack interfaces. However, comparable magnitudes of Q-factor for small and higher-amplitude waves indicate existence of some other, threshold-less dissipation mechanism.

Thermoelatic threshold-less mechanism of dissipation: its advantages and drawbacks.

In parallel to mechanisms based on frictional/adhesional hysteretic losses a threshold-less thermoelastic mechanism of dissipation in imperfect solids was proposed by Savage quite long ago [7]. Since for homogeneous solids thermoelastic losses are very low, by analogy, they are often neglected for imperfect materials as well. However, presence of cracks introduces an additional scale into the distribution of temperature variations in the elastic wave field, which may be essentially smaller than the elastic wave length. Thus the temperature gradients could be strongly increased, which results in strong increase of the thermoelastic attenuation. The frequency dependence of the dissipated amount of energy at a crack exhibits a relaxation character with a maximum occurring when temperature wave length is comparable with the crack diameter. Savage [7] has shown that reasonable amount of cracks may account for observed dissipation in seismic frequency range. Assuming a wide distribution of cracks over their size, a nearly constant Q-factor could be predicted for a rather wide band of seismic frequencies. However, since the amount of energy dissipated by a crack decreases rapidly with the crack size decrease (by a cubic law for the relaxation maximum), in order to expand the nearly constant Q up to kHz and ultrasonic frequency range, existence of very high (apparently unrealistic) densities of tiny micrometer-scale cracks should be assumed. Besides, amplitude-dependent dissipation, which is quite pronounced for the strain-range $e \sim 10^{-6}$, could not be readily incorporated in this mechanism since it seems quite improbable that such low strains may strongly affect the crack density. ...Below a possible way to resolve the formulated difficulties is outlined. The approach comprises not only the aforementioned "instantaneous" response of crack-containing solids, but the so-called slow-dynamics effects recently found for crack containing solids subjected to acoustic activation.

Main structural features of crack-like defects and their physical consequences.

For the further discussion it is essential to remind that cracks can be considered as planar objects with characteristic diameter L and opening d, their aspect ratio being normally very small, $d/L << 1$. In order to completely close a crack, it is enough to produce in the material an average strain roughly equal to the crack's aspect ratio, and this statement does not depend strongly on a particular crack model [1]. Usually this strain is rather small, $d/L \sim 10^{-3}..10^{-4}$, but nevertheless significantly larger than typical acoustic strains, $e \sim 10^{-5}..10^{-6}$, for which the aforementioned pronounced nonlinear elastic and dissipative effects were observed.

Further important statement is based on numerous direct images of crack-like defects in rocks and damaged solids, obtained by the methods of electron-, acoustic-, and atomic force microscopy, which indicate rather complex, wavy or zigzag interface shapes even for micrometer-scale cracks. These wavy interfaces are normally separated not only in the normal direction, but are laterally shifted, so that interface contacts are produced. Due to the wavy character of the contacting surfaces the cracks often have elongated (strip-like) inner contacts, rather than point like ones, as schematically shown in Fig.1 [8].

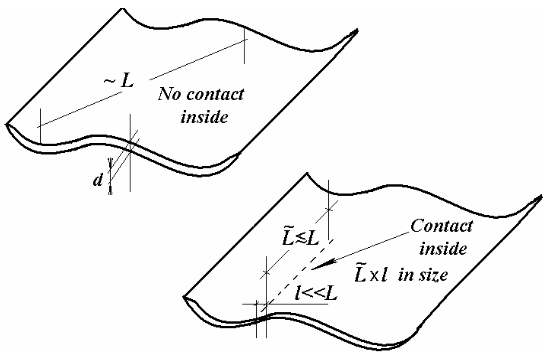

Fig. 1. A crack without and with an inner contact is shown here schematically. At $\tilde{L} \to l$ a strip-like contact reduces to a point-like contact.

Importantly that at these regions, local separation (or inter-penetration) $\tilde{d}$ of crack interfaces is much smaller than average separation d. Such contacts are especially stress-sensitive. Indeed, due to the described geometry they are strongly perturbed by the average strain, which can be orders of magnitude smaller (roughly $d/\tilde{d} >> 1$ times) than the typical magnitude $e \sim d/L \sim 10^{-3}..10^{-4}$ required to close the whole crack, in particular average strains $e \sim 10^{-5} - 10^{-6}$ can readily open or close such a contact. Another crucial point consists in the fact that defects of a material structure are regions of very effective energy dissipation even for elastic waves whose length is much greater than the crack size L. As mentioned above, this dissipation is usually attributed to friction or adhesion hysteresis at crack interfaces. However, in order to activate the adhesional/frictional losses, mutual displacement at interfaces should exceed the atomic size a. In this context, for a crack with diameter L, the average compressional or shear strain e can produce maximal lateral or normal interfacial displacement $\Delta \sim eL$ [8-10]. This estimate does not depend on the details of the crack model and agrees with the above statement that compressional strain $e \sim d/L$ produces $\Delta \approx d$, thus closing the crack completely. On the other hand, the requirement $\Delta > a$ determines the threshold strain $e_{th} > a/L$, below which the interfacial displacement is of sub-atomic scale. For a typical atomic size $a \sim 3 \cdot 10^{-10}$ m and a macroscopic crack with $L \sim 10^{-3}$ m, this yields $e_{th} \sim 0.3 \cdot 10^{-6}$, which should be exceeded in order to activate frictional and adhesional hysteretic losses. However, even at much smaller strains, the defects can efficiently dissipate elastic energy due to locally enhanced thermoelastic coupling. Indeed, near inhomogeneities, unlike the case of a homogeneous material, wave-induced temperature gradients are determined [11] not by the elastic wave length, but by the much smaller defect size L and the temperature wavelength d. When scales L and d coincide, the "global" (that is over the whole crack as considered by Savage [7]) losses per cycle reach their maximum, which is rigorously analyzed in [7] using the elliptical crack model. Alternatively, without specifying the crack model in detail, in order to estimate temperature gradients and the respective losses in the crack vicinity, one may use the approximate approach known for polycrystals [11]. In doing so simplified asymptotic expressions can be derived for the losses per cycle in the low-frequency limit (when $L << d$), the high-frequency limit ($L >> d$) and at the relaxation maximum (when $L \sim d$). With an accuracy of a factor of 2-3 these expressions are [8]:

$$W_{LF}^{dis} = 2pwT(a^2K^2/k)L^5e^2, \quad (1)$$

$$W_{HF}^{dis} = 2pT(aK/rC)^2[k/(rCw)]^{1/2}L^2e^2, \quad (2)$$

$$W_{crack}^{\max} = 2pT(a^2K^2/rC)L^3e^2, \; w = w_L \approx k/(rCL^2), \quad (3)$$

where w is the wave cyclic frequency, T is the temperature, a is the temperature expansion coefficient of the solid, K is the bulk elastic modulus; r is the density; C is the specific heat, e is the average strain, k is the thermal conductivity, and w_L is the relaxation frequency for defect scale L. For example, for $L \sim 10^{-3}$ m the relaxation frequency w_L falls between 10^{-1} -1 cycle/s for most rocks and metals. In the calculation of the low-frequency losses by analogy with [11] we took into account that crack size L is the characteristic scale of the transition from zero stress at the free interfaces to the applied average stress s. The derivation of the high-frequency expression took into account that different particular crack models consistently predict that at average applied stress s, the near-tip stress concentration has a universal form $s_{tip} \sim s/\sqrt{r/L}$ [12] (distance r is counted from the tip), and it is just this region which gives the main contribution to the high-frequency dissipation. The validity of the approximate expressions obtained is supported by the good agreement with rigorous analysis [7].

Analogous estimates for the dissipation at the inner contact of the crack take into account that the external applied stress is distributed between the arc crack-stiffness and the contact stiffness. The scale of the localization in the depth direction of the near-contact stress is roughly equal to the contact width $l << L$ [13]. At contacts that are soft (compared to the arc crack stiffness) the corresponding magnitude of the near-contact stress, s_c, is readily shown to be $s_c \sim s(L/l)$. These stress-distribution features, which do not depend on details of the crack and contact models, suffice for the estimations of the respective thermoelastic losses, which are similar to Eqs. (1)-(3), although the high frequency asymptotic dependence of the dissipation for contacts is w^{-1} instead of $w^{-1/2}$ for cracks:

$$W_{LF}^{dis} = 2pw\,T(a^2K^2/k)l^2\tilde{L}L^2e^2 \,, \tag{4}$$

$$W_{HF}^{dis} = (2p/w)kT(aK/Cr)^2\tilde{L}(L/l)^2e^2 \,, \tag{5}$$

$$W_{cont}^{\max} = 2pT(a^2K^2/rC)\tilde{L}L^2e^2 \,,\; w = w_l \approx k/(rCl^2) \tag{6}$$

Comparison of Eqs. (3) and (6) indicates the striking result that, for strip-like contacts with $\tilde{L} \sim L$, the maximum losses at the whole crack and at the small inner contact have the same magnitude, whereas the relaxation frequency for narrow, $l << L$, contacts can be 4-6 orders of magnitude higher and reaches the kHz or even the MHz band.

Consequences for the small-amplitude dissipation.

The obtained results indicate that the widely accepted opinion as to the low importance of thermoelastic coupling for seismic wave attenuation requires essential revision. Indeed, Eqs.(1)-(6) demonstrate that even a single crack with a few soft inner contacts can contribute to a weakly frequency dependent quality-factor in a very wide frequency range. Comparison of Eqs. (3) and (6) indicates that even a single inner contact of width l in a larger crack of size L produces the same dissipation at higher frequencies as a huge number (thousands and millions) of tiny cracks of size l. For example, for a reasonable ratio $L/l \sim 10^2$, even a point-like single contact with $\tilde{L} \sim l$ is equivalent to $(L/l)^2 \sim 10^4$ cracks, and a strip-like contact with $\tilde{L} \sim L$ can dissipate the same energy as $(L/l)^3 \sim 10^6$ small cracks. If, for example, a mm-scale crack contains several strip-like inner contacts, then according to Eqs.(1)-(6) such a crack may exhibit several overlapping relaxation maxima of comparable magnitude in the range from Hz- to ultrasonic frequencies. Further, taking into account crack and contact distribution over their size and realistic crack densities one may readily obtain in the conventional way [7] reasonable estimates of the magnitude of the resultant near-constant Q-factor. Such estimates indicate that, in contrast to widely accepted opinion based only on the "global" mechanism of the thermoelastic losses, they are not negligible at all from seismic to ultrasonic frequencies and in the low-strain amplitude range ($e \le 10^{-7} - 10^{-9}$) instead may strongly dominate.

Possibility of nonlinear effects at low strains.

…Another essential inference comes from the fact that quite moderate average strain, say $e \sim 10^{-5} - 10^{-6}$, which is too small to perturb the crack as a whole, can strongly perturb sizes l and $\tilde{L}$ of soft inner contacts. According to Eqs. (4)-(6) this may have a pronounced effect on the dissipation of a weaker probe wave, though neither adhesion-hysteresic, nor frictional losses are important for such a weak wave. In contrast, the complementary variation in material elastic moduli may remain very small, since the stiffness of such contacts is very low. In particular in crack-containing solids, favorable conditions should occur for the direct elastic-wave analogue of the so-called Luxemburg-Gorky (LG) effect, which represents one of the pioneering observations in nonlinear wave interactions [14]. It consists of the transfer of the modulation from the radiation of a powerful radio-station (originally, Luxemburg and Gorky-city stations) to another carrier wave. This cross-modulation is caused by variations in the absorption of the ionosphere plasma, which are induced by the amplitude-modulated stronger wave at frequencies on the same scale as the modulation frequency. The stronger wave thus produces pronounced amplitude-modulation of the weaker wave, whereas the role the complementary perturbations in the weaker wave velocity is of secondary importance for the considered phenomenon [15].

Below experimental demonstrations of the elastic wave analogue of the LG-effect and some other instructive observations of nonlinear elastic and dissipative effect are described for samples containing individual cracks.

Experimental demonstrations and interpretations.

Observation of the LG-effect for elastic waves [16,8]

In order to experimentally study the role of cracks (including the LG-effect) we observed interaction of resonant longitudinal modes in glass rods containing 1-3 corrugated thermally-produced cracks 2-3 mm in size (Fig. 2). An example of the observed modulation spectra of the initially sinusoidal probe wave under the action of the modulated stronger pump wave is shown in Fig. 3(a). In a reference rod without cracks, the modulation side lobes (existing due to residual parasite nonlinearities) were 25-40 dB lower than shown in Fig. 3 (a). Resonance curves for the probe wave [Fig. 3 (b)] obtained at different amplitudes of the second pump wave (without modulation) clearly demonstrate that primarily the dissipation, not the elasticity, was affected by the stronger wave in this experiment. Magnitudes and frequencies, at which the observed amplitude-dependent variations in dissipation were observed, agree well with estimates based on equations (4),(5) and (6). As argued above, for small enough strains $e \sim 10^{-8}$, estimated displacements eL of adjacent crack interfaces are subatomic in scale, so that neither hysteretic, nor

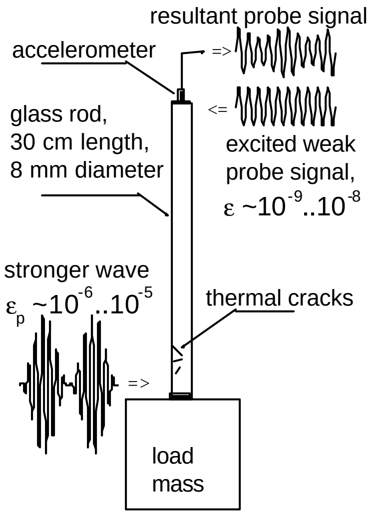

Fig. 2. Schematically shown experimental configuration.

frictional effects can be important for the probe wave dissipation. Indeed, careful experimental study [16] of the amplitude dependencies for the observed modulation confirmed linear character of the weak wave dissipation. Quantitatively, estimates based on Eq. (6) and typical parameters for glass show that even a single contact-containing crack of a few millimeters in size suffices to explain the observed ~ 10% variation in the initial magnitude of the quality-factor of about 300-350 for the probe wave. In this simple experiment we have used a transparent material in which the cracks are easily visible. Their parameters may be directly and non-destructively estimated. These cracks are the only defects present, and there is no doubt that only their presence is responsible for the observed effects.

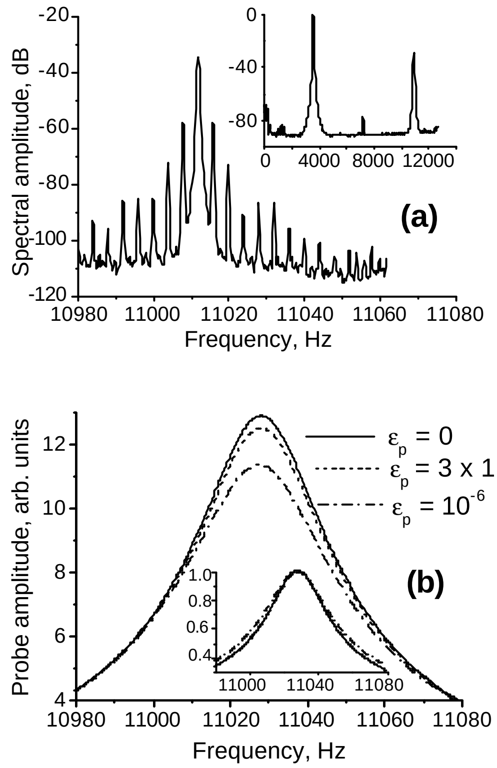

Fig. 3. Experimental observation of the elastic wave LG-effect. (a) – Modulation spectrum of weak 2nd mode near 11 kHz with $e \sim 10^{-8}$ caused by a stronger ($e_p \sim 10^{-6}$) 1st mode wave with carrier frequency ~3.6 kHz and slow amplitude modulation at 3 Hz. The inset shows the relative levels of the stronger and the weaker waves.
(b) – Resonance curves for the probe wave at different stronger-wave levels, clearly illustrating a greater than 10% variation in the probe wave Q-factor. In contrast, the resonance frequency shift is hardly noticeable. The inset shows the same curves in normalized form.

Induced transparency and induced dissipation [17].

In order to better understand the observations presented below it is important first to recall the following. As was argued above, for small amplitude probe waves (say with strains $e \sim 10^{-8}$) with subatomic displacements at the crack interface the frictional/hysteretic losses are not yet activated, whereas the thermoelastic coupling may be very efficient, especially at narrow inner contacts between the crack lips. For contacts of width $l << L$ the relaxation peak occurs at $w = w_l \approx D/l^2$, where $D = k/(rc)$ is the temperature diffusion coefficient. For the whole crack, its relaxation frequency w_L is determined by its scale $L >> l$, so that $w_L \approx D/L^2 << w_l$. For most rocks and glasses, typical w_L for a millimeter-scale crack corresponds to fractions of a cycle/s, whereas for micrometer-width inner contacts the respective frequency may lie from kHz up to MHz band. Further, since the local separation of crack lips in the contact vicinity may be orders of magnitude smaller than the average crack opening, waves with moderate strain $e \sim 10^{-6} - 10^{-5}$ may easily perturb the inner microcontacts (and even cause their clapping), although the average crack opening remains hardly perturbed. For example, for a contact whose initial strain e_0 and local applied stress s_0 are related by the Hertz-type [11] dependence $e \propto s^{2/3}$, the superimposed wave with oscillatory stress s_w comparable to s_0 may significantly reduce period-averaged contact strain $< e >$, as schematically illustrated in Fig. 4. It is worth mentioning that similar rectification (demodulation) effects are well documented in nonlinear acoustics of the interfaces [18] and provide the basis for the ultrasonic force mode in atomic force microscopy [19]. The resultant reduction of the averaged contact width shifts the maximum of the thermoelastic losses at the contact to a higher frequency w_l (see inset in Fig.4). This should cause an *increase* in the quality factor of the sample resonances located lower than w_l in the frequency domain, but simultaneously *decrease* the quality factor for the resonances located higher than w_l.

Among the fabricated samples we succeeded to choose one in which a thermally induced crack

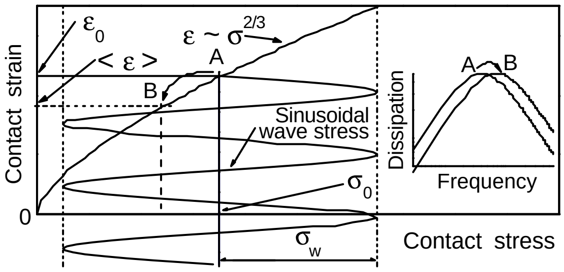

Fig. 4. Schematically shown softening of the contact by oscillating stress due to loading-unloading asymmetry. Initial static equilibrium $A = (s_0, e_0)$, and the perturbed time-averaged position $B = (<s>, <e>)$

demonstrated such a behavior. Figure 5 shows records of the probe-wave resonance peak below 4 kHz, which exhibited a pronounced *increase* in the quality factor under the pump-wave action, whereas the next mode above 10 kHz and a dozen or so other peaks within the observable band up to 100 kHz exhibited a *decrease* in the quality factor. In contrast to the opposite trends in the dissipation, the resonant frequencies for all peaks exhibited a *consistent decrease*, as should be expected owing to the time-average softening of the micro-contact(s) induced by the pump wave.

Further increase of the oscillation amplitude, $s_w > s_0$, transfers the contacts to the clapping regime, in which the averaged stiffness and size of near-Hertzian contacts begin to increase again. In this case both the decrement of the probe-peak and its position in the frequency domain may become non-

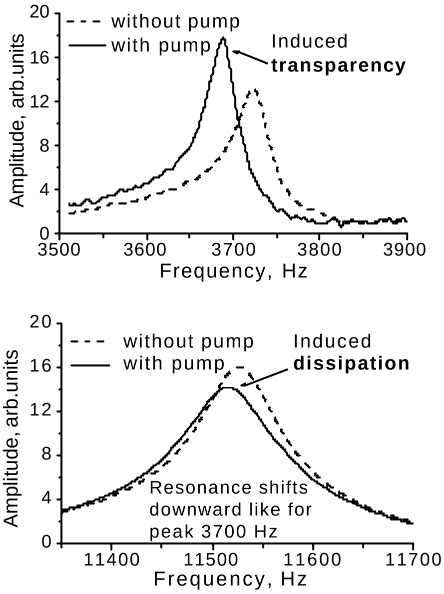

Fig. 5. Examples of the complementary pump-induced *decrease* in dissipation for the probe wave around 3.7 kHz and *increase* in dissipation for the probe wave around 11.5 kHz. The resonance frequency decreases in both cases. Pump wave strain is $e \sim 10^{-6}$.

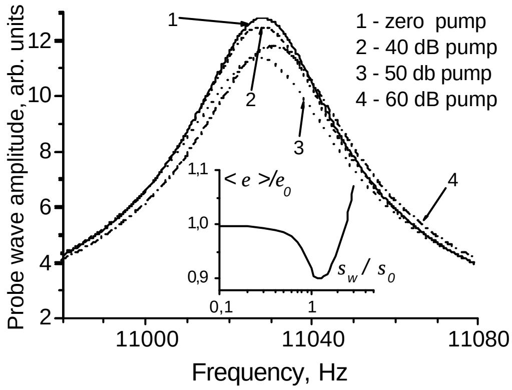

Fig. 6. Example of non-monotonous amplitude dependence of the pump-induced variations in dissipation and the probe wave resonance frequency. The inset shows the simulated averaged contact strain plotted against the pump-stress amplitude.

monotonous functions of the pump-wave amplitude. Resonance curves shown in Fig. 6 demonstrate such a non-monotonous dependence on the pump-amplitude both for the amplitude (that is dissipation) and the frequency shift of one of the probe-wave peaks. The inset shows the variation in the averaged strain $<e>$ simulated for a Hertzian contact as a function of normalized oscillating stress s_w / s_0, which indicates similar non-monotonous behavior.

Logarithmic in time slow dynamics.

Effects with slow dynamics that are logarithmic in time are remarkably common for many materials with granular and imperfect structure (logarithmic creep, ageing, magneto-relaxation, etc.). Recently logarithmic in time relaxation after acoustic stressing was found for rocks [20]. This universal log-time behavior is usually attributed to the complexity of dynamical processes in systems with wide distribution of energy barriers of some internal bonds which are, for example, broken by the activation and then are gradually restoring under action of thermal fluctuations. Note that logarithmic dynamics arises only for a suitable wide spectrum of the energy barriers, the origin of which and its relation to the material microstructure remains unknown [20]. Such a mechanism likely implies strong asymmetry between the relaxation and the bond breaking during the activation, which could be a fast process.

Recently, besides solids with numerous micro-defects [20], observations of slow relaxation and memory effects were reported for interaction of ultrasonic waves with a single crack [21]. For parametric generation of sub- and super-harmonics of the "reading-out" elastic wave, the threshold amplitudes remained perturbed up to minutes after activating the crack by another intense wave.

…In addition to studies of "instantaneous" effects, it seemed attractive to use the advantages of exploitation of independent pump and probe waves and to investigate whether individual cracks could exhibit slow-dynamic effects similar to those in [20] and to check the expected asymmetry between the activation and relaxation. To this end the experimental technique described above was supplemented by measurements of the slow evolution of the sample properties. We observed the temporal behavior of resonance peaks for a weak (probe) longitudinal wave with typical strain $e < 10^{-8}$ in glass rods subjected to action of another conditioning (pump) wave with typical strains up to $e \sim 10^{-6} - 10^{-5}$. The performed observations revealed that both the dissipation and the resonance frequencies exhibited pronounced *slow dynamics*. Figure 7a presents examples of variations in the resonance peak amplitudes plotted against the logarithm of time elapsed both after switching on or switching off the pump-wave. Figure 7b shows similar slow variations df / f_0 of the resonance frequency. Thus the observed logarithmic slow dynamics produced in the sample properties by *individual cracks* is strikingly similar to the slow relaxation effects found for rocks with inherent numerous defects [20]. A remarkable new revealed feature is that the log-time behavior during the crack-conditioning exhibits exactly *the same slope* (rate) as for the post-conditioning relaxation.

In the context of the above considered crack features, explanation for the slow logarithmic relaxation arrives rather naturally from the dynamics of thermal conduction in a cylindrical geometry. This geometry is specific for both the crack perimeter and the elongated inner microcontacts, which produce the main contribution to the dissipation and hence undergo local acoustic heating. The contact state may be strongly perturbed by nanoscale absolute distortions at the crack interfaces. Such distortions could be produced elastic oscillations as considered above for the "instantaneous" effects. Alternatively, similar distortions in the crack may be expected as a result of temperature inhomogenenities about $\Delta T \sim 0.1-1$ K. For a typical thermal expansion coefficient $a \sim 3\times10^{-6}$ K^{-1} and crack size $L \sim 3\times10^{-3} m$, the thermoelastic displacement is estimated as $aL\Delta T \sim 10^{-9} - 10^{-8}$ m, which is comparable with the instantaneous rectification effects. Note that direct infrared imaging of acoustically-induced heating of several degrees at the stress concentration areas at crack tips and lips is available [10]. The logarithmic slowing of the temperature rise (in the case of conditioning) in 2D-geometry is due to the diminishing of the temperature gradients with time. Indeed, for a step-like (in time) cylindrical thermal source $Q(r,t)$ localized in the area of a radius $r \le l$ $r \le l$, the 2D-equation for heat conduction $\partial T / \partial t - D\Delta_\perp T = Q/(rC)$ yields an asymptotically logarithmic law for the temperature increase ΔT in the source vicinity:

$$\tilde{T} \approx \frac{Q_F(k=0)}{4pr\,CD}\ln\frac{t}{l^2/D}, \text{ for } t >> l^2/D = w_l^{-1} \qquad (7).$$

Here r and C are the material density and specific heat, $Q_F(k)$ is the spatial Fourier transform of the source. For the subsequent cooling after switching off the source Q at time $t = t_0$, there is also a log-time approximate solution:

$$\tilde{T} \approx \frac{Q_F(k=0)}{4pr\,CD}\left[\ln\frac{t_0}{l^2/D} - \ln\frac{(t-t_0)}{l^2/D}\right], \qquad (8)$$

valid for $l^2/D << t - t_0 \le t_0$ and indicating *the same slope* as in (7), which is predetermined by the temperature spatial distribution produced by the initial heating. We already pointed out that equal slopes for the conditioning and relief were observed in the experiments (see Fig. 7), which is a strong argument supporting the heating-cooling mechanism. Indeed, the state of the stress-concentration areas is essentially different at rest and during intensive acoustic activation that causes contact clapping and

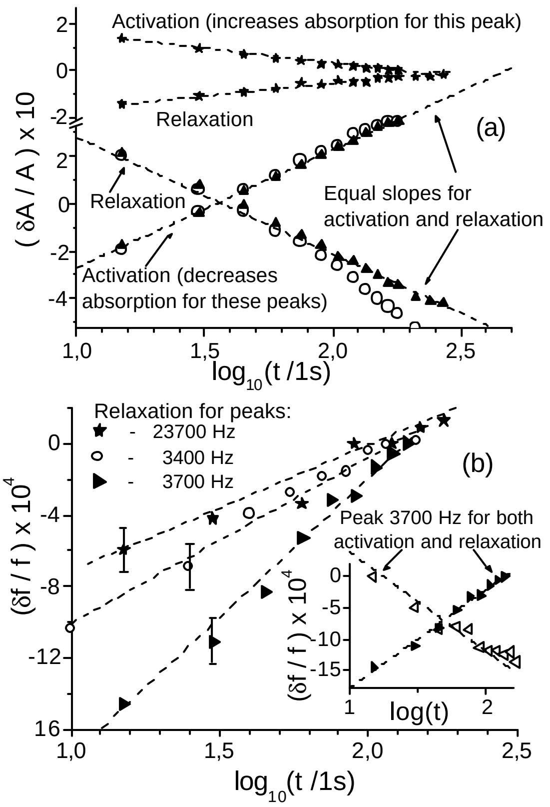

Fig. 7. Examples of the log-time dependence of the dissipation (a) and the resonance frequency shift (b) for different peaks.

Demonstration of memory in crack nonlinearity. Attempting to reveal slow dynamics in nonlinear crack-induced effects (for purposes of comparison with [21]), we faced a problem that the nonlinearly excited harmonics were essentially influenced by the system resonances, whose slow dynamics strongly masked possible memory in the nonlinear properties. We finally succeeded in observing a very clear manifestation of memory for the nonlinear effect of the cross-modulation [8,17] of the probe wave by a slowly-modulated pump wave. Figure 8 presents the modulation spectra obtained immediately after a few minutes of intensive conditioning of the sample and

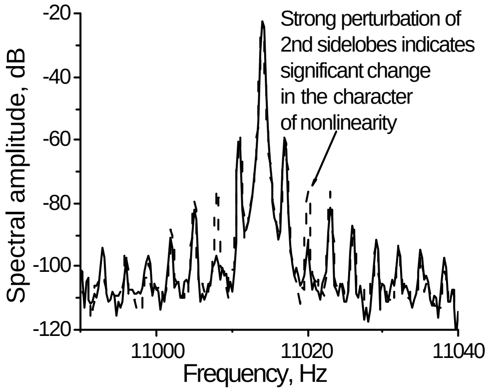

Fig. 8. Examples of cross-modulation spectra just after intensive conditioning (solid line) and 5 minutes later (dashed line).

after 5 minute pause. Since the stronger modulating wave also acted as a pump, after the pause we switched it on again only for the several seconds required to obtain the spectrum. There is a remarkable difference in the second side lobe amplitude indicating a significant change in the character of the crack-induced nonlinearity. In contrast, the amplitudes of the fundamental line and other side lobes are much more weakly perturbed, which assures that the whole resonance curve remained almost the same.

Conclusion.

The observed variety of effects is shown to be consistently explained basing on a few well established features of cracks. The proposed mechanism of the acoustic wave-crack interaction essentially revises the role of thermo-elastic coupling in imperfect solids. These findings provide a new physical insight into acoustic-wave-crack interaction at "instantaneous" and slow-dynamics time-scales, and suggest a way to resolve some difficulties (e.g. low-strain dissipation) faced by conventionally used interpretations. The nonlinear effects observed have proven to be very sensitive indicators of presence even of single cracks in the sample. In particular, the LG-effect is advantageous as a new nonlinear-modulation diagnostic method.

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