REFLECTION OF PLANE ELASTIC WAVES IN ACOUSTO-OPTIC CRYSTAL TELLURIUM DIOXIDE

N.V.Polikarpova and V.B.Voloshinov
Department of Physics, M.V.Lomonosov Moscow State University, Moscow, Russia
polikarp@osc162.phys.msu.ru

Abstract

The report presents results on investigation related to propagation and reflection of plane elastic waves in the acousto-optic crystal tellurium dioxide. The reflection of a slow shear elastic wave from a free and flat boundary separating the paratellurite crystal and vacuum was examined. The analysis was concentrated on the propagation and reflection of the waves in XY plane of TeO_2 in the case of glancing acoustic incidence on the crystal boundary. The analysis reveals the peculiarity that as much as two elastic waves are reflected from the crystal surface. Energy flow of one of the reflected waves is directed approximately backwards with respect to the incident energy flow so that two energy flows form an angle in the incidence plane as narrow as a few degrees. It was also found that relative intensity of the unusually reflected wave is close to a unit in a wide variety of crystal cuts. It is proved that strong elastic anisotropy of the material is responsible for the extraordinary behaviour of the acoustic waves. Possible applications in acousto-optic devices of the examined phenomena are discussed in the presentation.

Introduction

Single crystals of paratellurite TeO_2 are widely used in modern instruments of light beams control. Unique physical features of the crystal provide perfect operation parameters of acousto-optic instruments that have been designed on base of the crystal [1,2]. It is known that the material is characterized by extremely strong anisotropy of its elastic properties [1-5]. Utilization of the anisotropy made it possible to develop novel modifications of the acousto-optic instruments, e.g. the close to collinear tunable filters [4,5]. Moreover, it is expected that new types of acousto-optic instruments may be designed if one utilizes the unique elastic properties of the crystal. Therefore, the goal of the paper consists in the analysis of the elastic wave propagation and reflection in TeO_2 in order to design new devices of light beam control.

Acoustic properties of paratellurite

Fruitful information may be obtained if dependence of acoustic phase velocity on direction of sound propagation in a crystal is known. There exists a method to calculate the magnitudes of the acoustic phase velocities V in the crystalline materials. The method is based on application of the Christoffel equation [1,3]. It is known that the crystal of paratellurite belongs to the tetragonal crystalline structure. Substituting known values of the elastic constants c_{ij} in the Christoffel equation, in the case of XY plane of tellurium dioxide, it is possible to obtain the phase velocity values for the quasi-shear and the quasi-longitudinal waves

$$2\rho\, V_S^2 = \left[c_{11} + c_{66} - \sqrt{(c_{66}-c_{11})^2 \cos^2 2\varphi + (c_{12}+c_{66})^2 \sin^2 2\varphi} \right],$$
$$2\rho\, V_l^2 = \left[c_{11} + c_{66} + \sqrt{(c_{66}-c_{11})^2 \cos^2 2\varphi + (c_{12}+c_{66})^2 \sin^2 2\varphi} \right], \quad (1)$$

where ρ is the density of the material and the angle φ is measured relatively to the axis X [1]. Slowness curves in XY plane of TeO_2 are shown in figure 1.

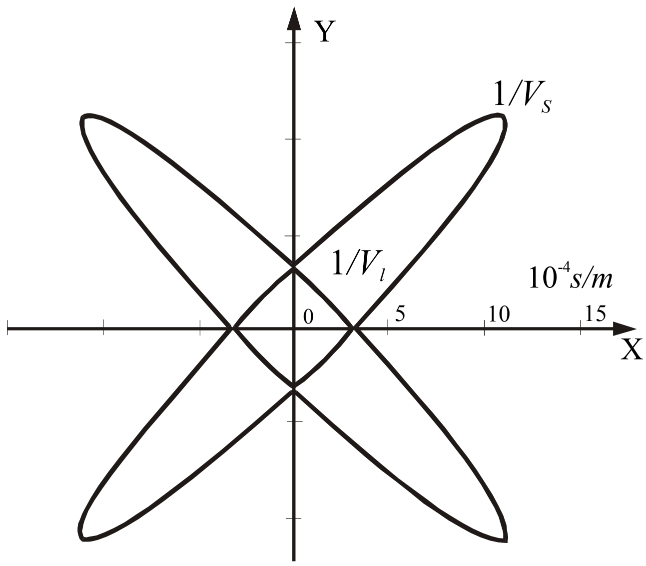


Figure 1: Acoustic slowness curve of paratellurite

Analysis of the figure proves that the crystal is characterized by the strong dependence of the phase velocity on the direction of propagation.

On base of equation (1) it is also possible to calculate the acoustic obliquity angle of the waves, i.e. the angle ψ between the phase and V and group V_g velocities of ultrasound. The dependence of the obliquity angles for the quasi-shear and quasi-longitudinal acoustic modes on the angle φ determining the propagation relatively to the axis X is plotted in figure 2. It seen that the obliquity angle for the slow shear acoustic mode in tellurium dioxide is as large as $\psi_1 = 74^0$. It will be shown that the large obliquity angles between the phase velocity vector and the Poyinting vector may be responsible for the unusual propagation and reflection of the elastic waves in the crystal [3,4].

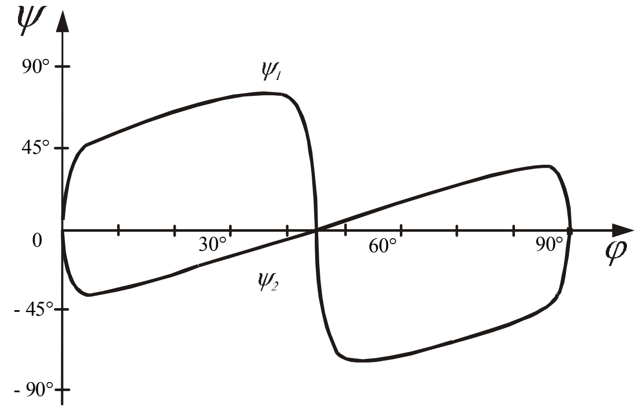


Figure 2: Acoustic obliquity angles in TeO_2

Reflection of elastic waves in rectangular sample

A particular case of the acoustic reflection is examined in this paper when the tellurium dioxide crystal is cut in form of a rectangular specimen and the propagation and reflection of the waves takes place in XY plane of the material. General scheme of the reflection is presented in figure 3. It is seen in the

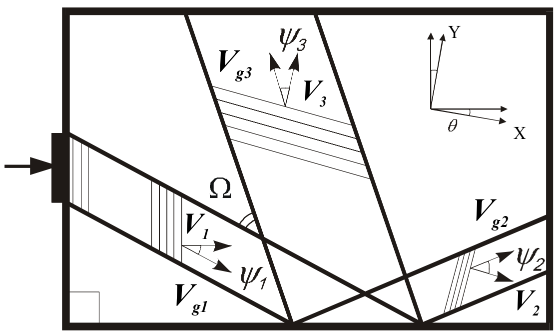


Figure 3: General scheme of reflection

figure 3 that the elastic waves are generated in the crystal by means of the piezoelectric transducer. Slow shear elastic waves are launched in the crystal at the angle θ formed by the axis X of paratellurite and the acoustic phase velocity vector $\boldsymbol{V_1}$. It is clear that the bottom facet of the crystal is also rotated at the angle θ with respect to the axis X . Consequently, the wave vector of the incident acoustic wave occurs parallel to the border separating the crystal and vacuum. It indicates that the incidence is glancing.

Energy flow and the group velocity vector $\boldsymbol{V_{g1}}$ of the initial wave are directed in the sample at the angle ψ_1 relatively to the phase velocity vector, as illustrated in the picture. The energy flow of the acoustic wave from the transducer propagates to the border separating the material and vacuum.

Analysis confirms that, after the reflection, as much as two elastic waves propagate form the bottom facet in the sample. One of the waves possesses the phase velocity vector $\boldsymbol{V_2}$ and the group velocity vector $\boldsymbol{V_{g2}}$ that forms the obliquity angle ψ_2 with the acoustic wave vector. This wave is reflected in the crystal in the traditional manner. On the other hand, there exists another reflected wave that is propagating approximately backwards with respect to the incident acoustic wave. The second reflected wave possesses the phase and group velocities vectors $\boldsymbol{V_3}$ and $\boldsymbol{V_{g3}}$ correspondingly. Energy flow of this wave forms the angle ψ_3 with the phase velocity vector.

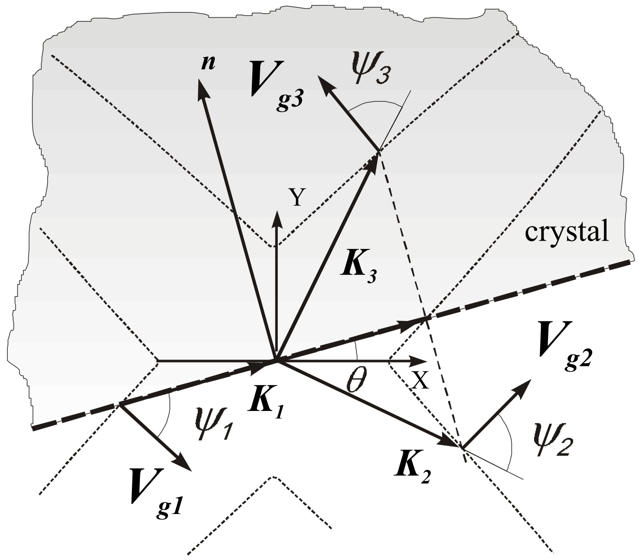


Figure 4: Mutual orientation of acoustic wave vectors

The angle Ω separates the energy flows of the initial and the second reflected waves. This angle may be seen in figure 3. It will be shown that this angle in the crystal may be as narrow as a few degrees.

Backward propagation of reflected acoustic wave

The unusual reflection of ultrasound may be explained by figure 4 where the directions of the wave vectors $\mathbf{K_2}$ and $\mathbf{K_3}$ corresponding to the two reflected beams are shown. As mentioned, the wave vector $\mathbf{K_1}$ of the incident beam is directed parallel to the bottom facet of the crystal. The length of the projection of the wave vector $\mathbf{K_1}$ on the boundary is equal to the length of the wave vector itself because the specimen is rectangular and the acoustic incidence is glancing.

According to the laws of wave motion, the directions of the two reflected wave vectors $\mathbf{K_2}$ and $\mathbf{K_3}$ may be found, in the traditional manner, from the intersections of the supplementary dash line with the dotted slowness curve [1]. As seen, the dash line drawn at the end of the acoustic vector with the length equal to **K_1** is orthogonal to the boundary between the crystal and vacuum.

The well-known requirement of equal projections of wave vectors representing the incident and the reflected beams is satisfied in this way in the crystal [1,3]. Therefore, during the propagation and reflection of the waves in paratellurite, the projections of the wave vectors representing in Fig. 4 the ordinary reflected beam and the extraordinary reflected ray appear equal to the length of the wave vector **K_1**.

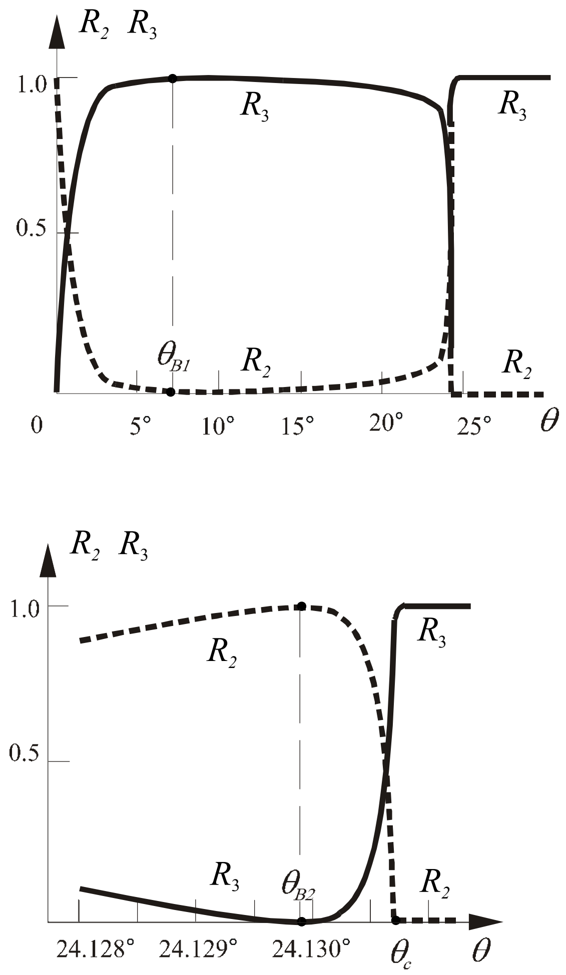
If the directions of all wave vectors are known then it is possible to determine the directions of the acoustic energy flow during the reflection. The picture also illustrates that the group velocity vector of the initial beam is orthogonal to the slowness surface of the material in the point where the vector **K_1** begins at the slowness surface.

As for the first reflected beam, data in figure 4 prove that the wave vector **K_2** is directed outside the crystal. It means that the acoustic wave fronts of this reflected beam are tilted clock-wise relatively to the direction of the boundary. On the contrary, the energy flow of the first reflected beam is propagating at the angle ψ_2 inside the crystal.

In order to understand the origin of the peculiar reflection at the border, it was necessary to find the direction of the acoustic group velocity of the second reflected beam. This beam is represented in figure. 4 by the wave vector **K_3** and the acoustic walkoff angle ψ_3. It is clear that the direction of the vector **K_3** in the crystal, similar to the direction of the vector **K_2**, may be found from the intersection of the slowness curve with of the dash line. As proved by figure 4, the wave vector **K_3** is directed inside the crystal and away from the bottom facet of the specimen. However, the energy flow and the group velocity vector V_{g3} of this beam are propagating approximately backwards relatively to the initial energy flow. It is evident that the revealed peculiarity originates from the extremely large value of the acoustic obliquity angle ψ_3 in tellurium dioxide while the large acoustic walkoff angle is the consequence of the strong elastic anisotropy of the material [3,4].

In this paper, the values of the acoustic obliquity angles of the three acoustic beams ψ_1, ψ_2 and ψ_3 have been calculated for the cuts of the crystal with the angle θ limited to $0 < \theta < 45^0$. During the analysis, the angle Ω separating in space the acoustic columns of the incident and the extraordinary reflected beam was also determined for the values of the angle θ. It was found that the angle between the two acoustic columns occurs amazingly narrow $\Omega < 10^0$ over the wide range of the propagation angles $4^0 < \theta < 32^0$.

Figure 5: Reflection coefficients in paratellurite



The performed calculations proved that the minimum value of the separation angle in tellurium dioxide occurs as low as $\Omega = 5.3^0$. It means that one of the reflected acoustic beams is propagating in the crystal like a boomerang, i.e. practically backwards relatively to the incident beam.

Calculation of reflection coefficients

In order to fulfill the analysis, mutual distribution of the incident elastic energy between the two reflected waves was evaluated. The reflection coefficients R_2 and R_3 of the two waves were calculated for the purpose. The coefficients were defined in the traditional manner [3.4, 0], where the value of each of the two coefficients was chosen equal to the ratio of normal projections of energy flows of the corresponding reflected beams and the initial beam.

Data in figure 5 illustrate the dependence of the reflection coefficients R_2 and R_3 on the angle θ. It may be seen in the figure that there exist as much as four intervals of the angle θ corresponding to different behaviour of the reflection coefficients in the specimen. These intervals are $0 < \theta \le 3^0$,

$3^0 < \theta < 24.13^0$, the interval near $\theta \approx 24.13^0$ and $24.13^0 < \theta < 45^0$.

The analysis confirms that if the crystal is cut with the angle θ close to zero then the major amount of the incident acoustic energy is reflected in form of the traditional elastic wave. It is seen in figure 5 that the coefficient R_2 in this case is close to a unit at $\theta \to 0$. On the other hand, the growth of the angle θ is accompanied by the fast increase of the energy in the extraordinary reflected wave while the intensity of the ordinary reflected wave vanishes. At the cut angle $\theta = 7^0$, the coefficients R_2 = 0 and R_3 = 1.0. Therefore, this angle may be defined as the Brewster angle $\theta_{B1} = 7^0$ because one of the reflected waves is absent. It should be emphasized that if the angle θ is limited by the interval $3^0 < \theta \leq 24.1^0$ then the major amount of the elastic energy is concentrated in the extraordinary reflected wave.

The carried out calculations confirm that the interval of the crystal cuts near $\theta \approx 24.13^0$ includes two characteristic angles one of which is the second Brewster angle θ_{B2} while the other angle is the critical angle θ_c. It was found that the values of the Brewster and the critical angles occur close to each other $\theta_{B2} \approx 24.13^0$ and $\theta_c \approx 24.131^0$.

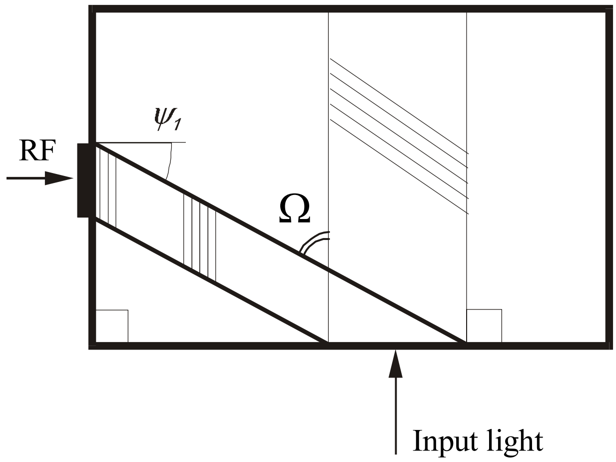


Figure 6: Collinear acousto-optic interaction

As for the energy flow, if the cut angle θ is growing from $\theta = 23^0$ to $\theta_{B2} \approx 24.13^0$ then the reflection coefficient R_2 is rapidly increasing to a unit while the energy flow of the extraordinary reflected beam is approaching the zero value. Consequently, one observes total reflection of the incident energy along the direction of the ordinary reflected wave at $\theta_{B2} \approx 24.13^0$.

Further increase of the cut angle θ from $\theta_{B2} \approx 24.13^0$ to the critical angle $\theta_c \approx 24.131^0$ is accompanied by the drop in the energy of the ordinary reflected wave so that $R_2 = 0$. It means that the elastic energy is reflected from the border at the critical angle $\theta_c = 24.131^0$ as the extraordinary wave. Finally, the case with $\theta \geq 24.131^0$ corresponds to a single reflected wave. As mentioned, this wave propagates in a wide interval of propagation angles practically backwards relatively to the energy of the incident beam.

Application of backward reflection in Acousto-Optic devices

The examined reflection phenomena may be applied in acousto-optic devices. One of these devices is schematically shown in figure 6. The acoustic reflection in the cell of a collinear filter takes place in such a manner that the reflected energy flow is propagating in the instrument orthogonally to the bottom facet of the sample. This peculiar reflection takes place in paratellurite if $\psi_1 + \Omega = 90^0$. Consequently, if an optical beam is sent normally to the bottom facet of the crystal then a collinear interaction of light and sound may be observed in the filter [1,2,5].

From the point of view of practice, the normal light incidence should be considered as an advantage of the design compared to the existing models of the collinear filters on paratellurite [1,2,5]. It may also be noted that only a single reflected wave is propagating in the cell of the filter because the crystal in figure. 6 is cut with the angle of propagation θ exceeding the critical angle. It provides total utilization of the elastic energy in the cell.

Acknowledgement

The investigation was in part supported by the Grant N 02-07-90448 of the Russian Foundation for Basic Research.

References

[1] E.Dieulesaint and D.Royer, Ondes Elastiques dans les Solides, Masson, Paris, 1974.
[2] A.Goutzoulis and D.Pape, Design and Fabrication of Acousto-Optic Devices, Marcel Dekker, 1994.
[3] M.J.P. Musgrave, "Reflexion and Refraction of Plane Elastic Waves at a Plane Boundary between Aeolotropic Media", Geophys. J. Royal Astron. Soc., vol. 3, pp. 406-418, 1960.
[4] V.Voloshinov and N.Polikarpova, "Application of Acousto-Optic Interactions in Anisotropic Media for Control of Light Radiation", Acustica Acta-Acustica, vol. 89, 2003.
[5] V.Voloshinov, "Close to Collinear Acousto-Optic Interaction in Paratellurite", Optical Engineering, vol. 31, pp. 2089-2094, 1992.