# THEORETICAL AND EXPERIMENTAL STUDY OF THE DIFFRACTION OF FOCUSED ULTRASONIC WAVES IN VISCOUS FLUIDS

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**Abstract**

The acoustic pressure emitted in a viscous fluid by a focused transducer of circular aperture, is calculated by using the impulse response method and the obtained waveforms are interpreted in terms of direct and edge waves. These results are extended to the case of the diffraction of focused transient ultrasonic waves by a small target immersed in a viscous fluid. A discussion will relate the influence of various parameters (frequency, position, aperture and focal distance, attenuation, ..) on the waveform of the detected ultrasonic pulses. The conclusions of this study have been used to interpret the pulses obtained in an experimental study of the diffraction of focused transient ultrasonic waves by a small target immersed in a viscous fluid (glycerin).

## Introduction

The impulse response method, initially suggested by Stepanishen [1], was developed for the calculation of transient fields radiated by plane transducers, then generalized to the case of slightly curved focused transducers for which the secondary diffraction can be neglected [2]. Because of the reciprocity of the wave equation, the impulse response method can be applied as well in transmit mode as in transmit-receive mode.

In this study, the acoustic field of the ultrasonic pressure radiated in a viscous fluid by a transducer focused and diffracted by a plane target of small size is calculated by using the impulse response method,. The problem of the modeling of the propagation in attenuating fluids is difficult to formulate when the dependence law of the attenuation versus the frequency is not a simple relation [3-6]. However, causal analytical expressions of the Green function corresponding to the wave equation have been calculated for media where the attenuation is proportional to the frequency [5] and for slightly attenuating viscous media where the attenuation is proportional to the square of the frequency [6-9].

## Transmit mode

The acoustic wave propagation in a viscous fluid is described by Stokes equation [7-9]:

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\beta\nabla^2\frac{\partial\phi}{\partial t}-\nabla^2\phi=0 \qquad (1)$$

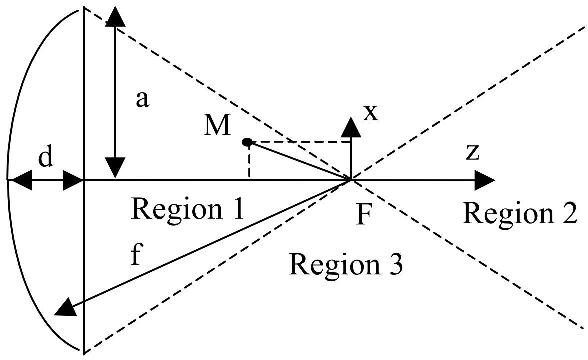


Figure 1. Geometrical configuration of the problem.

In this expression, $\phi$ is the acoustic potential, c is the propagation velocity and the relaxation time $\beta$ is a constant proportional to the kinematics viscosity coefficient of the fluid. $\beta$ is sufficiently small so that its square is negligible.

In the free field conditions, the causal Green function associated to this equation is [7,8]:

$$G(\vec{r},t;\vec{r}_0,t_0)=\frac{\exp[(\tau-Rc)^2/2\beta\tau)]}{4\pi^{3/2}R\sqrt{2\beta\tau}}H(\tau) \qquad (2)$$

where $\tau=t-t_0$ et $R=\|\vec{r}-\vec{r}_0\|$, $H(\tau)$ being the Heaviside function which translates the causality law. For a slightly curved transducer, the Rayleigh integral constitutes a good approximation for the evaluation of the radiated field [2]. For low values of $\beta$ and if all points of the transducer vibrate synchronously, the acoustic potential at a field point M can be written as a convolution in time [9]:

$$\phi(M,t)=v(t)\otimes\phi_i(M,t) \qquad (3)$$

where $\otimes$ represent the convolution in time. $\phi_i$ is called the impulse response for the potential. If the source is a uniformly vibrating focused transducer of surface $S_T$ embedded in a rigid baffle, $\phi_i(M,t)$ is defined by [9]:

$$\phi_i(M,t)=\iint_{S_T}\frac{\exp[(\tau-Rc)^2/2\beta\tau)]}{2\pi^{3/2}R\sqrt{2\beta\tau}}H(\tau)dS_T \qquad (4)$$

where $R=\left\|\overrightarrow{M_0M}\right\|$, $M_0$ being a point of the radiating surface where is located the elementary surface $dS_T$.

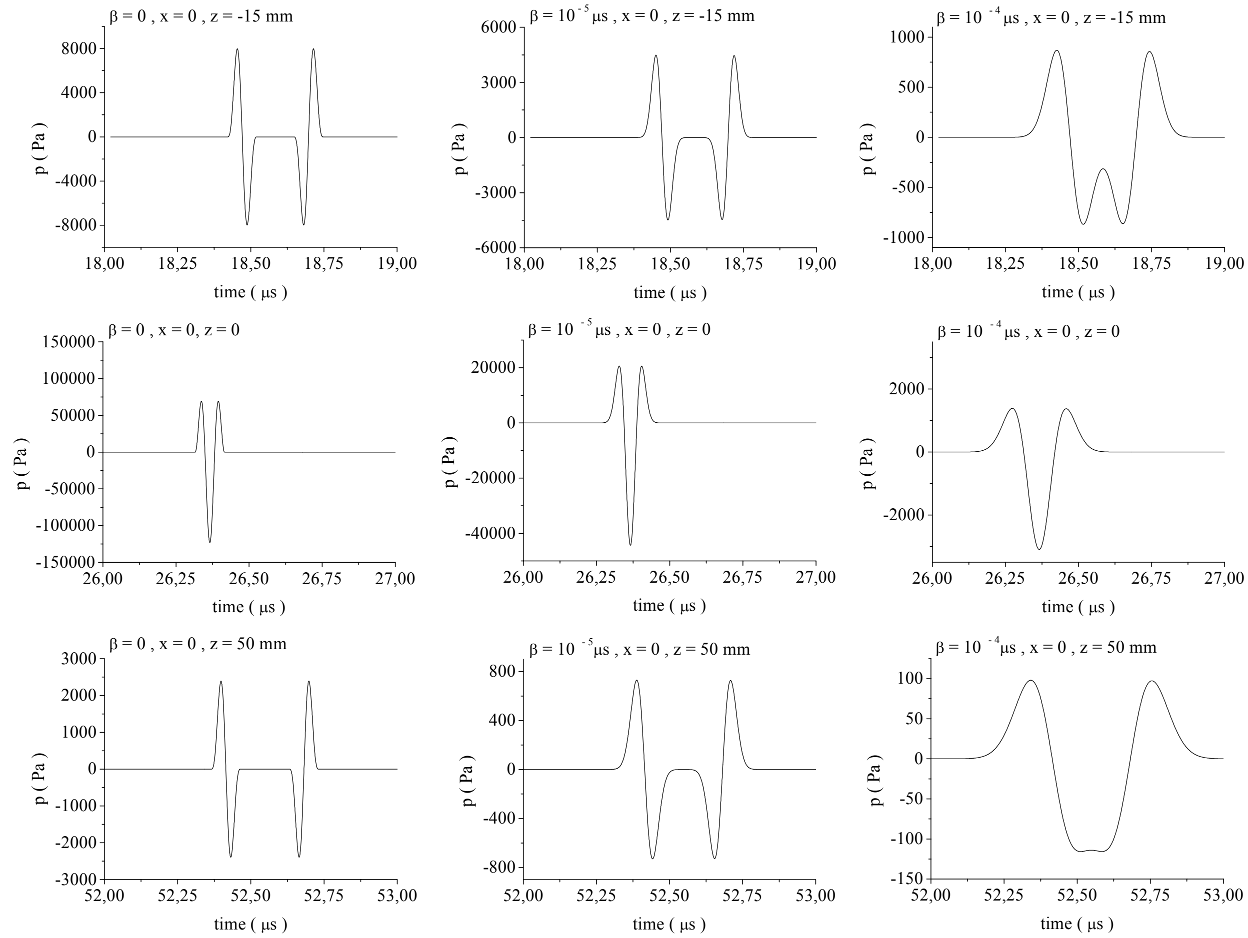


Figure 2 : Acoustic pressure for various values of β (0 µs; $10^{-5}$ µs; $10^{-4}$ µs).

The expression of $\phi_i$ depends on the position of the point M in the radiation field which is usually divided into three regions (fig. 1) [2].
When the point M is on the axis (M(0, z)) and if $t_0$=0, $\phi_i$ can be written under the form:

$$\phi_i(0,z,t) = \frac{f\ H(t)}{z\sqrt{2\pi\beta t}} \int_{R_{min}}^{R_{max}} \exp\left[-\left(t - R/c\right)^2 / 2\beta t\right] dR \qquad (5)$$

where $R_{min}$ and $R_{max}$ represent respectively the distance separating the observation point from the nearest point of the source and from the most distant point of the source. According to the location of the point in the field, these distances are defined by:

- If z<0: Region 1
  - $R_{min} = f + z$
  - $R_{max} = f - \sqrt{(f - d + z)^2 - a^2}$
- If z>0: Region 2
  - $R_{min} = f - \sqrt{(f - d + z)^2 - a^2}$
  - $R_{max} = f + z$

$\phi_i$ can be expressed simply according to the Error function Erf(.) :

$$\phi_i(z,t) = \frac{cf\ H(t)}{2z}\left\{ \mathrm{Erf}\left[\frac{t - R_{max}/c}{\sqrt{2\beta t}}\right] - \mathrm{Erf}\left[\frac{t - R_{min}/c}{\sqrt{2\beta t}}\right]\right\} \qquad (6)$$

The time dependence of the source velocity, v(t), used in the numerical simulations, is a function containing only one cycle of frequency 10 MHz and having an amplitude equal to $10^{-4}$ mm/µs .
Figure 2 represents the acoustic pressure at various points on the axis of a transducer having an aperture radius a=10mm, with a focal distance f=50mm, radiating in a fluid of density ρ= 1.26 $10^3$ kg.m$^{-3}$ in which the acoustic waves propagate with a velocity c=1.9x103 m.s$^{-1}$, and for three values of the β coefficient ( β = 0 µs, $10^{-5}$ µs and $10^{-4}$ µs ).
The results can be interpreted by using the concept of edge waves and direct wave. The influence of viscosity appears by a reduction in the amplitude and the presence of a precursor at the beginning of the pressure pulse [6]. When the value of β is significant, the attenuation is such that the concept of edge waves cannot be used to interpret the detected signal waveforms.

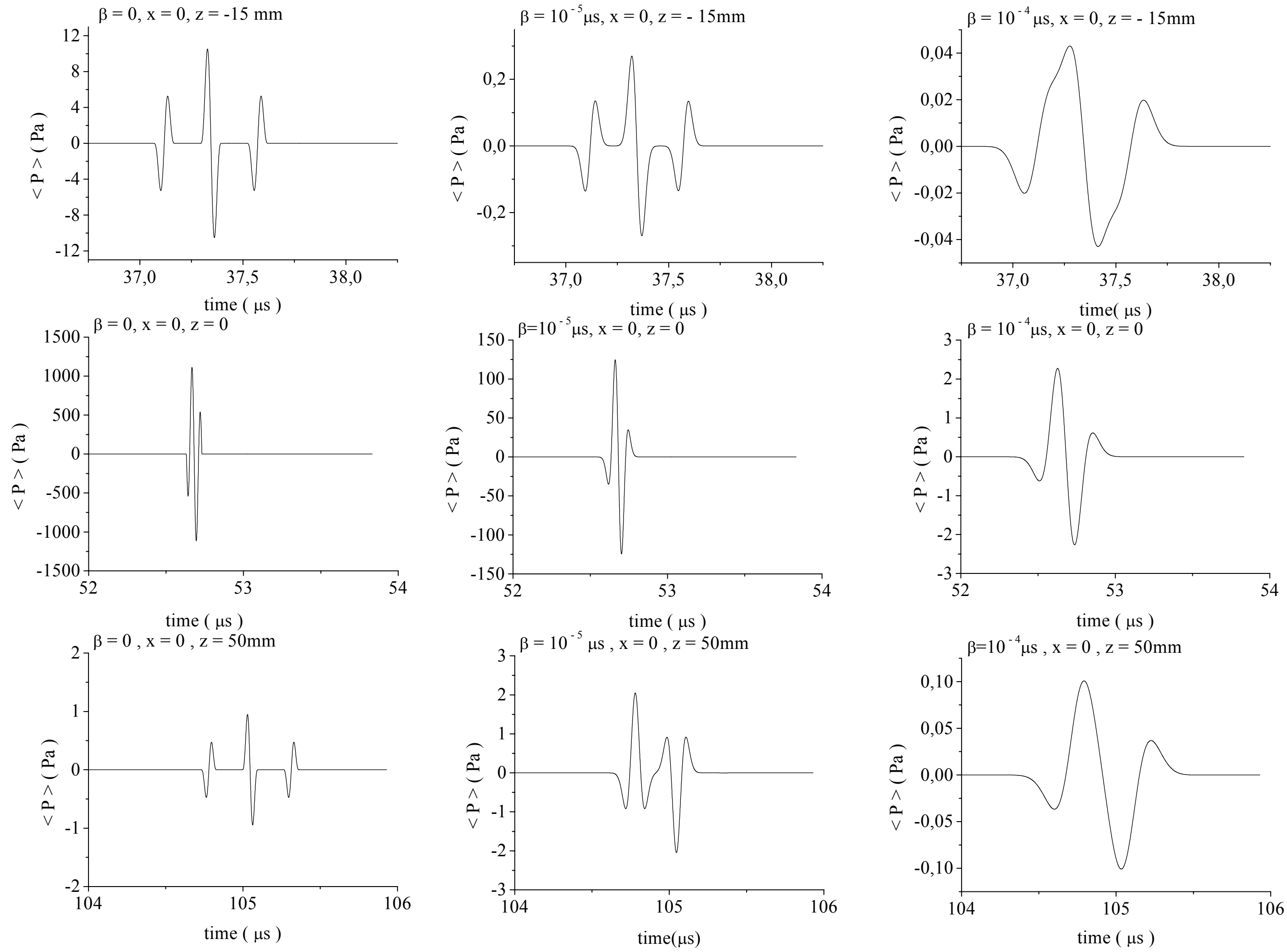


Figure 3 : Pressure detected for various positions of the target and various values of β ( 0 μs, 10 $^{-5}$ μs, 10 $^{-4}$ μs )

**Transmit-receive mode**

When a reflector (or target) is placed in front of the source, the incident wave is reflected and diffracted. The reflector behaves then as a radiating surface which vibrates with a velocity equal to the particle velocity corresponding to the reflected wave[10]. If the target is enough small, one can consider that the incident particle velocity is the same at any point of the target. If this later is a small circular reflector which center is denoted O, one can consider that the particle velocity at each target point is equal to the incident particle velocity at the target center O. If this target is located at the point (0, z) so that its surface is perpendicular to the z axis, the velocity of each point $M_C$ of the target is then given by:

$$v(M_c, t) \approx v(O,t) = v_0(t) \tag{7}$$

$v_0$ (t) is the velocity normal component at the target center. If the target has a very great impedance as compared to that of the fluid medium, the reflection is total and the particle velocity of the considered wave is opposite to the velocity normal component of the incident wave. The particle velocity can still be written as a gradient of the acoustic potential:

$$v_0 = -\frac{\partial \phi}{\partial z} \tag{8}$$

Knowing that the acoustic potential at the point O is expressed by:

$$\phi(O,t) = v(t) \otimes \phi_{iE}(O,t) \tag{9}$$

$v_0$ (t) can be deduced :

$$v_0(t) = -v(t) \otimes \frac{\partial \phi_{iE}}{\partial z} \tag{10}$$

This target behaves like a small source of surface $S_C$ located at the point O. Then, at point $M_T$, the acoustic potential emitted by this source:

$$\phi(M_T, t) = S_C \ v_0(t) \otimes \frac{\exp[-(t - R'/c)^2 / 2\beta t]}{2\pi^{3/2} R \sqrt{2\beta t}} \tag{11}$$

where. $R' = \left\| \overrightarrow{OM_T} \right\|$

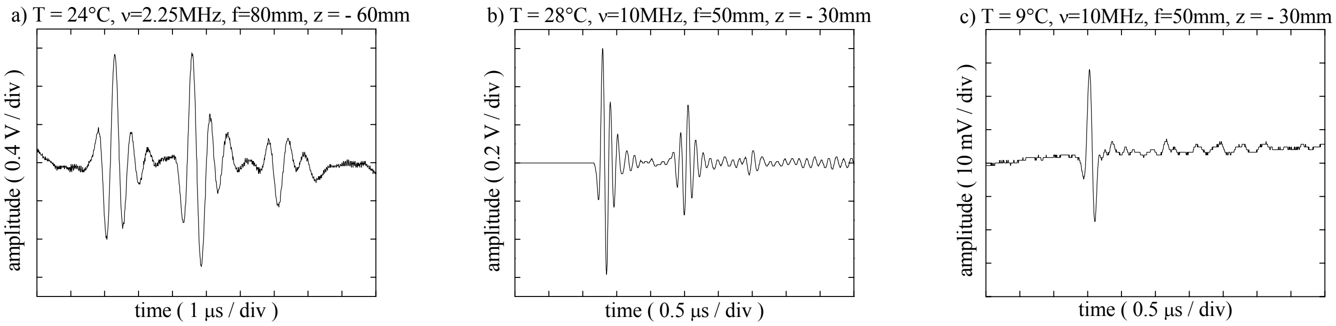


Figure 4 : Waveforms detected by two different focused transducers radiating in glycerin at different temperatures (T °C) for a target having a radius b = 0.4 mm. .

The wave reflected by the target is detected by the transducer functioning now as a receiver. The average pressure on its surface is given by :

$$\langle P(t)\rangle = \rho \frac{\partial\langle\phi(t)\rangle}{\partial t} \qquad (12)$$

where $\langle\phi(t)\rangle$ represents the average value of the acoustic potential over surface $S_T$ of the receiver. By taking account of Equation (11) :

$$\langle\phi(t)\rangle = \frac{S_C}{S_T} v_0(t) \otimes \iint_{S_T} \frac{\exp[-(t-R'/c)^2/2\beta t]}{2\pi^{3/2} R\sqrt{2\beta t}} dS_T \qquad (13)$$

$$\langle\phi(t)\rangle = -\frac{S_C}{S_T} v(t) \otimes \frac{\partial\phi_{iE}}{\partial t} \otimes \phi_{iE} \qquad (14)$$

Then the average pressure detected by the receiving transducer is obtained :

$$\langle P(t)\rangle = -\rho \frac{S_C}{S_T} \frac{\partial v(t)}{\partial t} \otimes \frac{\partial\phi_{iE}}{\partial z} \otimes \phi_{iE} \qquad (16)$$

Figure 3 represents the average pressure detected by the transducer used in the preceding paragraph for a target radius b=0.1mm located in various points on the source axis.

**Experimental results**

Figure 4 represents the experimental results obtained with a Duralumin target immersed in glycerin at various temperatures and with two focused transducers of different aperture and frequency. It will be noticed that when the attenuation is significant (T=9°C), the concept of wave of edge and direct wave cannot be used any more. Finally the amplitude of the second echo decreases appreciably in the case of the 10 MHz transducer since at this frequency the target cannot be any more considered as punctual.

**Conclusion**

The concept of wave of edge allows a better comprehension of the phenomenon of diffraction by the interpretation of the arrival instants of the detected pressure pulses. This concept is more delicate to use for viscous fluids because of the filtering effect resulting from the quadratic dependence from the attenuation versus to the frequency in such media.

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