MEASURING VISCOELASTIC ANISOTROPY IN DISPERSIVE MEDIA USING WAVEFORM RECONSTRUCTION

C. Aristégui [+], **N. Leymarie** [#], **O. Poncelet** [+], **and S. Baste** [+]
+ Laboratoire de Mécanique Physique, UMR CNRS 5469, Université Bordeaux 1, Bordeaux, France
# Commissariat à l'Énergie Atomique, CEA/DECS, Saclay France
c.aristegui@lmp.u-bordeaux1.fr

Abstract

An inverse method is proposed for the determination of the "viscoelastic anisotropy" of material plates from the plane-wave transmitted acoustic field. The original aspect lies in the lack of presuming choice for the rheologic model. The inversion technique allows one to evaluate, over a limited frequency band, the complex stiffnesses as frequency-dependent functions. The reliability of the process for measuring the dynamic properties of the medium is demonstrated using simulated data of which frequency-dependence ensures causal mechanical behavior. We demonstrate that this technique allows one to identify a global "viscoelastic anisotropy" of materials equally well for thin or thick specimens, and for dispersive as well as non-dispersive media. Finally a discussion is introduced concerning the sense of this measure on composite or heterogeneous materials and furthermore the homogenization validity.

Introduction

Ultrasonic waves are nowadays extensively used to investigate the internal microstructure of materials. Nevertheless, study of the acoustic phenomena generated by heterogeneities still remains an active research domain. When the wave propagation modeling is in development, it appears hazardous to think of an inversion process allowing the characterization of materials. However, observing that such media act on ultrasonic waves as dispersive and damping filters, a complex-valued stiffness tensor is usually introduced, assuming materials behave as linearly viscoelastic bodies.

Identification of viscoelastic properties commonly requires an *a priori* choice of the rheologic model. While the real parts of the stiffnesses are assumed to be constant, the imaginary parts are chosen with a polynomial-frequency dependence. Observe that this dependence for the stiffnesses does not satisfy the causality condition. In addition, when the material macroscopic behavior is unknown, the identification of the material properties cannot be performed by fixing beforehand a given frequency dependence. It is also important to note that, whatever the inspected material, the imaginary parts of all the stiffnesses are assumed to have the same frequency dependence. Thus, while the magnitude of all the stiffness tensor components may differ from each other (*i.e.* anisotropy on the magnitude of the components), these complex-valued stiffnesses follow the same frequency-variation (*i.e.* homogeneous frequency dependence).

The proposed technique, here called the "local" process, presents the advantage to make no assumption on the rheologic model. The identified viscoelastic behavior can thus be, in the most general case, anisotropic for the magnitude as well as for the frequency dependence of the stiffnesses.

The modeling of the material behavior is presented with the principles of the inversion process. Next, the reliability of the "local" characterization procedure is investigated by means of numerical simulations. Its interest, lying in its generality, with respect to the commonly-used approach, here called the "global" reconstruction, is then pointed out.

Modeling assumptions and method principle

Generalized constitutive law for viscoelastic anisotropic material

For a linearly viscoleastic medium, the time-dependent constitutive relation, relating the stress tensor (σ_{ij}) to the strain-rate tensor $(\dot{\varepsilon}_{kl})$, is written as [1]

$$\sigma_{ij}(t) = \left(\Re_{ijkl} * \dot{\varepsilon}_{kl}\right)(t), \tag{1}$$

where $(\Re_{ijkl})$ represents the tensor of relaxation functions and $*$ is the convolution product. By using the properties of the complex Fourier transform, Eq. (1) becomes, for any angular frequency ω,

$$\hat{\sigma}_{ij}(\omega) = C_{ijkl}(\omega)\,\hat{\varepsilon}_{kl}(\omega), \tag{2}$$

where $\hat{\sigma}_{ij}$ and $\hat{\varepsilon}_{kl}$ stand for the Fourier transforms (or spectra) of σ_{ij} and ε_{kl}, respectively. The functions C_{ijkl} are, in analogy with the elastic case, the components of the material stiffness tensor. However, in the viscoelastic case, these quantities are complex-valued and frequency dependent. Using the abbreviated subscripts notation, the stiffnesses are then expressed as

$$C_{IJ}(\omega) = C'_{IJ}(\omega) + \mathrm{i}\,C''_{IJ}(\omega), \tag{3}$$

where i is the imaginary unit, and C'_{IJ} and C''_{IJ} the real and imaginary parts, respectively. Observe that, from a physical point of view, these two functions cannot be independent. We know that a passive system does not respond before an excitation or "cause" is applied. The causality principle can then be expressed through the Kramers-Krönig relations [2], which establish a dependence between C'_{IJ} and C''_{IJ}.

Debye relaxation distribution model

In order to consider constitutive laws obeying the principle of causality, Wintle [3] proposes an alternative to the calculation of the principal value integrals relative to the KK relations, by introducing distributions of Debye relaxations

$$\begin{cases} \Re_{IJ}(t) = \\ \quad \Re_{IJ}(\infty)\left(1 + A_{IJ}\int_0^{+\infty} g_{IJ}(\tau)\, e^{-t/\tau}\, d\tau\right),\ t \geq 0 \\ \Re_{IJ}(t) = 0,\ \ t < 0 \end{cases} \tag{4}$$

where A_{IJ} is the relaxation strength and $g_{IJ}(\tau)$ is the probability distribution of the relaxation time τ with

$$\int_0^{+\infty} g_{IJ}(\tau)\, d\tau = 1\,. \tag{5}$$

Thus, by applying the Fourier transform to the time derivative of (4), we find

$$\begin{cases} C'_{IJ}(\omega) = C'_{IJ}(0)\left(1 + A_{IJ}\displaystyle\int_0^{+\infty} \frac{g_{IJ}(\tau)(\omega\tau)^2}{1+(\omega\tau)^2}\, d\tau\right) \\ C''_{IJ}(\omega) = C'_{IJ}(0)\, A_{IJ}\displaystyle\int_0^{+\infty} \frac{g_{IJ}(\tau)\,\omega\tau}{1+(\omega\tau)^2}\, d\tau \end{cases}. \tag{6}$$

Finally, by a judicious choice of the probability density functions, the system (6) can be evaluated analytically. We can then obtain expressions for the stiffnesses C_{IJ}, which ensure linear or quadratic asymptotic frequency-dependences [4].

Direct and inverse problems: recalls

The direct problem is based on the plate transmission coefficient for a plane wave, Fig. 1. The used formulation rests on the works from Deschamps and Hosten [5], generalized analytically for the case of any material symmetry [6].

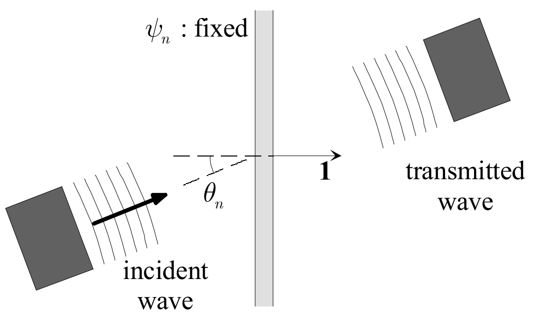

Figure 1: Plane-wave transmitted field through a solid plate immersed in a fluid; ψ_n defines the propagation plane (azimuthal angle) and θ_n the incidence angle.

By assuming plane wave propagation, the analytical expression of the transfer function H_n in transmission can be used to simulate the temporal waveform $y_n(t)$, for a given couple of angles (θ_n, ψ_n), by

$$Y_n(\omega, C_{IJ}(\omega)) = R(\omega)\, H_n(\omega, C_{IJ}(\omega))\,, \tag{7}$$

where $R(\omega)$ and $Y_n(\omega)$ are the spectra of the waveforms $r(t)$ and $y_n(t)$, respectively. Note that the reference signal $r(t)$ propagates in water without the plate.

The inverse method is directly inferred from the maximum-likelihood principle and the expression of ambiguity functions [7]. The writing of the analytic formulation of signals, suited especially to the assessment of difference between signals, is implicitly used in the following. The inverse problem consists of seeking the optimal parameters C_{IJ} corresponding to the best matching between the experimental and predicted, by using Eq. (7), waveforms. The maximum likelihood between two signals is reached when these signals have the same energy and when their interaction (defined by the modulus of the scalar product between these signals) is maximum. The signal interaction energy ε_n^{int} between the experimental $y_n^{exp}(t)$ and predicted $y_n(t, C_{IJ})$ signals, and the experimental signal energy ε_n^{exp} are defined, respectively, as [6, 8]

$$\begin{aligned} \varepsilon_n^{int}(C_{IJ}) &= \langle y_n, y_n^{exp} \rangle = 2\sum_{\omega_k \in D_\omega} H_n(\omega_k, C_{IJ})\, \chi_n(\omega_k), \\ \varepsilon_n^{exp} &= \langle y_n^{exp}, y_n^{exp} \rangle = 2\sum_{\omega_k \in D_\omega} \left|Y_n^{exp}(\omega_k)\right|^2, \end{aligned} \tag{8}$$

with $\chi_n(\omega) = R(\omega)\overline{Y_n^{exp}(\omega)}$. In Eq. (8), the symbols $|\cdot|$ and $\bar{\cdot}$ stand for the modulus and the conjugate of a complex value, respectively. The angular frequency domain D_ω represents the set of positive components of sampled analytic signals. However, without changing the significance of the energetic quantities (8), the range D_ω can be reduced to few frequency components around a given angular frequency. This process leads one to apply a sliding rectangular window to the signal spectra.

Concerning the solving procedure, the maximum-likelihood corresponds to the minimum of an objective function. Let N_S be the number of recorded experimental signals. We define the objective function F by

$$\mathrm{F}(C_{IJ}) = \frac{1}{N_S}\sum_{n=1}^{N_S}\left|1 - \frac{\varepsilon_n^{int}(C_{IJ})}{\varepsilon_n^{exp}}\right|^2. \tag{9}$$

The global minimum of this function is zero and is reached when the sets of experimental and predicted waveforms are similar. Finally the optimal parameters (C_{IJ}^{opt}) are defined such as

$$\left(C_{IJ}^{opt}\right) = \min_{(C_{IJ})} \left\{ \mathrm{F}\left(C_{IJ}\right) \right\}. \qquad (10)$$

Parameterization choice: the "local" formalism

Due to the frequency dependence of the optimal stiffnesses $\left(C_{IJ}^{opt}\right)$, it is necessary to define a procedure allowing one to identify the material viscoelastic response.

The rheologic models commonly used for ultrasonic frequencies have the form

$$C_{IJ}(\omega) = C'_{IJ}(\omega^*) + \mathrm{i}\left(\omega/\omega^*\right)^p C''_{IJ}(\omega^*). \qquad (11)$$

where p is equal to 0, 1 or 2 [5]. Over the entire frequency range, the parameters $C'_{IJ}(\omega)$ are independent of the frequency, while the imaginary parts of $C_{IJ}(\omega)$ are assumed to vary with the frequency as a fixed p^{th}-degree polynomial. The aim of this approach is to identify the parameters $C'_{IJ}(\omega^*)$ and $C''_{IJ}(\omega^*)$ of the chosen model (11). As already mentioned, all the functions $C_{IJ}(\omega)$ (11) have the same frequency dependence. However the energy loss mechanisms –related to the stiffness imaginary parts– may differ with the mechanical excitation, by example with the ultrasonic wave nature. This justifies the introduction of a local formalism which allows $C_{IJ}(\omega)$ frequency-dependences to be independent from each other.

A local parameterization of the stiffness variations with the frequency is then proposed by approximating the functions $C_{IJ}(\omega)$ around a given angular frequency ω^* by its 1st order Taylor expansion

$$C_{IJ}(\omega^* + \delta\omega) = C_{IJ}(\omega^*) + \frac{dC_{IJ}}{d\omega}(\omega^*)\delta\omega + \mathrm{O}(\delta\omega^2). \qquad (12)$$

Thus, the local variations of $C_{IJ}(\omega)$ around ω^* can be described by the following four real-valued parameters

$$\left\{ C'_{IJ}(\omega^*), C''_{IJ}(\omega^*), \; p'_{IJ}(\omega^*) = \frac{dC'_{IJ}}{d\omega}(\omega^*), p''_{IJ}(\omega^*) = \frac{dC''_{IJ}}{d\omega}(\omega^*) \right\}. \qquad (13)$$

Therefore, this local description allows one to make no assumption on the dependences of the stiffnesses over the entire frequency range.

Thus, instead of considering at the same time all the frequency components, the local approach considers each frequency component independently from each other. The aim being to characterize the viscoelastic properties of materials over the entire frequency range, identification of the stiffnesses is performed then at each frequency of the above mentioned range D_ω. Observe that the split of the domain D_ω into narrow bands allows the analysis of the frequency coherence between the sets of experimental and predicted signal.

Table 1: Viscoelastic properties of the composite material used for simulation at a fixed angular frequency ω^*. The frequency variation used for simulation are different for each stiffness constant.

$(\mathbf{1,3})$ plane	C_{11}	C_{33}	C_{13}	C_{55}
$C'_{IJ}(\omega^*)$ (GPa)	12	136	5.4	6,2
$C''_{IJ}(\omega^*)$ (GPa)	0.65	1.1	0.23	0.22

Inversion process validation

The sensitivity of the optimization technique to the frequency variations of the both real and imaginary parts of the stiffnesses is examined and quantified in this section. Signals propagating in physically realistic dispersive orthotropic materials are simulated for various directions in an principal plane, by using stiffnesses which satisfy the frequency-dependent model (6), and of which values at ($\omega^* =$) 2 MHz are reported in table 1. Observe that, due to the material symmetry, only the four complex-valued stiffnesses summarized in this table affect the wave propagation. The material density is around 1560 kg/m^3.

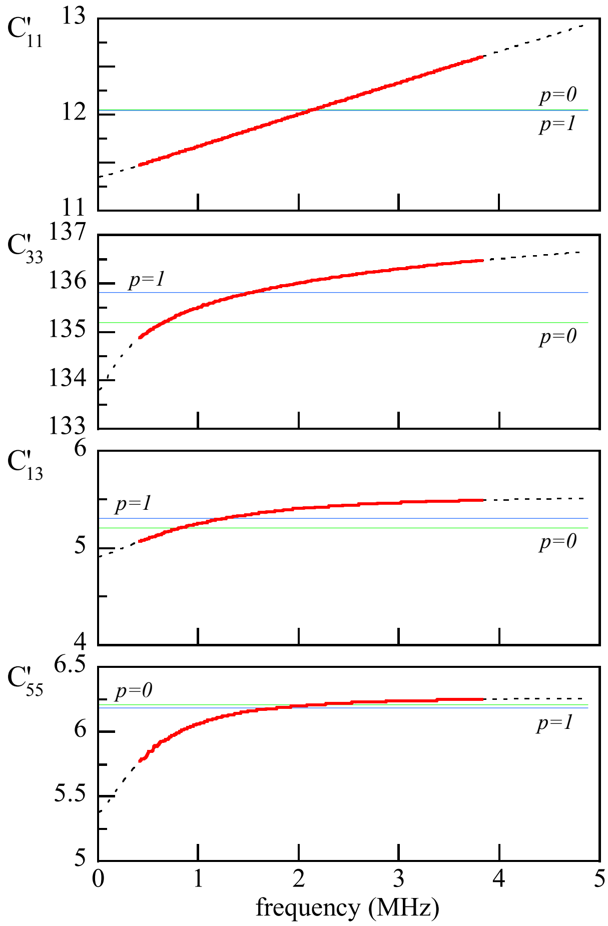

Figure 2: Real parts (in GPa) of all the stiffnesses as a function of the frequency. The dashed and solid lines correspond to the simulated and recovered dependences, respectively.

The simulated frequency dependences for the both real and imaginary parts of the stiffnesses, are represented by dashed lines in Figs. 2 and 3, respectively. All these frequency variations have been deliberately chosen different from each other. Finally, the "local" method is applied to a set of "causal" synthetic waveforms which have been generated from the chosen stiffness tensor $\left(C_{IJ}(\omega)\right)$. The "local" identification is then performed by using a sliding narrow frequency-band reduced to five frequency components. In addition, comparison with the "global" formalism is carried out by considering the non-causal rheologic model (11). The imaginary stiffnesses are assumed to be frequency independent $(p = 0)$ or even linear-frequency dependent $(p = 1)$. The results for the "local" and "global" identifications of the frequency-dependent stiffnesses are displayed Figs. 2 and 3, by bold and solid lines, respectively.

Except some numerical instabilities on the imaginary stiffness C''_{55} at low frequency, Figs. 2 and 3 show an excellent coincidence between the simulated material properties and those identified by the "local" method.

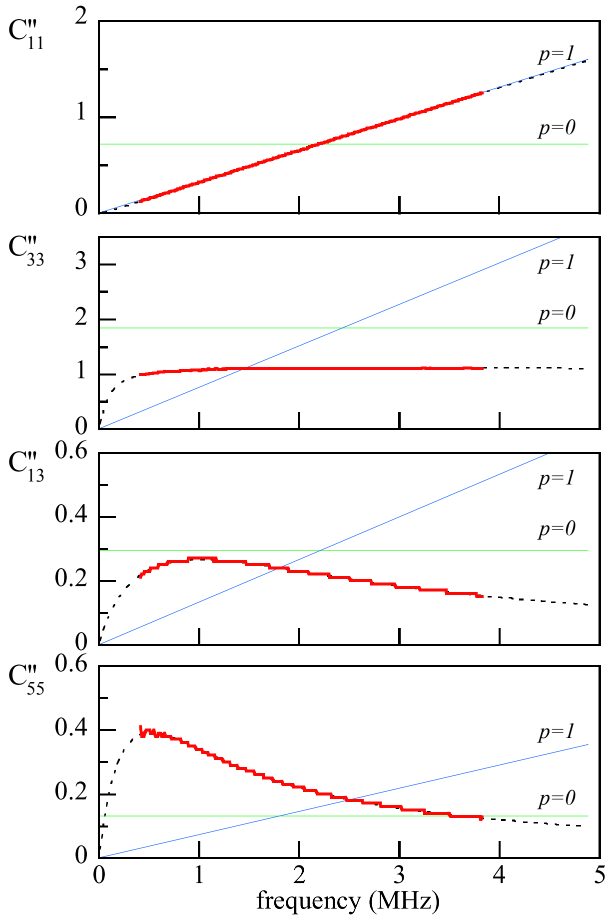

Figure 3: Imaginary parts (in GPa) of all the stiffnesses as a function of the frequency. The dashed and solid lines correspond to the simulated and recovered dependences, respectively.

Concerning the reconstruction obtained by the "global" method, the variations of the imaginary parts are never properly identified (except for the variable C''_{11}, when $p = 1$). As for the determination of the real parts, the relative errors remain lower than 4%. However the dispersion is not exactly characterized. Thus, in the case of complex viscoelastic behaviors, the "local" approach is more appropriate.

Conclusion

The set of performed simulations confirms the advantage of the "local" identification technique, which allows one the reconstruction of material constitutive laws without making any assumption about their frequency-dependence. However, the efficiency of this approach remains closely related to the match between the experimental conditions and the modeling assumptions. Noise in signals or disregard of plane wave condition indeed significantly affect the identification process.

In addition, one may wonder about the modeling of heterogeneous materials, such as unidirectional fiber-reinforced composites, by equivalent homogeneous media. As suggested by the continuous medium description, have the properties of shear waves propagating along the fiber direction, to be identical to those of shear waves propagating normally to the fibers?

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