ACOUSTOOPTIC COUPLING OF MODES IN QUASI-PERIODIC STRUCTURES

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Abstract

Counterpropagating modes of a single-mode guided electromagnetic wave (for example an optical fibre) can be coupled by a quasi-periodic structure with a period of the order of half the wavelength (Bragg grating). Co-propagating modes can instead be coupled by structures with a much longer period, possibly acoustic waves. Both these concepts have already been applied to the realization of a variety of optical fibres and integrated optical components. We propose a combination of the two concepts, and develop a numerical coupled-mode approach (based on $4 \times 4$ matrices, using Matlab or Scilab) for the simulation of spectral transmission and reflection characteristics of two waveguide modes, each with a Bragg reflector, coupled by an acoustic wave. We show that complex highly-selective spectral characteristics can be obtained for this 4-input, 4-output device, and can be controlled by the acoustic wave intensity.

Introduction

The concept of coupling of waves in waveguides or fibres (for an overview see [1], [2]) has a lot of applications in components for telecommunications and sensors. Co-directional coupling is used in optical fibre couplers and WDM (Wavelength Division Multiplexers), and the coupling between two waveguides in integrated optics can be controlled with electric fields, providing electro-optical switches. Co-directional coupling between waveguides of different propagation constant can be obtained with gratings, which can also be realized with acoustic waves, providing acousto-optical switches.

Contra-directional coupling, with the use of Bragg gratings with a period equal to half the wavelength of light, can provide narrow-band reflecting filters. Special Bragg gratings have also been produced with two parts separated by a half-period phase shift, which provide a very narrow transmission window at the center of the reflecting band. Bragg reflectors are used in fibres as narrow-band filters, laser mirrors, and in sensors due to their sensitivity to temperature and strain. With suitable variations of amplitude and phase along the fibre, various reflection/transmission spectral characteristics can be obtained [3].

In this numerical study we show that by combining co-directional coupling with an acoustic wave and contra-directional coupling with static quasi-periodic Bragg gratings in two waveguides or a two-mode fibre, a wide range of complex characteristics can be obtained, as function of acoustic intensity and of the wavelength of light. Devices based on this concept could provide complex switching patterns in the four outputs as a function of acoustic signal intensity for a fixed wavelength, or different spectra for each output.

Propagation equations

In general we consider a contra-directional coupling with a quasi-periodic structure, that is a Bragg grating with variable amplitude and phase, as this allows a wider spectrum of possibilities than a uniform grating. With reference to figure 1, we have a first guide with two traveling waves, 1 and 2, and a second one with waves 3 and 4.

![Figure 1: Example of a 4-mode device.](image)

Usually we will consider the input on port 1, i.e. $A_1(0) = 1$, with no input on ports 2,3,4: $A_2(L) = A_3(0) = A_4(L) = 0$. We observe the outputs $A_2(0)$ (reflection on the same guide) , $A_1(L)$ (transmission on the same guide), $A_4(0)$ (reflection on the other guide), $A_3(L)$ (transmission on the other channel). Then the equations describing the $z$-dependence of the four mode amplitudes are:

$$\frac{dA_1}{dz} = \alpha A_2 + k_1 A_3 , \quad \frac{dA_2}{dz} = \alpha' A_1 + k'_2 A_4 ,$$

$$\frac{dA_3}{dz} = k'_1 A_4 + \alpha' A_1 , \quad \frac{dA_4}{dz} = k_2 A_3 + \alpha A_2 ,$$

where $\alpha'$ is the complex conjugate of $\alpha$, $A_i$ are the amplitudes, $k_1 = i \kappa_1 \exp(-2i\delta_1 z)$, $k_2 = i \kappa_2 \exp(-2i\delta_2 z)$, $\alpha = -i \alpha_0 \exp(-i\delta_2 z)$ are the couplings and $\delta_i$ are the detunings.
The equations can be conveniently written in the matrix form:

\[
\frac{d\mathbf{A}}{dz} = \begin{pmatrix} 0 & \alpha & k_1 & 0 \\ \alpha' & 0 & 0 & k_2' \\ k_1' & 0 & 0 & \alpha' \\ 0 & k_2 & \alpha & 0 \end{pmatrix} \mathbf{A}, \quad \mathbf{A} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix}.
\]

**Numerical solution**

If the coupling coefficients were constant then the analytic solution of the system of equations is straightforward. However, for non-constant coupling coefficients we have to use a numerical procedure. The most part of standard numerical differential-equation solvers (e.g. “ode” function in Matlab) allow boundary conditions only on one side. Therefore we solve the 4 equations with all boundary conditions at \( z = 0 \), for 4 cases:

\[
\mathbf{A} = (1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1).
\]

We have now a linear system giving \( A_1(L), A_2(L), A_3(L), A_4(L) \) as combinations of \( A_1(0), A_2(0), A_3(0), A_4(0) \). If we have a complex Bragg grating, with different parts with a discontinuity (like a \( \lambda/4 \) shift) we just have to multiply the matrices corresponding to each section. But actually we know two conditions on one side and two on the other, while the unknown are also two at \( z = 0 \) and two at \( z = L \). We need then to “partially” invert the matrix of these coefficients. As we have found no such computer routine implemented in Matlab, Scilab or other numerical computer languages, we have written a Matlab/Scilab module which reshuffles data and unknowns in a linear system. We have then used these modules to write programs giving the outputs as functions of light wavelength or acoustic intensity or both.

We developed three program modules: **qua.m** (for the numerical solution of the matrix differential equation), **linear.m** (for solving a linear system of equations where some of the unknowns are actually constrained to some fixed value and some of the coefficients have to be determined accordingly) and a program to display outputs as functions of specific variables: wavelength, acoustic amplitude, or both.

**Examples**

We show here some examples illustrating the complex pattern of wavelength-selective outputs possible with such structures. We consider two “single-mode” waveguides with Bragg reflectors, coupled by an acoustic wave. The Bragg reflectors have a half-period shift in the middle, which induces a very narrow transmission band and a high field inside. In the absence of acoustic wave, wave 1 is coupled only with 2, and 3 with 4.

Figure 2 shows the spectrum of each of the 4 outputs for a suitable value of the acoustic intensity. In this case different (closely spaced) wavelengths are routed to each of the 4 outputs. Or a wide band input produces 4 narrow-band outputs, each on a different output port, with an efficiency of more than 80%.

In figure 3 light of a fixed wavelength is sent on the same system, and the output intensity of each of the 4 outputs is plotted as a function of acoustic intensity. We see that with suitable values of the acoustic amplitude, almost all the output can be directed to each of the 4 outputs (0,0.02,0.075,0.1); for other values we can have the light output equally distributed among the 4 (\( \sim 0.006 \) and \( \sim 0.09 \)), or equally distributed between the two backward channels (acoustic amplitude \( \sim 0.05 \)) or between the two forward channels (\( \sim 0.11 \) or \( \sim 0.13 \)).

We have also written a program for analyzing the outputs as functions of both light wavelength and sound intensity. A 3D plot of one of the outputs (\( A_2(L) \)) is
shown in figure 4. We see that the whole output spectrum varies as a function of acoustic power, giving either a narrow band or three equally spaced bands.

Figure 4: Spectrum (0..200) as a function of the acoustic amplitude (0..80). The first waveguide does not have any Bragg grating, the second waveguide contains a lambda/4-shift grating.

Conclusions
We have shown that in two waveguides with Bragg gratings, which can be coupled with an acoustic wave, the output configurations as functions of wavelength and of acoustic intensity can be various and complex, giving the possibility to tailor specific acousto-optical devices. Particular cases will then be studied for specific applications, as well as the generalization to multiple waveguides, and the effect of random imperfections.

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References