SOME THEORETICAL AND EXPERIMENTAL ASPECTS IN ULTRASONIC TRAVEL-TIME METHOD FOR DIAGNOSTIC OF GRID-GENERATED TURBULENCE.

Tatiana.A. Andreeva and William.W. Durgin
Mechanical Engineering Department, Worcester Polytechnic Institute, Worcester, MA
tatiana@wpi.edu

Abstract

The paper is devoted to the experimental investigation of the statistical characteristics of the grid-generated turbulence produced in a wind tunnel. Ultrasonic time-of-flight method using dual transducers is utilized to develop a methodology for determination of the correlation functions of turbulent velocity and sound speed fluctuations. The ultrasonic flowmeter equation is considered in the form that includes effects of turbulent velocity and sound speed fluctuations. The influence of temperature inhomogeneities on ultrasonic wave propagation is investigated using a set of experiments with a heated grid. Utilization of high-speed digital data acquisition cards and LabView software for the experiments allows collecting a significant amount of statistical data.

Introduction

The ultrasonic technique for measuring flows is offering great prospects for turbulent flow diagnostic[1]. There has been an intensive research work focusing on ultrasonic flow meters and their capabilities for measuring non-ideal flows[2],[3]. However, despite its advantages over traditional methods current ultrasonic applications fail to achieve theoretical accuracies. We believe the explanation lies in understanding the effect of turbulence on the ultrasonic wave propagation.

The fact that an acoustic wave carries some structure information of the turbulent medium after interacting with a medium makes it possible to use some statistical characteristics of the acoustic wave as a diagnostic tool to obtain some statistical information about the medium [5]-[10] Early theory and results on sound wave propagation in inhomogeneous moving medium are summarised in the books by Rytov [12] Tatarskii [11]. The modern theory of sound propagation in a moving random medium has been developing intensively since mid-1980s. The results are systematically described by Ostashev. [13]

We consider a locally isotropic, passive temperature field coupled with a locally isotropic velocity field, which is realized by introducing a heated grid in a uniform flow[14],[16]. The original goal of the present investigation was firstly, to develop a methodology for determination of correlation functions of turbulent velocity and sound speed fluctuations. Secondly, to demonstrate quantitatively that effect of thermal fluctuations is as much important as effect of velocity fluctuations on acoustic wave propagation. In order to do that the ultrasonic flowmeter equation is reconsidered, where the effects of turbulent velocity and sound speed fluctuations are included. The result is the integral equation in terms of correlation functions for travel time, turbulent velocity and sound speed fluctuations. Experimentally measured travel time statistic data with and without grid heating are approximated by Gaussian function and used to solve integral equation analytically. Turbulence spectral models for sound propagation in turbulent media were addressed by different authors and a summary of recent works presented in [15].

Methodology

In the experimental part of the study we utilize ultrasonic pulses traveling in straight paths as shown in Figure1. The sound propagates across a grid-generated turbulence from a transmitter to a receiver separated by a distance s. The flowmeter equation may be used to derive an expression for a travel time t of a wave traveling from the speaker to microphone.

$$t = \int_s \frac{dy}{c-u} \approx t_0 + \frac{1}{c^2}\int_s u\,dy; u = U\sin\beta + u' \qquad (1)$$

where t_0 is a travel time in the undisturbed media, U is a mean velocity, c is a sound speed, u' are fluctuations of the mean flow velocity. In Equation (1) we neglected the terms of order $U/c, U^2/c^2$.

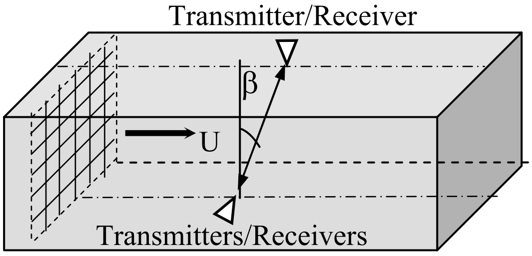

Figure 1. Sketch of wind-tunnel test section with ultrasonic flowmeter.

In the experiment the only parameter that is measured is a travel time of ultrasound pulses $t(s)$. The data from experiments performed for a finite number of different lengths s and s' are collected. For each of those lengths a value $t(s)t(s')$ is calculated. Averaging the latter through the entire ensemble results in a space correlation function of travel time t $K_t(s,s')$. Our objective here, then, is to construct spatial correlation functions for turbulent velocity u' and sound speed fluctuations c'. Perturbation analysis applied to (1) leads to the following expression:

$$t(s)=\frac{s}{\overline{c}}-\frac{U\sin\beta}{\overline{c}^2}-\frac{1}{\overline{c}^2}\int_s (u'+c')dx \qquad (2)$$

Here we assumed that $U \ll \overline{c}$ and terms $u'/\overline{c}$ and $c'/\overline{c}$ are the same order infinitesimal compare to unity. In order to construct the correlation function we introduce new variable $t^0(s)=t(s)-\langle t(s)\rangle$, where angular brackets mean an operation of averaging. Then,

$$t^0(s)t^0(s')=$$

$$\frac{1}{\overline{c}^4}\left[\int_s\int_{s'}\{u'(x)u'(x')+c'(x)c'(x')\}\,dxdx'\right]+ \qquad (3).$$

$$\frac{1}{\overline{c}^4}\left[\int_s\int_{s'}\{c'(x)u'(x')+u'(x)c'(x')\}\,dxdx'\right]$$

For the velocity measurement used here, the data were collected in the isotropic region of flow, so that u' and c' are not correlated. Consequently, the space correlation function of time can be defined as

$$K_t(s,s')=\frac{1}{c^4}\left[\int_s\int_{s'}(K_{u'}(x,x')K_{c'}(x,x'))dxdx'\right] \qquad (4)$$

It is important to emphasize that the space correlation function $K_{u'}(x,x')$ alone can be defined based on data from room temperature experiments. Then, data from heated air experiments can be used to identify $K_{c'}(x,x')$, knowing $K_{u'}(x,x')$ that is taken unchanged [[17]] from the room temperature experiment and $K_t(s,s')$.

Experimental arrangement

The experiments were carried out in the $1.75''\times 11.62''\times 45.25''$ test section of low turbulence, low speed open circuit type wind tunnel. The velocity and temperature fluctuations were generated simultaneously using a heated grid. The square-mesh biplane heated grid was composed of 16 round chromalox heating rods, model TSSM 14XX[16]. To insure the good quality of the grid, the heating rods were inserted in hollow aluminum rods with diameter of $0.25''$ positioned $1''$ between centers. The mesh, M, was therefore $1''$ and the grid solidity was 0.64. Nine cases of different distances L for two different temperatures $\mathrm{T}=59^\circ F$ and $\mathrm{T}=159^\circ F$, are studied. The defined temperatures correspond to the temperature of aluminum rods of the grid. The angle β is changed from 0 to 40 degrees with 5-degree step. The measurements were collected at $x/M=30$, where radiation effects are negligible. The mean flow velocity U was 3.5 m/s. The Reynolds number R_M based on M and U was about 10000 and the corresponding Péclet number $Pe_M=\Pr \mathrm{Re}_M$ ~4350; $\Pr=0.725$ for the working fluid air. . A more detailed description of the experimental particulars may be found in [10].

Experimental results and discussion

Following the strategy described in the Section II we have to convey to the flowmeter integral equation for the case of temperature of $59^\circ F$,

$$K_t^{F59}(s,s')=\frac{1}{c^4}\int_0^s\int_0^{s'}K_{u'}(x,x')\,dxdx' \qquad (5)$$

In many practical problems, the form of the correlation function is not known. However, its general shape is often approximated by a Gaussian function. It is very convenient for analytical studies and, besides, it allows taking into consideration the effect of the largest inhomogeneities in a medium on the statistical moments of a sound field[13]. We represent the correlation function in Equation (5) by

$$K_t^{59F}(s,s')=\sigma_t^2\Big|_{F59}\exp\left(-(s-s')^2/l^2\right)= \\ =\sigma_t^2\Big|_{F59}\exp\left(-\tau^2/l^2\right) \qquad (6)$$

Here σ_t^2 is a variance of travel time fluctuations. Selection of l is problematic. In some applications l is chosen to be equivalent to a Taylor microscale. A better procedure is to choose l on the basis of the integral length scale of the turbulence[13]. Since we are considering travel time fluctuations in rigorously speaking diffractive media in the approximation of ray acoustic, we should realize, that there is some uncertainty up to numerical coefficient[12]. Figure 2 demonstrate correlation function of travel time obtained using experimental data as a function of separation distance x compared with Gaussian curve providing the best fit. Experimental data allows us to determine unknown coefficients, $\sigma_t^2=9.85e-15$ and $l^2=0.0036$. Integration of Equation (5) with known leads to the following form of correlation function of turbulent velocity

$$K_{u'}^{F59^\circ}(\tau)=c^4\left[2\frac{\sigma_t^2\Big|_{F59^\circ}}{l^2}\exp\left(-\tau^2/l^2\right)\right]- \\ c^4\left[4\frac{\sigma_t^2\Big|_{F59^\circ}}{l^4}\tau^2\exp\left(-\tau^2/l^2\right)\right] \qquad (7)$$

It is apparent that correlation function of turbulent velocity is no longer Gaussian, although Gaussian part is present in the first term in Equation (7) and the second term is vanishing rapidly with distance.

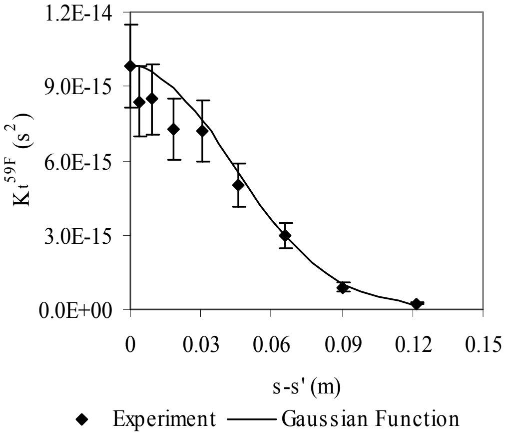

Figure 2 Correlation function of travel time along with Gaussian function providing the best fit.

Figure 3 shows the correlation function of turbulent velocity for our particular experimental data. Variance of velocity fluctuations is $\sigma_{u'}^2 = 2c^4 \frac{\sigma_t^2}{l^2} = 0.0801$. At the same time we know, that $\sigma_{u'} = \left\langle u'^2 \right\rangle^{0.5}$, meaning that for our experimental conditions we have very small values of $u'^2 / c^2 \sim 6.9 \cdot 10^{-7}$, which is in a very good correspondence with data[17]. The ratio of a turbulent velocity to the mean velocity is $\alpha = u'/U \cdot 100\% \sim 6\%$, which is typical for experiments performed in grid turbulence. Figure 4 shows the cross correlation function of travel time at temperature 159F, again along with Gaussian function providing the best fit

$$K_t^{F159}(s,s') = \sigma_t^2\Big|_{F159} \exp\left(-\tau^2 / l^2\right) \qquad (8)$$

Unknown coefficient is determined to be $\sigma_t^2\Big|_{F159} = 2.05\text{E-}13$

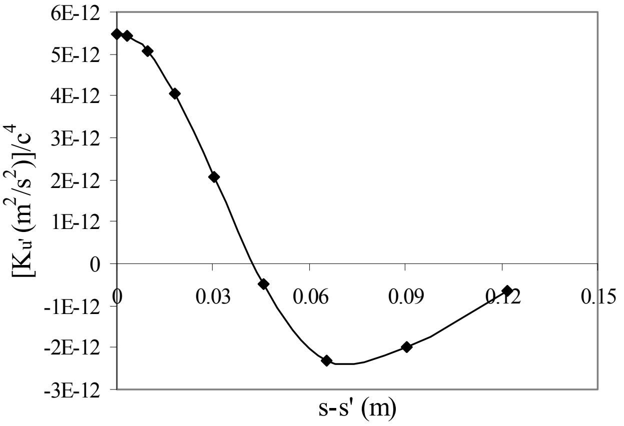

Figure 3 Experimentally obtained correlation function of turbulent velocity.

In accordance with the methodology, the next step is to find the correlation function of sound speed fluctuations. Correlation function of sound speed fluctuations can be found from the following equation

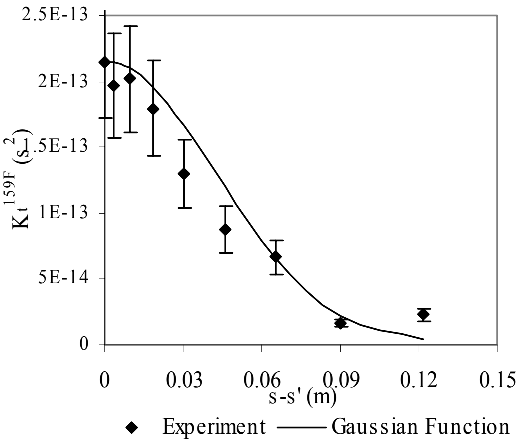

Figure 4. Correlation function of travel time obtained from experimental data collected at temperature of $159^\circ F$.

$$K_t^{F159^\circ}(s,s') - K_t^{F59^\circ}(s,s') = \frac{1}{c^4} \int_s \int_{s'} K_{c'}(x,x') dx dx' \quad (9),$$

where

$$K_t^{159F}(s,s') - K_t^{59F}(s,s') = (\sigma_t^2|_{159F} - \sigma_t^2|_{59F}) \times \times \exp\left(-\frac{(s-s')^2}{l^2}\right) = \Delta\sigma_t^2 \exp\left(-\frac{\tau^2}{l^2}\right) \qquad (10)$$

Substitution of Eq.(10) into Eq. (9) yelds

$$K_{c'}(s,s') = c^4 \left[2 \frac{\Delta\sigma_t^2}{l^2} \exp\left(-\frac{\tau^2}{l^2}\right) \right] - - c^4 \left[4 \frac{\Delta\sigma_t^2}{l^4} \tau^2 \exp\left(-\frac{\tau^2}{l^2}\right) \right] \qquad (11)$$

Figure 5 shows the correlation function of sound speed fluctuations.

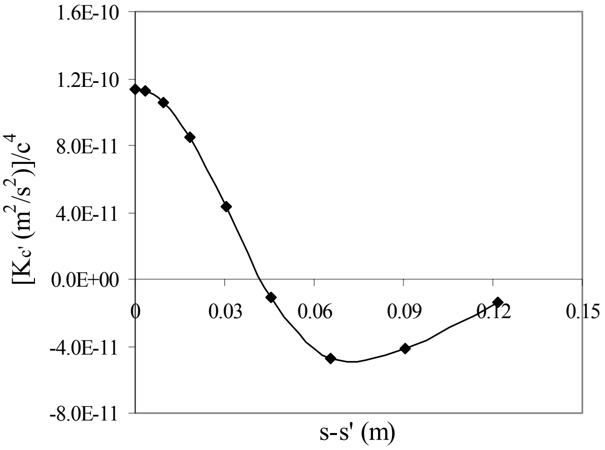

Figure 5 Correlation function of sound speed fluctuations.

The variance of the sound speed fluctuations is $\sigma_{c'}^2 = c^4 2\Delta\sigma_t^2 / l^2 \approx 1 m^2/s^2$. Neglecting humidity fluctuations, a speed of sound fluctuation is given to the first order by $\langle c'^2 \rangle^{0.5} = (c_0 / 2T_0) \langle T'^2 \rangle^{0.5}$ [15], where T_0 is a representative value of the temperature. Turbulence-level measurements made with heating the grid are consistent with results by Yeh and Van Atta [14]], namely

$$\beta \langle u'^2 \rangle^{0.5} / C_p R \sim 10^{-9}; \beta \langle T'^2 \rangle^{0.5} / R \Pr \sim 10^{-5} \quad (12),$$

where β is a coefficient of thermal expansion for air, C_p is a specific heat and R is a universal gas constant.

Conclusions

The importance of the presented paper is that we attempted to show the influence of turbulence on ultrasound wave propagation by solving the integral flowmeter equation that included the sound speed fluctuation term.

Developed a methodology for determination of statistic characteristics of isotropic, homogeneous turbulence. The methodology involves experimental and theoretical parts. Experimental part utilizes ultrasonic time-of-flight method using dual transducers in a wind tunnel. Utilization of high-speed digital data acquisition cards and LabView software for the experiments allows collecting a significant amount of statistical data.

The theoretical part of the methodology deals with ultrasonic flowmeter equation, which is reconsidered in order to include the term corresponding to sound speed fluctuations, which previously were neglected. The result is an integral equation for the corresponding correlation functions. The influence of temperature inhomogeneous on ultrasonic wave propagation is investigated using a set of experiments with a heated grid. Experimentally measured travel time statistic data with and without grid heating were approximated by Gaussian function and used to solve integral equation analytically in terms of correlation functions of turbulent velocity and sound speed fluctuations. During the experiment we observed that thermal fluctuations have significant influence on the sound speed propagation and thus it may not be neglected as has often been supposed previously both in experiment and theory.

References

[1] Lynnworth, L.C., "Ultrasonic Measurements for Process Control," Academic Press, San Diego, CA, 1989.

[2] Johari, H. and Durgin, W.W., "Direct Measurements of Circulation using Ultrasound," *Experiments in Fluids*, Vol. 25, 1998, pp.445-454.

[3] Yeh, T.T. and Espina, P.I., "Special Ultrasonic Flowmeters for In-situ Diagnosis of Swirl and Cross Flow", *Proceedings of ASME Fluids Engineering Division Summer Meeting*, FEDSM2001-18037, 2001.

[4] L.A. Chernov, Wave Propagation in a Random Medium, (McGraw-Hill Book Company, Inc, 1960).

[5] V.E. Ostashev, "Sound propagation and scattering in media with random inhomogeneities of sound speed, density and medium velocity," *Waves in Random Media* Vol. 4, 1998, pp. 403-428.

[6] G.A. Daigle, T.F.W. Embleton and J.E. Piercy, "Propagation of sound in the presence of gradients and turbulence near the ground," *Journal of Acoustical Society of America,* Vol. 79, 1986, pp.613-627.

[7] H.E. Bass, L.N. Bolen, R. Raspet, W. McBride, and J. Noble, "Acoustic propagation through a turbulent atmosphere: Experimental characterization," *Journal of Acoustical Society of America,* Vol. 90 (96), 1991, pp.3307-3313.

[8] M. Karweit, Ph. Blanc-Benon, D. Juvé, and G. Comte-Bellot, "Simulation of the propagation of an acoustic wave through a turbulent velocity field: A study of phase variance $^{a)}$," *Journal of Acoustical Society of America,* Vol. 89 (1), 1991, pp.52-62.

[9] B. Iooss, Ph. Blanc-Benon and C. Lhuillier, "Statistical moments of travel times at second order in isotropic and anisotropic random media," *Waves in Random Media* Vol. 10, 2000, pp. 381-394.

[10] T.A. Andreeva &W.W. Durgin, "Ultrasound Technique for Prediction of Statistical Characteristics of Grid-Generated Turbulence," *AIAA Journal* to appear in 2002.

[11] Tatarski, V.I., "The Effect of the Turbulent Atmosphere on Waves Propagation," Israel Program for Scientific Translation, Jerusalem, 1971.

[12] Rytov, S.M., Kravcov, Yu.A., Tatarskii, V.I., "Introduction to Statistical Radio physics, Random fields," Moscow, *Science* , 1978.

[13] V.E. Ostashev, "Acoustics in Moving Inhomogeneous Media," (E & FN SPON, London, UK, 1997).

[14] Yeh, T.T. and Van Atta, C.W., "Spectral Transfer of Scalar and Velocity Fields in Heated-grid Turbulence," *Journal of Fluid Mechanics*, Vol. 58 (2), 1973, pp. 233-261.

[15] Wilson, D.K., "A Turbulence Spectral Model for Sound Propagation in the Atmosphere that Incorporates Shear and Buoyancy Forcings," *Journal of Acoustical Society of America*, Vol. 108 (5), 2000, pp. 2021-2038.

[16] Sreenivasan, K.R., Tavoularis, S., Henry, R. and Corrsin, S., "Temperature Fluctuations and Scales in Grid-generated Turbulence," *Journal of Fluid Mechanics*, Vol. 100(3), 1980, pp. 597-621.

[17] Sepri, P., "Turbulence in Constant Density Fluids with Thermal Fluctuations," Ph.D. Dissertation, University of California, San Diego, 1971.