WAVE PROPAGATION IN MICROSTRUCTURED SOLIDS

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Abstract

Microstructured solids are characterized by intrinsic space scales that introduce the scale-dependence into the governing equations. Based on the Mindlin model, the simplest scale-dependent 1D equation is derived taking into account the proper scalings against the wavelength (or frequency) of the initial excitation. The fundamental properties of such a model are: (i) hierarhical structure of the governing equation distinguishing macro- and microbalance; (ii) changes in the wave speed due to microstructure; (iii) definite influence of dispersion (due to the inertia of microstructure the double dispersion appears). Nonlinearities in both scales (macro- and microscale) can be taken into account. Numerical analysis by the finite volume method supports theoretical considerations.

Introduction

In recent years much attention is devoted to the stress analysis of microstructured materials. This concerns granular materials, polycrystalline solids, ceramic composites, functionally graded materials, alloys, damaged materials, etc. Such materials are characterized by the existence of intrinsic space-scales in matter, like the lattice period, the size of a crystallite or a grain, the distance between the microcracks that introduce the scale dependence into the governing equations. From the theoretical viewpoint, several mathematical theories of continua have been elaborated in order to catch the influence of certain inclusions in media.

Beside the theory, experiments provide often only indirect data on physical effects related to the behaviour of materials with microstructure. In terms of wave motion, there are several physical effects that should be taken into account like attenuation, dispersion, possible localisation of damage, stress-induced phase-transition etc. In addition, the influence of nonlinearities causes nonadditivity of physical effects and possibly also the emergence of localised waves (solitons). That is why the mathematical models should be elaborated with care in order to be consistent with experiments. Correctly formulated and solved inverse problems make a backbone for all the methods of nondestructive testing (NDT). In NDT, acoustic methods have an important role but again, the theory and experiments should be consistent.

In this paper, a rather general mathematical model is described in order to demonstrate the role of dispersion for waves in microstructured materials [1,2]. The modelling results will be compared with those based on the lattice theory or theory of periodic structures. Speaking about NDT at least 2D models should be derived for wave beams. Nevertheless, the on-axis distribution of the wave field is rather fully described also by 1D model if diffractive effects are small (see [3]). At least, if a 1D model is satisfactorily described, its generalisation to a 2D model could be done easily [4].

We shall first describe a basic 1D model following [1,2] and show then its place among the other models. The main physical effects governed by the model are discussed. The role of nonlinearities is analysed separately. Finally, the analytical considerations are supported by numerical simulation using the finite volume method.

The basic model

According to [1], the fundamental balance laws for microstructured materials can be formulated separately for macroscopic and microscopic scales. Here we use the simplest 1D model and get [2]

$$\rho\, u_{tt} = \sigma_x, \tag{1}$$

$$I\, \varphi_{tt} = \eta_x + \tau, \tag{2}$$

where u is the macrodisplacement, φ is the microdeformation, σ is the macrostress (Piola stress), η is the microstress and τ is the interactive microforce; ρ is the macrodensity, I is the microinertia and indices x and t denote the differentiation. As usual, we have [1,2]

$$\sigma = \frac{\partial W}{\partial u_x}, \quad \eta = \frac{\partial W}{\partial \varphi_x}, \quad \tau = \frac{\partial W}{\partial \varphi}, \tag{3}$$

where W is the free energy. At this moment, we neglect the dissipation.

Now the simplest free energy function describing the influence of a microstructure is a quadratic function

$$W = \frac{1}{2}\,\alpha\, u_x^2 - A\,\varphi\, u_x + \frac{1}{2} B\,\varphi^2 + \frac{1}{2}\,C\,\varphi_x^2, \tag{4}$$

with α, A, B, C - constants.

For further analysis we introduce dimensionless variables and use the slaving principle for eliminating φ. Omitting the details (for those see [2]), the final governing equation in terms of nondimensional displacement U reads:

$$U_{TT} = \left(1 - \frac{A^2}{\alpha B}\right) U_{XX} + \\ + \delta \frac{A^2}{B^2} \left(I^* U_{TT} - \frac{C^*}{\alpha} U_{XX}\right)_{XX}. \qquad (5)$$

Here $\delta = l^2\ L^{-2}$, l is the scale of the microstructure and L, for example, is the wavelength of the excitation, while X and T are nondimensional space and time, respectively. The quantities I^* and C^* are determined by $I = \rho\ l^2\ I^*$ and $C = l^2\ C^*$.

Equation (10) is the sought "skeleton" of the wave equation for microstructured solids.

Nonlinearities

In NDT, physical nonlinearities are also of importance. It means that the free energy function (4) should be implemented by cubic terms $\frac{1}{6}\ \beta\ u_x^3$ and $\frac{1}{6}\ M\ \psi^3$. In this case the nonlinearities are in effect only for macro- and microstructure and not for the interacting forces. It seems to be rational to look for nonlinearities separately: either in the macroscale or in the microscale. In the macroscale ($\beta \neq 0, M = 0$), an additional term

$$+k\ U_X\ U_{XX} \qquad (6)$$

appears in the r.h.s. of Eq.(5) while in the microscale ($\beta = 0,\ M \neq 0$), the addition to the r.h.s. of Eq.(5) reads

$$-\delta^{3/2}\ \epsilon\ \frac{A^3\ M^*}{\alpha\ B^3} \left[\frac{1}{2}\ (U_{XX})^2\right]_{XX}, \qquad (7)$$

where $M = l^2\ M^*$, $k = \epsilon\ \beta\ \alpha^{-1}$, $\epsilon = U_0\ L^{-1}$ and U_0 is related to the initial excitation.

Discussion

The basic model (5) for waves in microstructured materials reflects the following physical phenomena:

(i) it describes the wave hierarchy in Whitham's sense including two wave operators - one for macrostructure, another for microstructure; if the scale parameter δ is small then the last two terms, i.e. influence of microstructure can be neglected; if δ is large then on contrary, the influence of first two terms, i.e. influence of macrostructure is weaker and the process is governed by the properties of the microstructure;

(ii) the wave speed in the compound material is affected by the microstructure (1 versus $A\ \alpha^{-1}\ B^{-1}$) and clearly only $A = 0$ excludes this dependence.

(iii) the influence of the microstructure on wave motion is, as expected, characterized by dispersive terms; however, the double dispersion occurs due to the different higher order terms (U_{TTXX} and U_{XXXX}).

Among the other models, eq. (5) is more complete. In order to carry out comparison, we rewrite eq. (5) in a form

$$U_{TT} = c^2\ U_{XX} + \delta\ (\eta\ U_{TT} - \gamma\ U_{XX})_{XX}, \qquad (8)$$

where c is the velocity ($c < c_0 = 1$) and $\delta,\ \eta,\ \gamma$ are positive constants.

From lattice theory (see [5] and references therein) the well-known Born-Karman model yields

$$U_{TT} = c_0^2\ U_{XX} - \frac{1}{12}\ c_0^2\ a^2\ U_{XXXX} + \\ + b\ U_{XXXXXX} + ... \qquad (9)$$

where c_0 is the velocity and a - the spacial lattice constant.

From the theory of periodic structures [6], the corresponding wave equation reads

$$U_{TT} = \Omega_1^2\ U_{XX} + \frac{1}{3}\Omega_1\ \Omega_3\ U_{XXXX}. \qquad (10)$$

Here $\Omega_1 < c_0$ and $\Omega_3 < 0$.

If in a periodic structure only microinertia is taken into account, then the governing equation yields [7]

$$U_{TT} = c^2\ U_{XX} + h\ U_{TTXX}. \qquad (11)$$

First, the wave speed. Clearly all the models, except lattice theory (9) take the influence of the microstructure into account while the speed is definitely different from the wave speed c_0 in the macromaterial.

Second, the dispersion. Clearly the basic model (5) takes into account two phenomena: inertia of the microstructure (term U_{TTXX}) and velocity in microstructure (term U_{XXXX}). Other models are in this sense less general, paying the attention only to one or another phenomenon. The further analysis should show the accuracy of models (9), (10), and (11). The double dispersion is also important for describing longitudinal strain waves in rods [8], but the wave speed is then not affected.

We support our results by numerical simulation based on the finite - volume method [9] that permits to assign all the physical parameters to every volume element in a material (or a specimen). As an example, we

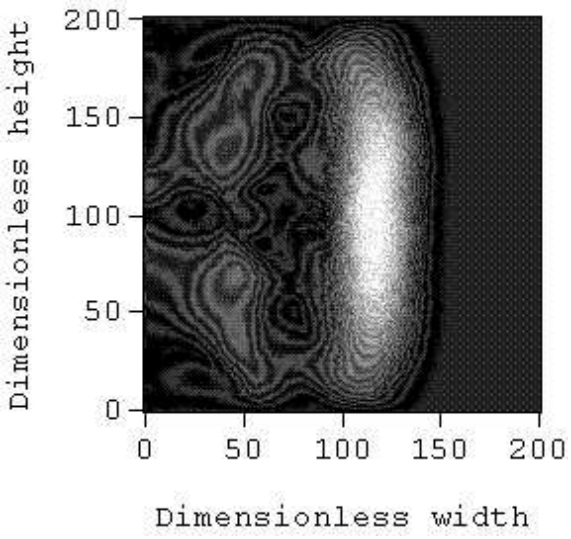

Figure 1: Wavefronts in metal-ceramic composite with ceramic reinforcement at 3 μs, $f = 0.75$.

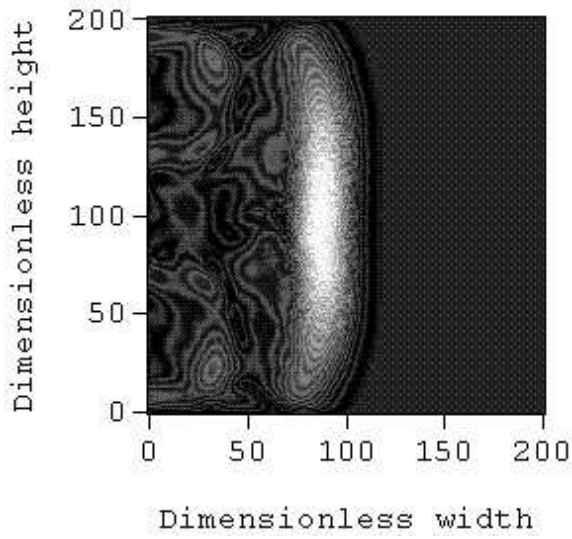

Figure 2: Wavefronts in metal-ceramic composite with ceramic reinforcement at 3 μs, $f = 0.25$.

consider the wave propagation in metal-ceramic composites. The elastic properties of the metal matrix and ceramic reinforcement are the following [10]: Young modulus 70 GPa and 420 GPa, Poisson ratio 0.3 and 0.17, and density 2800 kg/m^3 and 3100 kg/m^3, respectively. Volume fraction $f = V_c/V$, where V_c is the volume of ceramic particles and V is the total volume is varied. A Gaussian-type excitation was generated at the left boundary of a 2D specimen between 40 and 160 space steps (see Fig. 1)

$$\sigma_0(0,t) = \sigma_0 \sin^2(\pi(t-2t_r)/2t_r), \qquad (12)$$

where $\sigma_0 = 125\ MPa$ and $t_r = 10$. The initial wave length corresponds to 20 space steps. In Figs. 1,2 the wavefronts in metal-ceramic composite for volume fractions $f = 0.75$ and $f = 0.25$ are shown. The differences in wave speeds are obvious. This supports all those analytical models that foresee such changes (models (5), (10), (11)).

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