RESONANT BEHAVIOR OF N-SHELL CLUSTERS EMBEDDED IN WATER

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Abstract

Resonances of a single thin hollow cylindrical aluminum shell embedded in a fluid are well known. At low frequency, they are due to external circumferential A waves, that may easily couple two or more shells to each other, provided they are close enough. This induces a new resonant behavior of the N-shell cluster (N>1) that we are interested in. We study different clusters. We show that each resonance of a single shell splits into M resonances, with M depending on both the number of shells and the cluster geometry.

Introduction

A few authors have been interested in the comparison of the resonant behavior of a limited number of scatterers with that of one single scatterer. Huang and Gaunaurd[1,2] have studied the acoustic scattering by two spherical shells and shown a shift of low resonant frequencies to lower frequencies as the distance between shells decreases. Our work follows that of Kheddioui[3] and Lethuillier[4,5] who studied aligned thin cylindrical shells. For such shells, the low-frequency resonances are well separated, and correspond to an A-wave, which can easily couple two close shells, as most of its energy is in the surrounding fluid. When coupling occurs, the A-wave resonances of one single shell split into new ones. That split is the subject of this paper.

We first recall the method to determine the scattered field by N cylinders in a fluid that are insonified by a harmonic plane wave. Then we look at the resonances of different N-shell clusters, with different values of N and different spatial arrangements of the shells.

Analytic determination of the acoustic scattering by N shells

The analytic derivations used to determine the pressure scattered by a N-shell cluster are similar to Varadan's [6]. The details may be found elsewhere [7], along with the determination of the scattering S-matrix of the cluster.

We consider N hollow cylindrical elastic shells embedded in water. An incident harmonic plane wave propagates in the (x_1Ox_2) plane, with an incident angle α, and the shells are infinite in the Oz direction, so the problem is a two dimensional one. The observation point P is placed at (r,θ) in the global coordinates system and at (r_h,θ_h) in the local coordinates system of cylinder h, centered at (d_h,χ_h). Figure 1 presents the geometry and the notations.

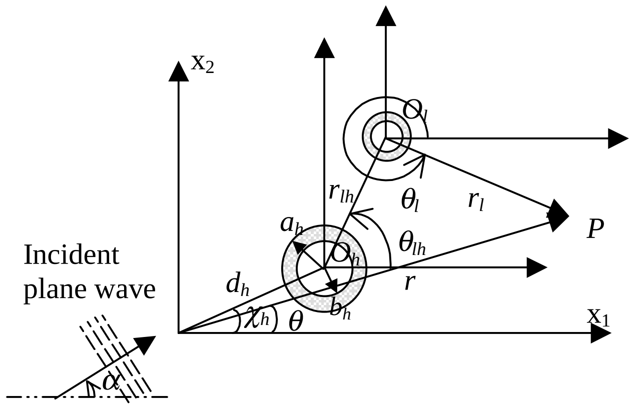

Figure 1: Acoustic scattering by N cylindrical shells. Geometry of the problem

A $e^{-i\omega t}$ time dependence is assumed. The wave number k is equal to ω/c where c is the sound velocity in the fluid. The incident wave is then written as :

$$\phi_{inc}^{(0)}(\alpha,\theta) = e^{i\vec{k}.\vec{r}} = e^{ikr\cos\alpha\cos\theta}e^{ikr\sin\alpha\sin\theta} = \sum_{n=-\infty}^{+\infty} i^n J_n(kr)e^{in(\theta-\alpha)} \quad , \qquad (1)$$

where $J_n(kr)$ is the Bessel function of order n and argument kr. The wave incident on cylinder h is composed of the incident wave and of the waves scattered by all other cylinders. So, using the transition matrix $T^{(h)}$ of shell h, , the wave scattered by cylinder h in its local coordinates system is:

$$\phi_s^{(h)} = \sum_{m=-\infty}^{+\infty} C_m^{(h)} H_m^{(1)}(kr_h)e^{im\theta_h} \quad ,$$

$$C_m^{(h)} = \sum_{m=-\infty}^{+\infty} T_{mm}^{(h)} \left[e^{ikd_h\cos(\chi_h-\alpha)} i^m e^{-im\alpha} + \sum_{\substack{l=1 \\ l\neq h}}^{N} \sum_{n=-\infty}^{+\infty} G_{nm}^{lh} C_n^{(l)} \right] \qquad (2)$$

where the $C_m^{(h)}$ are unknown coefficients, $H_n^{(1)}$ the Hankel function of the first kind, and G_{nm}^{lh} the elements of the Graff matrix defined by:

$$G_{nm}^{hl} = (-1)^{m-n} e^{i(n-m)\theta_{lh}} H_{m-n}^{(1)}(kr_{lh}) . \qquad (3)$$

The second equation in relation (2). is the system to be solved in order to express the total scattered field Φ_s :

$$\Phi_s = \sum_{h=1}^{N} \phi_s^{(h)} = \sum_{h=1}^{N} \sum_{n=-\infty}^{+\infty} C_n^{(h)} H_n^{(1)}\left(kr_h\right) e^{in\theta_h} . \quad (4)$$

Use of the asymptotic development of the Hankel function allows the determination of the far field scattered amplitude $f(\theta,\alpha)$ defined as:

$$\Phi_{s\infty} = \underset{r\to\infty}{limit}\left(\Phi_S\right) = \frac{e^{i\left(kr-\frac{\pi}{4}\right)}}{\sqrt{kr}} f\left(\theta,\alpha\right). \quad (5)$$

Numerical study of resonances

The shells we consider are in aluminium with a density ρ=2790 kg/m^3, a longitudinal velocity c_l=6120 m/s and a shear velocity c_s=3020 m/s. The ratio of the inner radius b to the outer one, a, is equal to 0.9.

The resonance spectra are obtained from the backward form function F_∞, by plotting the frequency derivative *ds/dx* of the curvilinear abscissa *s* of the form function Argand diagram [8]:

$$\frac{ds}{dx} = \sqrt{\left(\frac{dF_\infty'}{dx}\right)^2 + \left(\frac{dF_\infty''}{dx}\right)^2} , \quad (6)$$

with

$$F_\infty = F_\infty' + iF_\infty'' = \sqrt{\frac{2}{\pi ka}} f\left(\alpha+\pi,\alpha\right), \quad (7)$$

and $x=ka$.

We look only at the frequency domain in which all resonances are due to the A-wave. No interesting phenomenon occurs at higher frequencies.

N aligned shells

Lethuillier showed that for N aligned and equally spaced shells in the eclipse configuration, *i.e.* when the incidence direction is parallel to the direction of alignment of the shells, each resonance of a single shell splits into N new ones when the shells are close enough *i.e.* β<2.3, with βa the distance between two centers.

Figure 2 shows the split of one resonance of the single shell into three different ones in the case of three aligned shells with β=2.06.

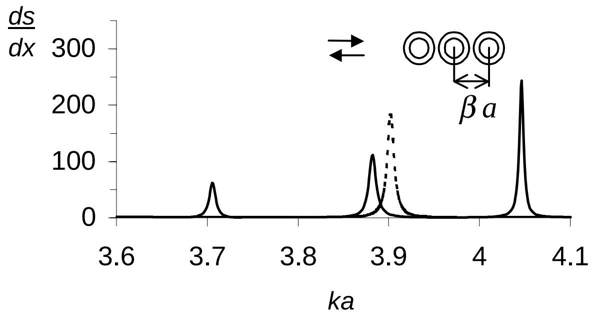

Figure 2: Zoom of the resonance spectrum of three aligned shells, β=2.06. The one shell resonance (dashed line), is split into three well-separated ones (solid line).

Lethuillier has shown [5] that the low frequency resonances of the N-shell grating are those of the N-1 shell grating, shifted towards lower frequencies because of an added mass effect, and that the highest frequency resonance is the only one due to the coupling of all N shells together.

Variation of the incidence angle, yet, induces a new split of the resonances. Each of the N resonances of the eclipse case may split, at one value of the incidence angle, into two new overlapping ones. Figure 3 shows the evolution of the resonance spectrum with the incidence angle. Amplitudes are coded in grayscale; light colors correspond to low amplitudes and dark colors to high ones. The solid line represents the position of the single shell resonance.

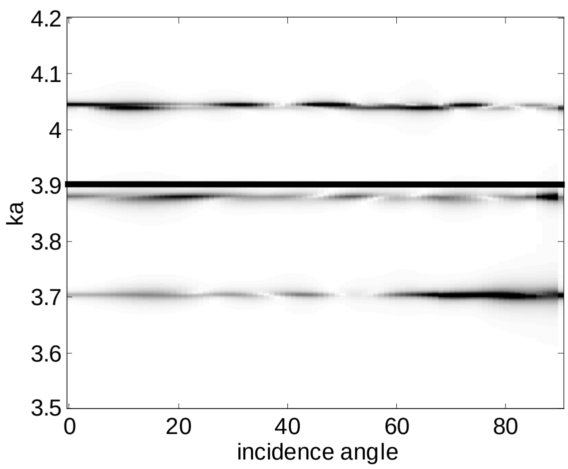

Figure 3: Three aligned shells, β=2.06. Resonance spectrum evolution with the incidence angle.

The same phenomenon occurs whatever the number N of shells: the A-wave resonances of one shell split into N well separated ones, as in the eclipse configuration, and each of these N resonances will split, for some value of the incidence angle, into two overlapping ones.

The situation is still more complicated for other shell configurations, as shown in the next section.

N non-aligned shells

The resonance spectrum evolution, with the incidence angle, for three shells placed at the apexes of an equilateral triangle is shown in Figure 4. In this case, each single shell resonance splits into four well-separated resonances, as may be seen in figure 4, and none of them ever splits again into overlapping ones.

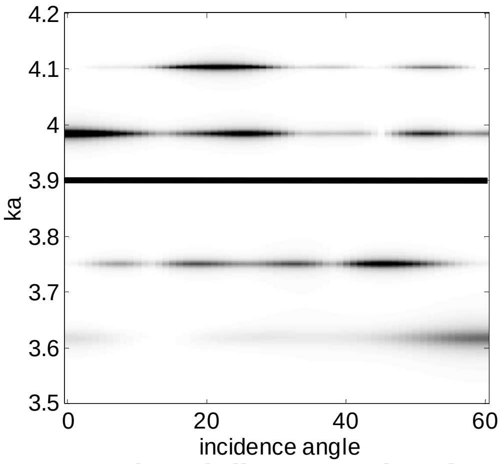

Figure 4: Three shells in an equilateral triangle configuration, β=2.06. Resonance spectrum evolution with the incidence angle. 0° corresponds to incidence on an apex.

The relation we found in the last section between the number of resonances and the number of shells is no more valuable in that case.

The resonance spectrum evolution for four shells placed at the apexes of a square is shown in figure 5.

In this case, each single shell resonance splits into three well separated new ones, and each one of them may split again, at some value of the incidence angle, into two overlapping ones, same way as we found for aligned shells. We have now four shells, and six resonances.

At this point, there seems to be no simple relation, contrary to the case of aligned shells, between the number of shells and the number of resonances each resonance of a single shell splits into.

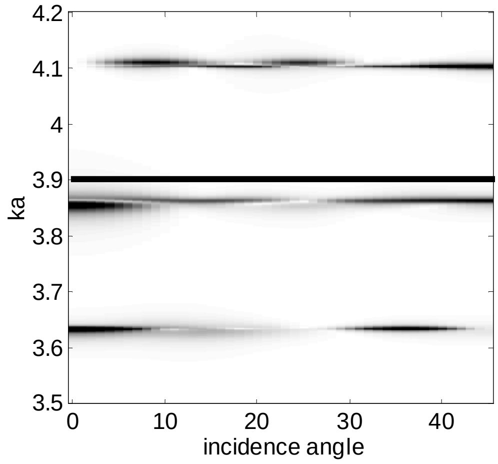

Figure 5: Four shells in a square configuration, β=2.06. Resonance spectrum evolution with the incidence angle. 0° correspond to incidence on an apex.

Now, on a single shell, all resonances are associated to the propagation of the A-wave with such a velocity that an integer number of wavelengths fits over the circumference of the shell. In case of clusters, of course, it is also rather tempting to associate each resonance to a closed propagation path corresponding to a standing wave. However, such an association is not so easy to find, and Decanini *et al.*[9]have proposed, as a first step, to explain the number of resonances of a given cluster from symmetry considerations. Each cluster they studied belongs to a different symmetry group, characterized by M irreducible representations. Using character tables, they decompose the far field form function into M sub-sums, each one corresponding to an irreducible representation. For two shells, there are four of them, as well as for three shells in an equilateral triangle configuration. Four shells in a square configuration correspond to a symmetry group with six irreducible representations. So far, then, the number of irreducible representations is equal to the number of resonances.

However, two shells and three aligned shells belong to the same symmetry group, but we found six resonances for the three aligned shells. It seems, then, that the number of irreducible representations of a symmetry group is indeed the number of resonances, provided two conditions are met : the cluster belongs to that symmetry group, and it is composed of a minimum number of shells.

Using the form function decomposition given in[9], we have plotted the resonance spectrum associated to each sub-sum, for a few clusters. Figure 6 corresponds to the equilateral triangle configuration, for which we found only four well-separated resonances. This figure shows that each sub-sum is not associated to one (and only one) resonance, even in that particular case for which the number of sub-sums is indeed the same as the number of resonances.

The same conclusion may be drawn for the case of overlapping resonances [7].

The way the symmetry properties of the cluster influence its resonant behavior is still, then, not understood.

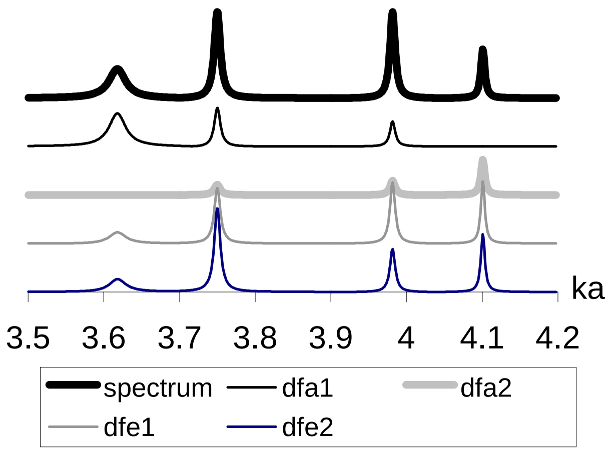

Figure 6: Three shells in an equilateral triangle configuration, β=2.06; α=50°. comparison between the resonance spectrum on top and the frequency derivative of the curvilinear abscissa of sub-sums labeled as in [9].

Conclusion

We have studied the resonances of clusters made of N hollow cylindrical elastic shells embedded in a fluid. For close enough shells, each low-frequency resonance of one single shell, due to the external A-wave, splits into M new ones.

M depends both on N and on the spatial arrangement of the shells. For N aligned shells, there are, for each single shell resonance, N well-separated cluster resonances. Each one of them may also, depending on the incidence direction, split into two overlapping resonances, so that M=2N.

For non-aligned shells, we found no simple rule for the value of M.

For those clusters composed of the minimum number of shells so that they belong to a given symmetry group, however, it seems that M is equal to the number of irreducible representations of that group. As soon as N is increased, with the cluster still in the same symmetry group, M increases, in a way we still do not understand.

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