Acoustics of nonequilibrium media with the negative second viscosity

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Abstract

This paper is a brief review of new acoustical properties of nonequilibrium media, which are caused by the inversion of the second (bulk) viscosity at some nonequilibrium degree. Among them there are new dispersion characteristics; the sound beam self-focusing; new properties of parametric interactions of sound waves with thermal waves and vortices. Media with a negative viscosity are acoustically active. In such media it is possible an existence of stationary structures that are essentially different from the shock wave monotonic structures. These structures were usually investigated with using nonlinear equations, obtained for low-frequency and high-frequency acoustical disturbances separately. The disadvantage of these equations is their disability to describe a nonstationary evolution of disturbances with a wide spectrum. In the present paper it is investigated the solutions of general nonlinear equation, describing a wide spectrum acoustical disturbance evolution in nonequilibrium media with the exponential relaxation model.

Introduction

The gasdynamics of nonequilibrium media (e.g. nonisothermal plasmas, chemical and optical active gases, reactors, atmospheric condensation) is investigated in numerous works. An interest to this problem is caused both the wide region of nonequilibrium gas applications and the unusual gasdynamic phenomena (the amplification of sound waves, a new shock waves structure and precursors, changes of aerodynamical forces, the intensive vortex generation etc.). Their clear theoretical explanations do not exist yet. It is necessary to look into the root of these phenomena and to investigate an acoustics of nonequilibrium media in more detail.

The vibrationally excited gas with the exponential relaxation model

$$\frac{dE}{dt} = \frac{E_e - E}{\tau(T,\rho)} + Q, \tag{1}$$

is the simple model of nonequilibrium media with the negative second viscosity. The pressure disturbance in this model has form $\tilde{P} = u_S^2\tilde{\rho} - \xi div\vec{v}$, where

$$u_S = \sqrt{\frac{C_{V0}^2 u_0^2 + \omega^2\tau_0^2 C_{V\infty}^2 u_\infty^2}{C_{V0}^2 + \omega^2\tau_0^2 C_{V\infty}^2}},$$

$$\xi = \frac{\xi_0 C_{V0}^2}{C_{V0}^2 + \omega^2\tau_0^2 C_{V\infty}^2} \tag{2}$$

are the sound speed and the second viscosity coefficient (its real part). Here $\xi_0 = \rho_0\tau_0 C_{V\infty}(u_\infty^2 - u_0^2)/C_{V0}$ is the low-frequency second viscosity coefficient; $u_\infty = \sqrt{\gamma_\infty T_0/m}$, $u_0 = \sqrt{\gamma_0 T_0/m}$ – are the frozen ($(\omega >> \omega_0 \equiv \tau_0^{-1}\sqrt{C_{P0}C_{V0}/C_{P\infty}C_{V\infty}}$) and equilibrium ($\omega<<\omega_0$) sound speeds; $\gamma_\infty = C_{P\infty}/C_{V\infty}$, $\gamma_0 = C_{P0}/C_{V0}$; $C_{P\infty}$, $C_{V\infty}$ are the frozen specific heats at constant pressure or volume; $C_{P0} = C_{P\infty} + c + S(\tau_T - \tau_\rho)$, $C_{V0} = C_{V\infty} + c + S\tau_T$ are the equilibrium (low-frequency) specific heats in vibrationally excited gas with relaxation law (1)[1]; $S = Q\tau_0/T_0$ is the nonequilibrium degree; $c = dE_e/dT$; $\tau_T = \partial\ln\tau_0/\partial\ln T_0$; $\tau_\rho = \partial\ln\tau_0/\partial\ln\rho_0$;

Acoustics of nonequilibrium media

In nonequilibrium media the sound speed depends on the nonequilibrium degree S. There are regions of S, where the dispersion is positive $u_0 < u_\infty$ like dispersion in an equilibrium medium, negative $u_0 > u_\infty$ or where $u_0^2 < 0$ and the low-frequency sound does not propagate [1-4].

The dependence of the sound speed on S leads to the different acoustical density of the nonequilibrium and the equilibrium media. It can be shown the possibility of the anomalous sound waves reflection with $R > 1$ on the equilibrium-nonequilibrium gas boundary. Moreover, the strong difference between the frozen and equilibrium speed can be one of the reasons of the known drag and lift aerodynamical forces changes in nonequilibrium media.

In vibrationally excited gases the second viscosity coefficient ξ_0 become negative at $S > c/\left|C_{V\infty}\tau_\rho + \tau_T\right|$. This condition corresponds to the positive feedback between the gasdynamic disturbance and nonequilibrium heating: in compression regions the nonequilibrium heating increases and such medium is acoustically active. The acoustical increment has simple form

$$\alpha = \frac{\omega^2 \xi}{2\rho_0 u_s^3} \qquad (3)$$

It is received also the second viscosity coefficients and investigated the sound dispersion at other models of the vibrational and rotational relaxation, in chemically active mixtures, nonisothermal plasma and some others nonequilibrium media, including the many relaxation and many components media [1-4]. In last cases the frequency dependencies $\xi(\omega), u_S(\omega)$ become more complicated, but increment keeps its simple form (3). The general condition of acoustically instability is

$$\xi(\omega) < 0 .$$

In high- and low-frequency limits the increment (3) equals to the superposition of partial coefficients $\alpha_{\infty i} = \xi_{0i} C_{V0} / 2u_\infty^3 \rho_0 \tau_{0i}^2 C_{V\infty}^2$, $\alpha_{0i} = \omega^2 \xi_{0i} / 2u_0^3 \rho_0$, where ξ_{0i} is the low-frequency coefficient of the second viscosity, forming by i-relaxation process. It simplifies significantly the stability analysis of such complicated nonequilibrium media and gives useful classification of the sound propagation regimes. In result it is both systematized (and simplified) the known instability conditions in different nonequilibrium media and found new instabilities regions. For example in [2] we present the most detail classification and critical investigation of the main mechanisms of the glow discharge atomic gas acoustically instabilities (in homogeneous approximation). It is important to emphasize that the conditions of the plasma acoustics activity coincide with the inversion of the second viscosity. Acoustical increment in nonequilibrium media with slow varying parameters is equal to the sum of α in the form (3) and the quasi-stationary or the inhomogeneous addition, which is different for the pressure, density and velocity disturbances [5,6].

The negative second viscosity leads not only to the sound instability, but also to the decrease of the threshold (the critical Reynolds number) of the laminar-turbulent nonequilibrium flow transition [7].

In acoustically active media there are new properties of the parametrical interactions. Among them the vortex and temperature waves parametrical amplification without the threshold [8]. Moreover, the vortex and temperature waves increment is proportional to the sound gain α, if the sound field is weak, or ~ to the $\exp\alpha$ in the strong sound field. It can lead both to the intensive whirl excitation and to the temperature stratification of such media.

There is another interesting property of the acoustical active media. As is known, high-power acoustic beams spreading in a liquid or gaseous medium give rise to an aperiodic motion of the medium called the acoustic wind. One of the reasons of this flow is caused by the viscous losses of the acoustic wave momentum and the appearance of the radiation force $F \sim \alpha$. In acoustically active medium $\alpha < 0$. In result the radiation force and the acoustic wind directions must change to opposite[9]. A nonequilibrium medium gives energy to the propagating acoustic wave and moves in the opposite direction. Such flows can lead to the selffocusing of sound beam. Unlike the equilibrium gaseous and liquid media, where the sound wind is one of the defocusing reason. In [10] it is investigated two mechanisms of the sound beam self-interactions in quasi- stationary acoustically active gas media: the acoustical wind excitation, which propagates oppositely to the beam moving and the temperature cooling in the strong sound field. The both mechanisms lead to self-focusing sound beam on the length

$$L_F = -\ln(1 - \alpha L_F^0)/\alpha ,$$

where $L_F^0 \approx (4{,}7/\sigma\int N dt)^{1/2}$, N is the sound power, $\sigma = -4\alpha / \pi a_0^4 u_\infty^2 \rho_0 (2 + \gamma_\infty / C_{P0})$, a_0 is the beam radius.

Shock wave structures in an acoustical approximation

In the second order perturbation theory the nonlinear evolution of gasdynamic disturbances in media with one relaxation processes is described by equation, obtained in [11]

$$C_{V\infty}\tau_0(\mathrm{v}_{tt} - u_\infty^2 \mathrm{v}_{xx} - u_S \Psi_\infty \mathrm{v}_{xx}^2 - \frac{\mu_\infty}{\rho_0}\mathrm{v}_{xxt})_t +$$
$$+ C_{V0}(\mathrm{v}_{tt} - u_0^2 \mathrm{v}_{xx} - u_S \Psi_0 \mathrm{v}_{xx}^2 - \frac{\mu_0}{\rho_0}\mathrm{v}_{xxt}) = 0. \quad (4)$$

Here $\mu_{0,\infty} = 4\eta/3 + \chi m(1/C_{V0,\infty} - 1/C_{P0,\infty})$, η, χ are the shear viscosity and the thermal conductivity coefficients; $\Psi_\infty = (\gamma_\infty + 1)/2$; $\Psi_0 = \Psi(S)$. Coefficient Ψ_0 can have both the positive and the negative signs.

For waves, traveling in one direction ($\tilde{v} = v/u_s$, $\varsigma = (x - u_s t)/u_s\tau_0$, $y = \theta t/\tau_0$), Eq. (4) reduces to

$$(\tilde{v}_y + \frac{(u_\infty^2 - u_s^2)}{2u_s^2}\tilde{v}_\varsigma + \frac{\Psi_\infty}{2}\tilde{v}_\varsigma^2 - \tilde{\mu}_\infty \tilde{v}_{\varsigma\varsigma})_\varsigma - \quad (5)$$

$$-\frac{C_{V0}}{C_{V\infty}}(\tilde{v}_y - \frac{u_s^2 - u_0^2}{2u_s^2}\tilde{v}_\varsigma + \frac{\Psi_0}{2}\tilde{v}_\varsigma^2 - \tilde{\mu}_0 \tilde{v}_{\varsigma\varsigma}) = 0,$$

where $\tilde{\mu} = \mu / 2\tau u_s^2 \rho_0$.

In a low frequency approximation ($\partial\tilde{v}/\partial y \sim \theta\tilde{v}$) Eq. (5) reduces with an accuracy to $\sim\theta^3$ to the modified Kuramoto-Sivashinsky equation

$$\tilde{v}_y + \Psi_0 \tilde{v}\tilde{v}_\varsigma = (\tilde{\mu}_0 + \tilde{\xi})\tilde{v}_{\varsigma\varsigma} + \tilde{\beta}\tilde{v}_{\varsigma\varsigma\varsigma} + \tilde{\kappa}\tilde{v}_{\varsigma\varsigma\varsigma\varsigma} \quad (6)$$

Eq. (6) is earlier used for a description of wave processes in inclined liquid films, reagent concentration disturbances at chemical reactions and a fusion; electrostatic potential waves in toroidal systems etc. In the present work Eq. (6) is obtained for small low frequency disturbances, propagating in a gas with the relaxation process (1). In (6) $\tilde{\xi} = \xi_0 / 2\rho_0\tau_0 u_0^2$ is the second viscosity coefficient, $\tilde{\kappa} = C_{V0}\tilde{\beta}/C_{V\infty} = C_{V0}^2\tilde{\xi}/C_{V\infty}^2$ (with neglect of $\sim\tilde{\mu}_0^2, \tilde{\mu}_0\tilde{\xi}$). All these coefficients are negative if $C_{V0} > 0$ and $\tilde{\xi} < 0$.

In a high frequency approximation ($\partial\tilde{v}/\partial y \sim \theta^{-1}\tilde{v}$) Eq. (5) reduces (with an accuracy to $\sim\theta^2$) to Burgers equation with a source and a integral dispersion

$$\tilde{v}_y + \Psi_\infty \tilde{v}\tilde{v}_\varsigma = \tilde{\mu}_\infty \tilde{v}_{\varsigma\varsigma} - \tilde{\alpha}_\infty \tilde{v} - \tilde{\beta}\int \tilde{v} d\varsigma, \quad (7)$$

where $\tilde{\alpha}_\infty = \xi_0 C_{V0}^2 / C_{V\infty}^2 \rho_0\tau_0 u_\infty^2$ is the dimensionless gain (at $\xi_0 < 0$) of a high frequency sound, $\tilde{\beta} \approx C_{V0}\alpha_\infty / C_{V\infty}$ is the dispersion coefficient.

An evolution of a small disturbance with a wide spectrum must be investigated basing on Eq.(4). At a weak dispersion $\tilde{m} = (u_\infty^2 - u_0^2)/u_\infty^2 \sim \theta << 1$ and Eq. (4) can be written in form

$$(\tilde{v}_y + \frac{\Psi_\infty}{2}\tilde{v}_\varsigma^2 - \tilde{\mu}_\infty \tilde{v}_{\varsigma\varsigma})_\varsigma - \frac{C_{V0}}{C_{V\infty}}(\tilde{v}_y + \frac{\tilde{m}}{2}\tilde{v}_\varsigma +$$

$$+\frac{\Psi_0}{2}\tilde{v}_\varsigma^2 - \tilde{\mu}_0 \tilde{v}_{\varsigma\varsigma}) = 0 \quad (8)$$

Numerical solutions of Eq. (8) are found with a help of splitting method in suggestion of the negative total viscosity $\tilde{\mu}_\Sigma = \tilde{\xi} + \tilde{\mu}_0 = \tilde{\mu}_0 - \tilde{m}C_{V\infty}/2C_{V0} < 0$ and $C_{V0} > 0, \Psi_\infty > \Psi_0 > 0$.

An initial disturbance has the form of a step-like function. Evolution ways depend on an initial amplitude of disturbance $\tilde{v}_1$.)

If $\tilde{v}_1 > \tilde{v}_{cr}$ the evolution leads to a rounded front of the step (Fig. 1). At $\tilde{\mu}_\infty = 0$ it is obtained $\tilde{v}_{cr} = 2|\tilde{\mu}_\Sigma| C_{V0}/C_{V\infty}(\Psi_\infty - \Psi_0)$.
Such rounded front is typical also for relaxing media at $\tilde{\mu}_\Sigma > 0$ and a nonlinear effect predominating.

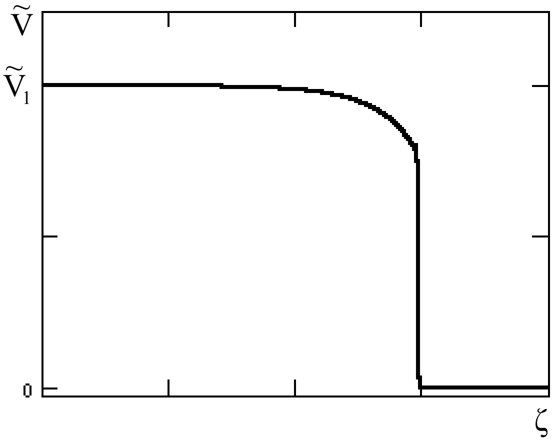

Figure 1:The shock wave in relaxing media

At $\tilde{v}'_{cr} < \tilde{v}_1 < \tilde{v}_{cr}$, where $\tilde{v}'_{cr} = 2|\tilde{\mu}_\Sigma| C_{V0}/C_{V\infty}(2\Psi_\infty - \Psi_0)$, one (as in Fig. 2) or a few maximums and minimums were

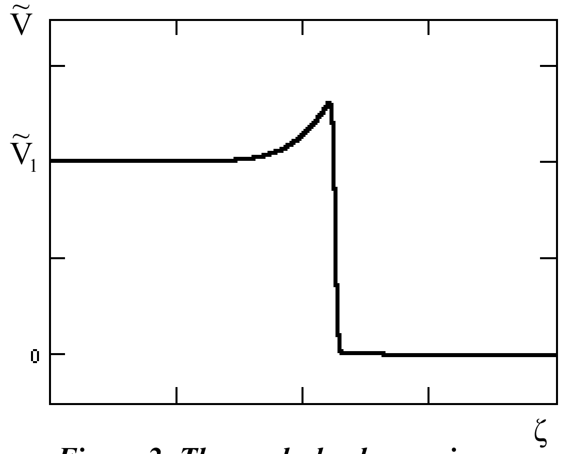

Figure 2: The weak shock wave in nonequilibrium media

observed on the disturbance front.

At $\tilde{v}_1 < \tilde{v}'_{cr}$ the step becomes unstable and transforms to a periodical series of stationary impulses [12].

All structures, described in the present paper, can be also obtained by an analytical solution of an automodel form ($z = \varsigma - Wy$) of Eq. (8):

$$\tilde{v}_{zz} + \tilde{v}_z[(\frac{W}{\tilde{\mu}_\infty} - \frac{C_{V0}}{C_{V\infty}}\frac{\tilde{\mu}_0}{\tilde{\mu}_\infty}) - \frac{\Psi_\infty}{\tilde{\mu}_\infty}\tilde{v}] +$$
$$+ \frac{C_{V0}}{\tilde{\mu}_\infty C_{V\infty}}[(-W + \tilde{m}/2)\tilde{v} + \frac{\Psi_0}{2}\tilde{v}^2] = 0 .$$

Their existence is possible only at $\tilde{\mu}_\Sigma < 0$.

Fig.3 shows a form of a solitary impulse, corresponding to the motion along a separatrix on the phase plane. This impulse is exactly similar to the stationary impulses of Eq. 8.

At $|\tilde{\mu}_\Sigma| / \tilde{\mu}_\infty >> 1$ the solitary impulse has a strongly asymmetrical form. The leading shock front has width $\sim (2\Psi_\infty - \Psi_0) C_{V\infty} \tilde{\mu}_\infty / 2\Psi_\infty C_{V0} |\tilde{\mu}_\Sigma|$ and trailing front exponentionally decaying with decrement $\sim \Psi_0 C_{V0} / 2\Psi_\infty C_{V\infty}$. The impulse amplitude is exactly $\tilde{v}_{imp} = 2\tilde{v}'_{cr}$.

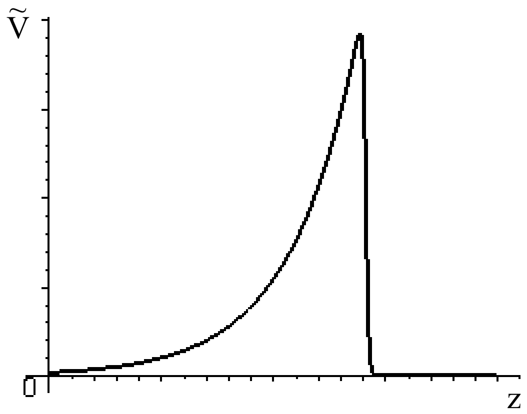

Figure 3: Autowave impulse

Thus, in the present paper it was predicted the existence of stationary shock waves with round or nonmonotonous fronts and, also, the strongly asymmetrical solitary impulses in media with a negative viscosity. We obtained also very interesting result, that at negative Ψ_0 the compression step like autowave with shock front can exist. Its amplitude $\tilde{v}_1 = \tilde{v}_{cr}$.

Conclusion

There are tremendous many unsolved problems in acoustics of nonequilibrium media. For example the influence of the medium inhomogeneity is very weakly investigated. But even in the simplest homogeneous models, permitting the analytical approaches, we have the essential changes of nonequilibrium medium dynamics. They must be taken into account in more complicated models.

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