SUPER-FAST INTERFACIAL LEAKY WAVES IN PIEZOELECTRIC MEDIA

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Abstract
A study is made of the existence of leaky waves propagating along a layer inserted into a homogeneous medium faster than the quasi longitudinal bulk wave in this medium. It has been shown that a sufficient condition for such super high-velocity leaky solutions to originate is the presence of concavity on the slowness surface of the quasi longitudinal waves; the concavity required can exist in piezoelectric crystals. As an example, computations for $KNbO_3$ crystal containing a thin diamond film have been performed. We also discuss the occurrence of localized leaky solutions at the interface between two media that differ only by the sign of piezomoduli and at an infinitesimal thin metallic layer inserted into a homogeneous medium.

Introduction
A thin layer inserted into a homogeneous solid can guide a localized interfacial wave (IW). This wave can be viewed as originating from a bulk wave that would propagate in the medium in the absence of the layer with a group velocity parallel to the plane becoming the interface; such a bulk wave is referred to as a limiting bulk wave (LBW). Certain relations of the type of inequality between the material constants of the layer and the medium embedding it decide whether such an IW appears.

IW can be a truly localized one, i.e. propagating without attenuation caused by the energy leakage away from the layer-medium interface, or leaky (pseudo-localized) one. This depends on the velocity $\hat{v} = \omega/\hat{k}$ of LBW, where $\hat{k}$ is the projection of the wave vector on the interface.

In this paper, we discuss the existence of IW propagating faster than the quasi longitudinal LBW. The occurrence of such a super high-velocity IW does not seem possible in non-piezoelectric media, since no non-uniform modes exist above the quasi longitudinal velocity threshold in this case (with reservation that that the quasi longitudinal wave is the fastest one). On the other hand, in piezoelectrics the wave equation still has non- uniform partial solutions beyond the limiting velocity of quasi longitudinal waves. These are Coulomb modes coupled with mechanical displacement and stresses via the piezoelectric effect. As an example, we describe super high-velocity leaky IW in $KNbO_3$ crystals. However, we begin by discussing some general aspects of the existence of IW in piezoelectric media in the presence of an internal plane interface.

Acoustoelectric waves in piezoelectric media
Consider first a semi-infinite piezoelectric medium of general symmetry. We shall be making use of the fact that the characteristics of partial modes ($\propto exp[ik(x + p_\alpha z - vt)]$) labelled by the subscript α, namely, the decay factor p_α, the polarization vector $\mathbf{A}_\alpha$, the electric potential Φ_α, the traction $\mathbf{f}_\alpha$, the normal projection of electric displacement D_α, can be found from an eigenvalue problem for an 8×8 real matrix

$$\hat{\mathbf{N}} = -\left\{ \begin{array}{cc} \hat{\mathbf{N}}_{11} & \hat{\mathbf{N}}_{12} \\ \hat{\mathbf{N}}_{21} & \hat{\mathbf{N}}_{11}^t \end{array} \right\}, \qquad (1)$$

where $\hat{\mathbf{N}}_{12} = (nn)^{-1}$, $\hat{\mathbf{N}}_{11} = \hat{\mathbf{N}}_{12}(nm)$, $\hat{\mathbf{N}}_{21} = (mn)\hat{\mathbf{N}}_{11} - (mm)$, and the superscript $()^t$ means transposition. The symbols of the type (ab) stand for 4×4 matrices with components $(ab)_{IJ} = a_k E_{kIJl} b_l$, $I, J = 1, \ldots, 4$, where $\mathbf{a}$ and $\mathbf{b}$ is a pair of three-component vectors and $E_{kIJl} = c_{kIJl} - \rho v^2 m_k m_l \delta_{IJ}$, $I, J = 1, 2, 3$, $E_{k4Jl} = e_{kJl}$, $J = 1, 2, 3$, $E_{kI4l} = e_{lIk}$, $I = 1, 2, 3$, $E_{k44l} = -\varepsilon_{kl}$. The unit vector $\mathbf{n}$ is perpendicular to the surface and the unit vector $\mathbf{m}$ lies in the surface indicating the direction of wave propagation, $(nn)^{-1}$ is the inverse of the matrix (nn), $v = \omega/k$.

Let us compose eight-component vector columns $\xi_\alpha = (\mathbf{A}_\alpha, \Phi_\alpha, \mathbf{L}_\alpha, Q_\alpha)^t$, where $\mathbf{L}_\alpha = ik^{-1}\mathbf{f}_\alpha$ and $Q_\alpha = ik^{-1}D_\alpha$ and the symbol $()^t$ stands for transposition. Generally the matrix (1) is not degenerate and it can be shown that $\hat{\mathbf{N}}\xi_\alpha = p_\alpha \xi_\alpha$, $\alpha = 1, \ldots, 8$, at arbitrary values of v [1, 2].

Of our concern is the vicinity of the limiting velocity $\hat{v}$. The matrix $\hat{\mathbf{N}}$ is known to become non-semisimple degenerate at $\hat{v}$ [3]. In this case two eigenvalues merge into one real eigenvalue; let it be $p_3(\hat{v}) = p_7(\hat{v}) = p_{d3}$. The degenerate eigensolution (p_{d3}, ξ_{d3}) corresponds to LBW. The situation $v =$

$\hat{v}$ is referred to as a transonic state (TS) of type 1 [4].

At $v = \hat{v}$ one has

$$\hat{\mathbf{N}}\xi_\alpha = p_\alpha \xi_\alpha, \ \hat{\mathbf{N}}\xi_{d7} = p_{d3}\xi_{d7} - \xi_{d3}, \qquad (2)$$

where ξ_{d7} is a generalized eigenvector and $\alpha = 1, 2, 4, 5, 6, 8, d3$. Since $\left(\hat{\mathbf{T}}\hat{\mathbf{N}}\right)^t = \hat{\mathbf{T}}\hat{\mathbf{N}}$, where $\hat{\mathbf{T}}$ is an 8×8 matrix with elements $T_{ij} = T_{i+4,j+4} = 0$ and $T_{i,j+4} = T_{i+4,j} = \delta_{ij}$, ξ_α at $\hat{v}$ can be introduced such that

$$\xi_\alpha \cdot \hat{\mathbf{T}}\xi_\beta = \delta_{\alpha\beta}, \ \xi_{d3} \cdot \hat{\mathbf{T}}\xi_\alpha = \xi_{d7} \cdot \hat{\mathbf{T}}\xi_\alpha = 0,$$

$$\alpha \neq d3, d7, \ \ \xi_{d3} \cdot \hat{\mathbf{T}}\xi_{d7} = 1, \ \xi_{d7} \cdot \hat{\mathbf{T}}\xi_{d7} = 0. \qquad (3)$$

The vectors $\xi_{d3,d7}$ are real at positive curvature TS ($TS^{(+)}$) and purely imaginary at negative curvature TS ($TS^{(-)}$).

In the vicinity of $\hat{v}$ the non-degenerate p_α and ξ_α varies linearly with $v - \hat{v}$ to the lowest approximation while $p_{d3,d7}$ and $\xi_{d3,d7}$ depends on $(v-\hat{v})^{1/2}$ rather than on $v - \hat{v}$. By analogy with the non-piezoelectric case [5, 6]

$$\xi_{3,7}(v) \approx \xi_{d3} \mp \Delta p \xi_{d7}, \ p_{3,7}(v) \approx p_{d3} \pm \Delta p, \qquad (4)$$

where the upper sign corresponds to $\alpha = 3$, $\Delta p = \sqrt{\rho}|\mathbf{A}_{d3}|f(v)$ near $TS^{(+)}$ and $\Delta p = i\sqrt{\rho}|\mathbf{A}_{d3}|f(v)$ near $TS^{(-)}$ with $f(v) = \sqrt{v^2 - \hat{v}^2}$ at $v > \hat{v}$ while $f(v) = i\sqrt{\hat{v}^2 - v^2}$ at $v < \hat{v}$.

After the above introductory remarks we pass on to considering three types of the interface inside a piezoelectric.

Thin dielectric layer

Let a dielectric layer of thickness h be sandwiched between two halves of a medium. We assume perfect bonding along the interface. Similar to the case of non-piezoelectric structures considered in Ref. [7], the determinant of boundary conditions Δ_{BC} reduces to

$$\Delta = ||\ldots||\{\Delta p - i\Xi + \ldots\}, \qquad (5)$$

with $\Xi = -(\hat{N}_l)_{33}H$, where $H = kh \ll 1$, $(\hat{N}_l)_{33}$ denotes the contraction $\xi_{d3} \cdot \hat{\mathbf{T}}\hat{\mathbf{N}}_l\xi_{d3}$, and $\hat{\mathbf{N}}_l$ is matrix (1) of the layer; $||\ldots||$ is a determinant which is of no importance for us.

The value of $(\hat{N}_l)_{33}$ and, hence, of Ξ are necessarily real. In view of the definition of Δp the solution exists if $\Xi > 0$. This inequality can be fulfilled both at $TS^{(+)}$ and $TS^{(-)}$, depending on the relations between the material constants of the layer and the "main" medium.

Once the wave appears, its velocity v_S will differ from $\hat{v}$ by a quantity of the order $\hat{v}H^2$. It is worth noting for further use that v_S is smaller than $\hat{v}$ in the case of $TS^{(+)}$ and greater than $\hat{v}$ in the case of $TS^{(-)}$.

If $\hat{v}$ is not the slowest TS, than the second- and higher-order terms in H in (5) are complex, indicating that near such a TS leaky waves, rather than pure localized ones, generally emerge. The imaginary part of the leaky wave velocity has the order $\hat{v}H^3$.

"180^o-domain wall"

Let the upper and lower halves of the bi-crystal have piezomoduli of opposite sign, with the other material constants being identical. The two halves are perfectly bonded.

Assuming the piezoeffect to be weak we consider its influence using perturbation theory. As a basis for perturbation expansions, ξ_α's calculated at $e_{ijk} = 0$ will be taken.

Representing the corrections $\delta^{(n)}\xi_\alpha$ of the order $(e_{ijk})^n$ to ξ_α at $\hat{v}$ as $\delta^{(n)}\xi_\alpha = \hat{\mathbf{M}}^{(n)}\xi_\alpha$ where $\hat{\mathbf{M}}^{(n)} \propto (e_{ijk})^n$ is a real 8×8 matrix independent of the suffix α, and using (2)-(4) allows the appropriate Δ_{BC} to be brought into the expression similar to (5) in which

$$\Xi = (\hat{M}^{(1)})^2_{F3} - (\hat{M}^{(1)})^2_{G3}, \qquad (6)$$

where $(\hat{M}^{(1)})_{F3} = \mathbf{F} \cdot \hat{\mathbf{T}}\hat{\mathbf{M}}^{(1)}\xi_{d3}$ and $(\hat{M}^{(1)})_{G3} = \mathbf{G} \cdot \hat{\mathbf{T}}\hat{\mathbf{M}}^{(1)}\xi_{d3}$; the real vectors $\mathbf{F}$ and $\mathbf{G}$ read as

$$\mathbf{F} = (\mathbf{0}, 0, \mathbf{0}, \sqrt{2p})^t, \ \mathbf{G} = (\mathbf{0}, \sqrt{2/p}, \mathbf{0}, 0)^t, \qquad (7)$$

where $p = [\varepsilon_{nn}\varepsilon_{mm} - (\varepsilon_{mn})^2]^{1/2}$ and $\varepsilon_{nn} = \varepsilon_{ij}n_i n_j$ and $\varepsilon_{nm} = \varepsilon_{ij}n_i m_j$. Explicitly

$$\begin{aligned} (\hat{M}^{(1)})_{F3} &= \Gamma\Big\{\varepsilon(\hat{N}^{(1)})_{F3} + p(\hat{N}^{(1)})_{G3}\Big\}, \\ (\hat{M}^{(1)})_{G3} &= \Gamma\Big\{\varepsilon(\hat{N}^{(1)})_{G3} - p(\hat{N}^{(1)})_{F3}\Big\}, \end{aligned} \qquad (8)$$

where $\hat{\mathbf{N}}^{(1)}$ is the correction to $\hat{\mathbf{N}}$ linear in e_{ijk}, $\varepsilon = \varepsilon_{nn}p_{d3} + \varepsilon_{nm}$ and $\Gamma = \varepsilon_{nn}/[\varepsilon^2 + p^2]$.

The quantity Ξ (6) is real so that the solution exists when $\Xi > 0$. This inequality can hold true again both at $TS^{(+)}$ and $TS^{(-)}$. The Ξ value has the order κ^2, where κ is the electromechanical coupling coefficient.

If TS under consideration is not the slowest one, then the wave generally becomes leaky. The complex corrections to Δp appear starting from the term $\propto \kappa^4$ so that the attenuation of leaky IW $v_l'' \propto \hat{v}\kappa^6$ while the change in the real velocity $|v_l' - \hat{v}| \propto \hat{v}\kappa^4$.

Note that the odd powers of e_{ijk} cannot appear in the expansion of the determinant of boundary conditions, because the value of Δp cannot depend on the sign of e_{ijk}.

Infinitesimally thin metallic layer
The piezoeffect is still assumed to be weak. After some evaluations, one obtains (5) with

$$\Xi = (\hat{M}^{(1)})^2_{F3}. \tag{9}$$

Due to (7) Ξ is real and positive at $\mathrm{TS}^{(+)}$ independently of the material constants. But $\Xi < 0$ at $\mathrm{TS}^{(-)}$. Hence, internal "metallization" necessarily leads to (pseudo) IW near $\mathrm{TS}^{(+)}$ unless the limiting bulk waves remains non-piezoactive when the piezoeffect is "switched on". The waves cannot arise in the vicinity of $\mathrm{TS}^{(-)}$.

On the other hand, if the thickness of the metallic insertion is taken into account, then $\Xi = -(\hat{N}_l)_{33}H + (\hat{M}^{(1)})^2_{F3}$; here $(\hat{N}_l)_{33}$ is the contraction of $\xi'_{d3} = (\mathbf{A}_{d3}, \mathbf{L}_{d3})^t$ with the non-piezoelectric "analog" (6×6 matrix) of $\hat{\mathbf{N}}$ (1) describing the elastic properties of the layer. The solution exists near the negative curvature TS starting from $H \approx (\hat{M}^{(1)})^2_{F3}/(\hat{N}_l)_{33} \sim \kappa^2$ provided that $(\hat{N}_l)_{33}$ is negative.

Super high-velocity IW in $KNbO_3$
It has been pointed out that the velocity of IW originating from LBW exceeds $\hat{v}$ in the case of $\mathrm{TS}^{(-)}$. The slowness surface of quasi longitudinal waves can be concave only in piezoelectrics. The piezoelectric properties must be strong enough to "suppress" convexity that elastic properties provide.

Crystals having concavity on the quasi longitudinal branch are known. In particular, this is $KNbO_3$ ($mm2$) where concavity is the most pronounced and exists for $(0, 90, 90 \pm 82)$ cuts (Euler angles are understood). We discuss the case when a dielectric layer is inserted.

The existence criterion for IW is fulfilled, e.g., for a diamond film. Fig. 1 shows the H-dependence of the real part v'_l of the leaky IW velocity and attenuation. Here v'_l is the inverse real part of the complex slowness s_l (in section 2, for the sake of simplicity, we considered the complex velocity $v_l = 1/s_l$; these definitions are equivalent within the precision of our estimates). Starting from the value $v'_l(0) = \hat{v}_L \approx 7736\ m/s$, where $\hat{v}_L$ is the velocity of the longitudinal wave along [001], v'_l increases with H. After reaching the maximum value, v'_l goes down and becomes equal to $\hat{v}_L$ at $H_0 \approx 0.2$. The leaky wave does not exist for $H > H_0$. Contrary to v'_l, attenuation steadily increases with H.

As has been pointed out, to the lowest approximation, $v'_l - \hat{v}_L \propto \hat{v}_L H^2$ while $v''_l \propto \hat{v}_L H^3$. In the case under consideration, these dependences are observed enough well up to $H \approx 0.03$. Note also that "physically" IW disappears at H smaller than H_0. The matter is that the singularities of Green's function associated with the leaky solution become strongly smoothed for H exceeding 0.1, suggesting that the leaky wave can barely be generated when $H > 0.1 - 0.12$ (in particular, the existence of leaky IW will not show up in the behavior of the coefficients of plane mode conversion, see [7, 8, 9]).

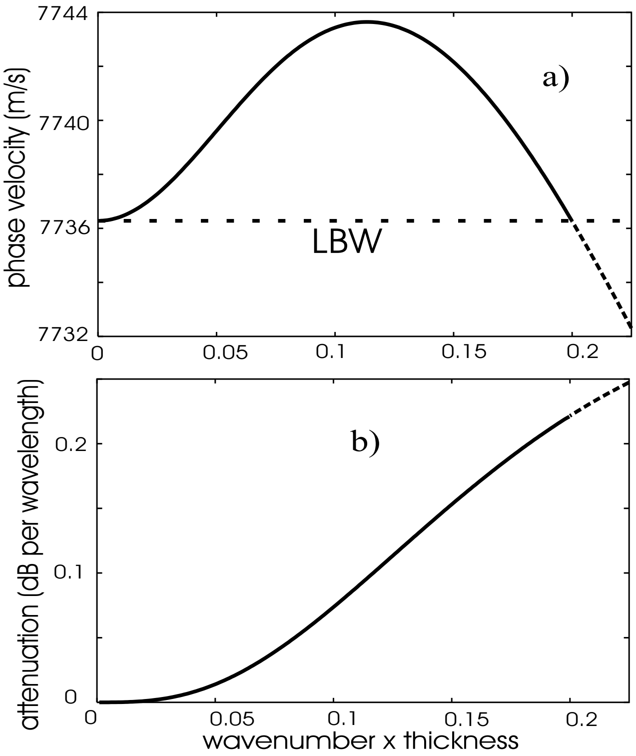

Figure 1: Real part of the leaky wave velocity and attenuation of the leaky wave as functions of the diamond film thickness; the orientation of $KNbO_3$ is $(0, 90^o, 90^o)$.

In Fig. 2 the difference $v'_l - \hat{v}_L$ and attenuation as functions of angle ψ are depicted (ψ is the angle between the direction of propagation $\mathbf{m}_\psi$ still lying in the plane (010) and the axis [100]; $\hat{v}_L$ is the quasi longitudinal wave velocity along $\mathbf{m}_\psi$). One sees that the $v'_l - \hat{v}_L$ value and attenuation decreases and increases abruptly enough, respectively, with ψ approaching the critical value at which the curvature of the slowness surface changes sign. The value of v'_l reaches v_L at ψ's smaller than this critical angle. However, "physically" the leaky IW disappears still earlier for the reason already mentioned.

Conclusion
Concavity of the slowness surface of quasi longitudinal bulk waves in a medium is a sufficient condition for the existence of leaky IW propagating faster than the quasi longitudinal bulk

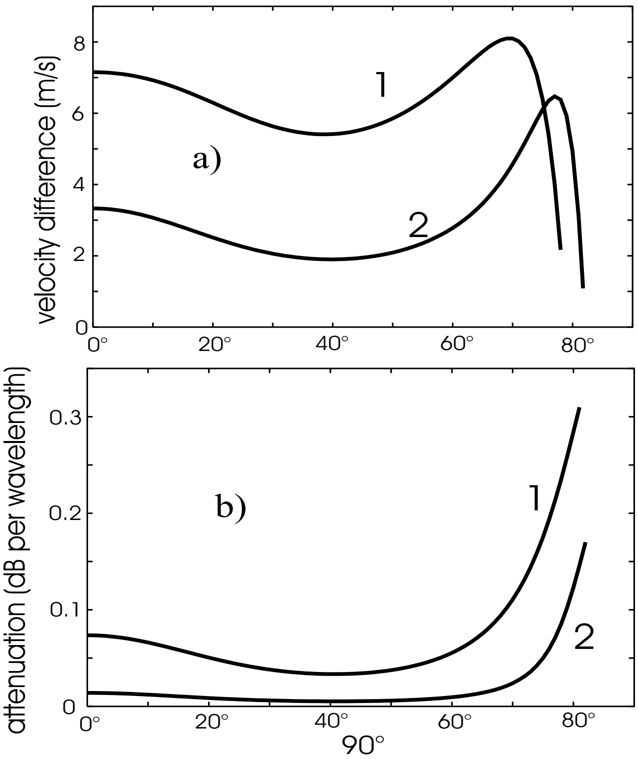

Figure 2: Difference $v'_l - \hat{v}_L$ (a) and attenuation (b) as functions of angle ψ; the orientation of $KNbO_3$ is $(0, 90^o, \psi)$. $1 - H = 0.1$; $2 - H = 0.05$.

wave. If this condition is fulfilled, then one should choose a layer with material constants securing the existence of such IW. There is no simple relation between the material constants of the medium and the layer under which super fast leaky waves appear. However, as a rough criterion, one can consider the statement that the longitudinal wave velocity in the layer should be greater than that in the medium.

In addition, we have discussed the localization of an LBW at the interface between two medium that differ only by the sign of piezoelectric moduli. In this case, IW may or may not originate near a TS of any curvature sign, depending on the material constants. We have also considered the localization of LBW caused by an infinitesimally thin metallic layer inserted into a homogeneous medium. It appears that IW originates independently of material constants. But this occurs only in the vicinity of $TS^{(+)}$.

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