ON THE RESPONSE OF A LIMITED CHAIN OF WELDED SPHERICAL BEADS

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Abstract

This paper deals with the propagation of waves along a one-dimensional chain made up of welded spheres. First, a theoretical analysis allows the vibration modes of the chain to be quantitatively described. It has been validated by a comparison with numerical and experimental results. It is numerically and experimentally verified that the peaks associated to the Rayleigh modes broaden out as the mode number increases and that the pass-band structure is strongly influenced by the characteristic of the welding between the cells of the periodic structure. The interest of such an approach is then illustrated by the examination of the inverse problem, in which the analytical model is used to deduce the characteristics of the welding.

Introduction

The propagation of waves through aggregates and layered structures was thoroughly studied in the recent past [1-3]. To go forward in the comprehension of vibration phenomenon related to aggregates and layered structures, the study of linear chains of spheres has been proposed. This simplified model has revealed the existence of allowed and forbidden frequency bands [3, 4]. Recently, we have proposed a qualitative interpretation of the chain vibrations with the help of an analogy with phonons of solid state physics [4]. The structure of the acoustic band gaps depends on the coupling between the basis unit of the chain. It is shown for example that the stronger is the coupling and the wider is the pass-band.

In this paper, a model is first presented to describe quantitatively the frequency bands, by the writing of the dispersion relations. Then, the model is validated both by the measurement of the vibration modes of a chain of spheres and by a full calculation using the finite element method. One of the main interest of the model is that a coupling factor is introduced in the equations, directly related to the mechanical coupling between spheres. Therefore, it is possible to use the model to solve the inverse problem on periodic chains of welded spheres: with the help of the experimental measurement of vibration frequencies of the chain, one can give a quantitative information on the mechanical bind between spheres. Finally, experimental results on various chains are compared with theoretical results, allowing the inverse problem to be solved.

Theoretical approach

The vibration modes of the single sphere have previously been presented in details [4]. Three types of vibration modes exist : the Rayleigh modes R(n>1,1) where a wave is propagating around the sphere whereas the center does not move. In this paper, one distinguishes the odd Rayleigh modes R(2n+1,1), in which the displacement of two points diametrically opposed is in phase, and the even Rayleigh modes R(2n,1), in which the displacement of two points diametrically opposed is out of phase. For the whispering gallery modes [WG(n>0, ℓ=2,3,4..)], theoretically, the surface of the sphere does not move whereas a vibration is observed in the heart of the sphere. Finally, at higher frequency, the breathing mode corresponds to a radial vibration of the sphere.

In the next we focus our attention to the vibrations of a system composed of identical spheres regularly spaced.

The first part is devoted to the case of an infinite chain of identical spheres, with in particular the equations of the dispersion curves. Then, the vibration modes of a finite chain of identical spheres are presented as a particular case of the vibration modes of the infinite chain with specific boundary conditions.

Case of an infinite chain of identical spheres

Consider an infinite chain of identical spheres of radius *a*, regularly spaced of a distance *2d*. The welding between spheres is taken into account by introducing the two parameters (r_w, r'_w) described in Fig. 1 ($d = \sqrt{a^2 - r_w'^2}$). Due to the periodicity of the system, the wave number *k* is introduced for designating the vibration modes. Furthermore, one knows that *k* should belong to the first Brillouin zone, which is given by [$-\pi/2d$, $\pi/2d$]. For each *k* value, a vibration frequency ω exists that leads to the notion of dispersion curve $\omega(k)$.

Four kinds of collective vibrations would result of the association of spheres in a linear chain. First, each sphere can be seen as a rigid "atom" which can be translated with respect to its neighbors. The dispersion relationship is [3]:

$$\omega(k) = (4K/m)^{1/2} \sin kd \qquad (1)$$

where *K* designates the coupling constant and *m* the mass of an individual sphere. These modes are low frequency modes (LF) due to the nature of the

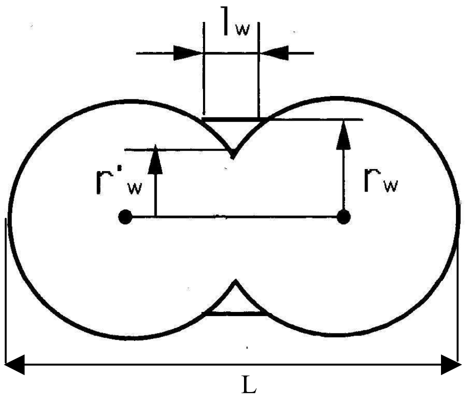


Figure 1: Definition of the characteristic distances for two welded spheres (L, l_w, r_w and r'_w).

displacement of the whole sphere around its equilibrium position. If the sphere was perfectly stiff, these modes would be the only permitted modes. In solid-state physics these vibrations are designated as phonons belonging to the *acoustical branch*.

A second kind of collective vibrations appears since each sphere is elastic and able to vibrate. Depending on the nature of this vibration we distinguish three kinds of such collective modes, which may be designated as *molecular phonons* due to their origin. Considering first the **odd** Rayleigh modes, the equation for the branch of mode n is written as:

$$\omega_n^2(k) = \omega_{0,n}^2[1+(4K_n/m\omega_{0,n}^2)\sin^2(kd)] \qquad (2)$$

where $\omega_{0,n}$ $(=2\pi f_{0,n})$ designates the resonance frequency of the mode for a single sphere and K_n measures the coupling between neighbors spheres for the mode n.

For **even** Rayleigh modes, the equation for the branch of mode n is written as:

$$\omega_n^2(k) = \omega_{0,n}^2[1+(4K_n/m\omega_{0,n}^2)\cos^2(kd)] \qquad (3)$$

Finally, in the case of WG modes, the displacement is localized inside the sphere. Only weak coupling exists between spheres, leading to a flat dispersion curve.

The numerical propagation of plane acoustic waves in an infinite and periodic structure is studied using only the mesh of one unit cell, thanks to Bloch-Floquet relations. It provides dispersion curves from which results of physical interest can be easily extracted: identification of propagation modes, cutoff frequencies, pass-bands and stop-bands [5,6]. Fig. 2 presents the first branches of the dispersion curve of an infinite chain of steel spheres, the diameter of which is equal to 10 mm. The welding between spheres is characterized by r_w = 1.46 mm and r'_w= 1.04 mm. On Fig. 2, full lines correspond to a finite element calculation [5], dotted lines correspond to the equation of each branch (Eq. 1, 2 and 3), determined with the help of the frequencies at $k = 0$ and $k = \pi/2d$. It shows that the equations fit well the dispersion curves. The frequency of the single sphere is always the lowest value of the branch.

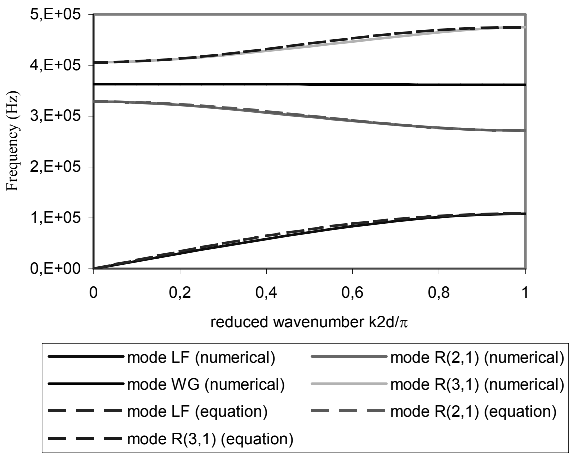


Figure 2: First branches of the dispersion curves of an infinite chain of steel spheres in the first Brillouin zone, with reduced scale. Full line: numerical results, dashed line: equation of the branches (Eq. 1, 2 and 3).

Case of a finite chain of identical spheres

A finite chain can be considered as a particular case of an infinite chain by applying appropriate boundary conditions. Considering N particles with both ends free, which simulates experimental conditions, the discrete values of k corresponding to the N modes are $k = s\pi/2Nd$ with $s = 1 \ldots N\text{-}1$ for odd modes, and with $s = N\text{-}1 \ldots 1$ for even modes. For a finite chain of N spheres, we obtain N discrete frequencies for each branch (acoustical and molecular branches). Thanks to the quantification of the wave number k, one can deduce the vibration modes of a chain of N spheres, using the previous dispersion curves (Fig. 2). The mesh of only one sphere is enough.

The vibration modes in the particular case of a chain of two identical spheres are presented by marks on Fig. 3.

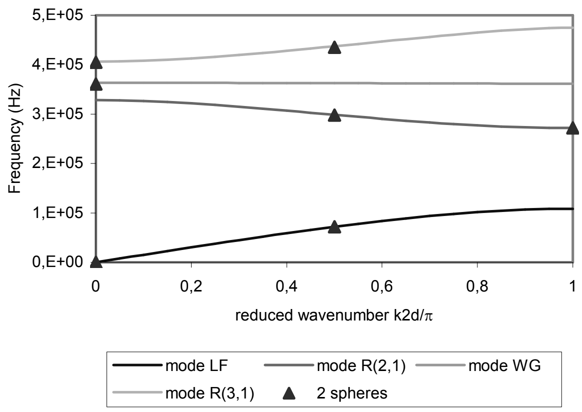


Figure 3: Numerical results (marks) of a chain of two welded spheres, positioned on the dispersion curves of the infinite chain of welded spheres.

Experimental set-up and specimen

The experimental set up includes a pair of broadband transmitters longitudinally polarized [7]. The emitter is driven by short ultrasonic pulses and a frequency analysis of the initial section of the transmitted acoustic signal is achieved via a FFT algorithm. The axial static force applied along the axis of the specimen does not alter the position of the peaks (resonance) in the experimental frequency spectra. The systems under consideration consist of N welded very well steel calibrated spheres. In a first approach, the dispersion of the diameters and asphericity of the beads are neglected. The spheres are coupled by using a spot welding process the principle of which is shown in Fig. 4. By adjusting the intensity applied to the electrodes it is possible to modify the radius of the contact area between two adjacent spheres. It was experimentally verified that the contact area are identical in the case of samples including three or four spherical unity.

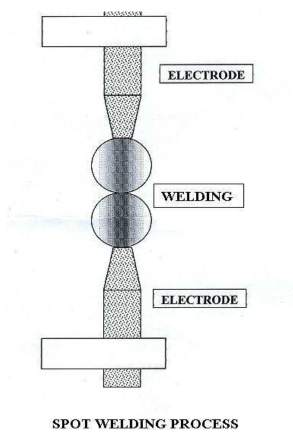


Figure 4: Sketch of the spot welding process.

The welding between two adjacent beads may be characterized by the distance r_w which designates the radius of the external circular contact area in which the roll around the welding is included. Several specimen showing different values of r_w have been built. Two examples of frequency spectrum are given in Fig. 5-a and 5-b. The first sample (r_w = 1.0 mm, L = 19.90 mm) is made up of two welded spheres of 10mm in diameter. The second sample (r_w = 0.8 mm and L = 29.88 mm) is made up of three welded spherical beads of 10mm in diameter.

Analysis of the results

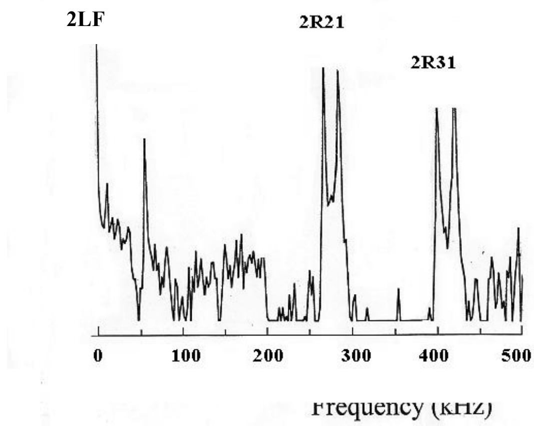
The experimental frequency spectra given in Fig. 5 reveal the following features:

-- A "splitting" of each normal mode of the Rayleigh series *R(n,1)*. In the case of two welded spheres we notice the existence of two main peaks; in the case of three welded beads we observe three peaks the frequency distribution of which is not symmetric. The lowest peak of each multiplet exists at the same frequency value whatever are the welding conditions and corresponds exactly to the resonance frequency observed with a single sphere of same material and diameter.

-- The N low frequency modes are detected; notice that one of these modes (f = 0) exists for all the samples.

These observations are in very good agreement with the numerical approach developed in the previous section: a finite chain gives rise to discrete values of the wave number k according to the dispersion curves. From additional experiments, it was checked that when the contact increases between the spheres, the frequency range *(Δf)* between the first and the last peaks of a given multiplet enlarges. Moreover it was also verified that *Δf* increases with the order of the mode.

(a)



(b)

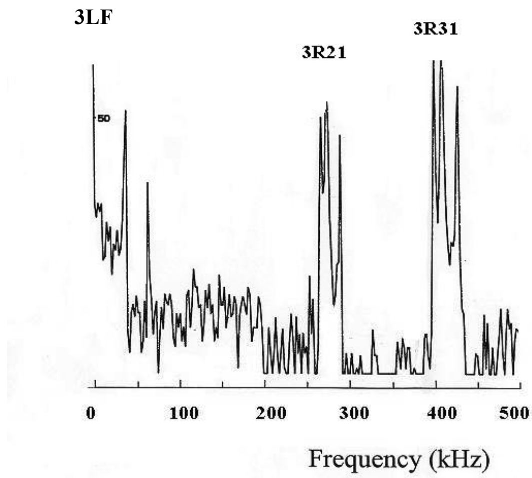
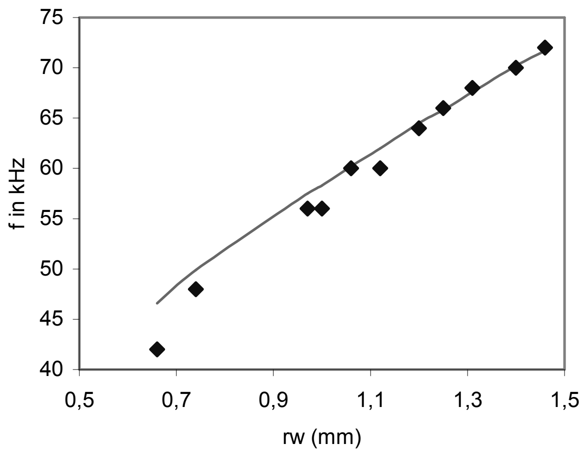


Figure 5: Examples of frequency spectra obtained with a specimen made up of two (a) and three welded spheres (b) respectively.

To see the influence of the contact on the "splitting effect", Fig. 6-a presents the variations of the frequency of the second low frequency mode in case of two welded spheres as a function of r_w, the frequency of the first low frequency mode being equal to zero. Similarly, in Fig. 6-b the variations of the frequency of the same mode are plotted versus L. On both figures, marks correspond to experimental results, full line to numerical results [6]. Taking into account that the frequency resolution of the experimental set-up is 2kHz, one can conclude that there is a very good agreement between numerical and experimental results. Thanks to Fig. 6 one can use the predicting model to solve the inverse problem: the characteristic distance r_w and the length L of a chain of welded spheres can be deduced using the frequency of the low frequency mode.

(a)



(b)

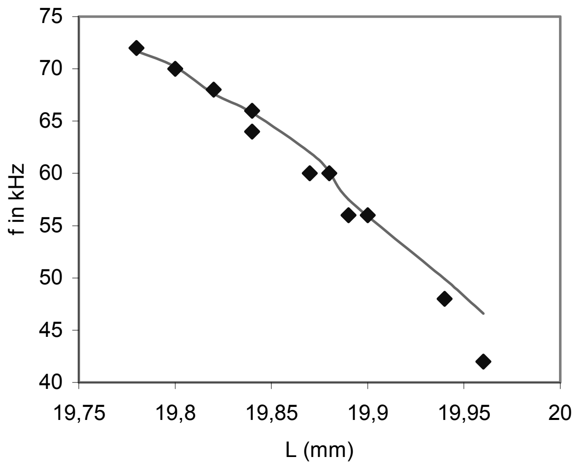


Figure 6: Experimental and numerical variations of the frequency of the second low frequency mode versus r_w (a) and versus L (b).
Full line: numerical results,
marks : experimental results.

Conclusion

In this paper, the propagation of short acoustic pulses in a chain made of welded spheres has been numerically and experimentally studied. Parametric study shows that one can use the predicting model to solve the inverse problem, it means to deduce the characteristics of the welding between the spheres using the discrete frequencies.

Finally, one may question oneself about the extension of this work to a more complicated case. The case of three welded spheres with two identical weldings has been considered and gives a nice agreement between numerical results and measurements. One interesting case could be also a chain of spheres with a mixed coupling between the spheres. First experimental results have shown that the detected frequencies are strongly affected by such a change in the coupling. Therefore, in the future we will try to improve the theoretical study in order to solve more realistic problems, like a default in a chain.

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