ACOUSTO-OPTIC CELLS WITH TRANSDUCERS OF VARYING THICKNESS: ELECTRICAL, ACOUSTIC AND ACOUSTO-OPTIC CHARACTERISTICS

V.I.Balakshy
Department of Physics, M.V.Lomonosov Moscow State University, Moscow, Russia
balakshy@osc165.phys.msu.ru

Abstract

Acousto-optic cells with piezoelectric transducers of varying thickness are investigated theoretically and experimentally. Relationships describing electrical, acoustic and acousto-optic properties of the cells are obtained in the approximation of a small thickness of the piezoelectric plate. Most attention is given to wedge-shaped transducers. Features peculiar to this type of transducers are discussed on the basis of numerical examples. It is shown that a complicated phase structure of acoustic field excited by the wedge-shaped transducer influences essentially acousto-optic cell characteristics. This effect can be used for improvement of acousto-optic devices.

Introduction

More than twenty years ago, measuring characteristics of acousto-optic (AO) cells made in our laboratory, we unexpectedly revealed that some cells demonstrated a perfectly strange frequency dependence of the Bragg angle [1]. Instead of a straight line inherent to AO isotropic diffraction, we obtained in some frequency range the curves shown in Figure 1. It is seen that the discrepancy between theoretical (ϑ_B^{theor}) and experimental (ϑ_B^{exp}) values of the Bragg angle was not negligibly small; it exceeded 3 times. If the AO cell was turned through 180^0, the difference ϑ_B^{exp} - ϑ_B^{theor} changed only in sign.

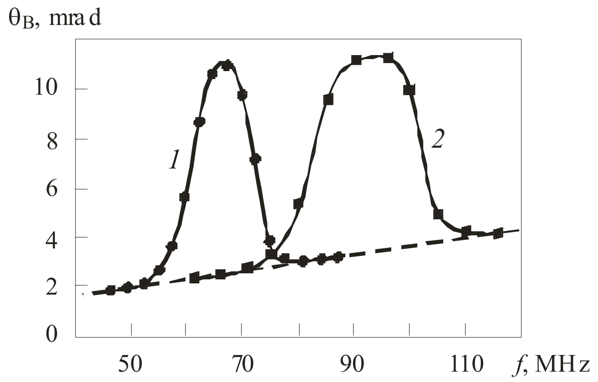


Figure 1. Theoretical (dashed line) and experimental (curves *1* and *2*) frequency dependencies of the Bragg angle

Analysing these results, we proposed that the emerged effect was caused by an acoustic wave front rotation with frequency that resulted, in its turn, from a wedge-like form of the transducers. To verify this proposition, we made a few small outer electrodes instead of a big solitary one. Measuring resonant frequencies of these small transducers, we convinced ourselves that the thickness of the piezoelectric plates really changed from one edge to the other.

Formerly, the revealed effect could be considered only as a hindering one which had to be eliminated. At the present time, the situation has changed. The technology of transducer making has become so perfect that transducers of almost either form can be fabricated. This circumstance opens up new possibilities for applied acousto-optics. Acousto-opticians know very well what an important role is played by the Bragg angle frequency dependence in AO devices. It is not overstated to say that all advantages of anisotropic diffraction in comparison with isotropic one result from quite another and very diverse form of this dependence [2]. The implementation of transducers with varying thickness enables us to get one more possibility for regulation of the Bragg angle or, in other words, for apodization of the AO cell to a specific problem to be solved. Therein lies an ideological aspect of the given work.

It should be noticed that in acoustics the transducers of this kind have already been studied in the context of the problem of ultrasonic frequency band broadening. The application of similar transducers in acousto-optics has specific features. Here, the knowledge of integral transducer characteristics, such as frequency band and efficiency of electric-to-acoustic power conversion is not sufficient for evaluation of transducer quality. The acoustic field structure and its changing with frequency are of great importance as well.

Theoretical background

The problem under discussion is formulated as follows. A piezoelectric plate of a varying thickness $h(x)$ is attached to a flat surface of an AO medium (Figure 2). The plate is fed by an alternating voltage of a frequency Ω from a HF generator having an electromotive force E_0 and an internal resistance R_i. For definiteness, we assume that no matching elements are between the generator and the transducer, implying, nevertheless, that such elements can additionally improve characteristics of the transducer.

At high frequencies, the piezoelectric plate is in effect a layer and its thickness variations are very

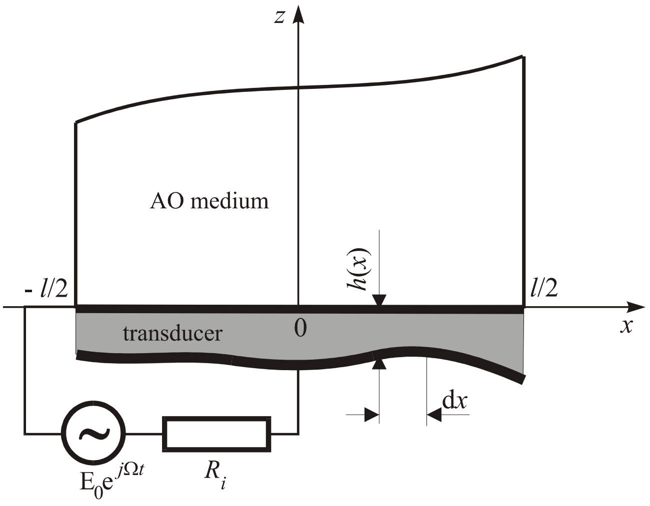


Figure 2. Excitation of ultrasound by a transducer of varying thickness

small. In this case, we can use a well-known solution of the problem of homogeneous piezoelectric plate excitation [3] and write the following expression for the complex admittance of a small part dx of the plate:

$$\mathrm{d}Y = j\frac{\Omega^2 \varepsilon b}{V_0 F(x)}\left\{1 - \frac{k^2}{F(x)} \cdot \frac{Z_a \sin(F(x)) + 2j[1-\cos(F(x))]}{Z_a \cos(F(x)) + j\sin(F(x))}\right\}^{-1} \mathrm{d}x \tag{1}$$

where $F(x) = \Omega h(x)/V_0$ is the normalized frequency, $l \times b$ are the dimensions of the piezoelectric plate, ε is the dielectric permittivity, k is the piezoelectric coupling coefficient, $Z_a = \rho_1 V_1/\rho_0 V_0$ is the relative acoustic impedance, V_0 and V_1 are the sound velocities in the transducer and AO medium respectively.

A total admittance of the inhomogeneous plate is determined by the formula

$$Y = \int_{-l/2}^{l/2} dY = \frac{1}{R(\Omega)} + j\Omega C(\Omega) \tag{2}$$

where R and C are the resistance and the capacitance in the parallel equivalent scheme of the transducer. Equation (2) makes it possible to calculate the voltage applied to the transducer

$$U = \frac{E_0}{1 + YR_i}, \tag{3}$$

the acoustic power radiated into the AO medium

$$P_a = \frac{E_0^2 \operatorname{Re}(Y)}{\left|1 + YR_i\right|^2} \tag{4}$$

and the conversion coefficient

$$\kappa = \frac{P_a}{P_{match}} = \frac{4R_i \operatorname{Re}(Y)}{\left|1 + YR_i\right|^2}. \tag{5}$$

The deformation in the acoustic wave close to the transducer is defined by the expression

$$a(x) = \frac{e}{\rho_0 V_0^2 V_1} U G(x) \tag{4}$$

where e is the appropriate piezoelectric constant and the function $G(x)$ is determined as

$$G(x) = \Omega[1-\cos(F(x))]\{F(x)\sin(F(x)) - 2k^2[1-\cos(F(x))] + jZ_a\left[k^2 \sin(F(x)) - F(x)\cos(F(x))\right]\}^{-1}. \tag{5}$$

It should be emphasized that the function G is complex. It means that there exists a phase shift between the voltage U and the acoustic strain a. This phase shift changes along the piezoelectric plate as well as the strain amplitude. Thus, the transducer with varying thickness excites an acoustic wave having a complicated amplitude and phase structure. In addition, this structure strongly depends on the frequency Ω.

Characteristics of AO diffraction can be calculated based on Equations (4) and (5). In the regime of small diffraction efficiency, the diffracted light intensity is expressed as

$$I_d = const \cdot \frac{\left|\int_{-l/2}^{l/2} G(x)\exp\left[-j\frac{\Omega}{V_1}(\vartheta_0 - \vartheta_B)x\right]dx\right|^2}{\left|1 + YR_i\right|^2} \tag{6}$$

where ϑ_0 is the incidence angle.

Simulations

With employing the computational procedure described above, three types of inhomogeneous transducers have been analysed:

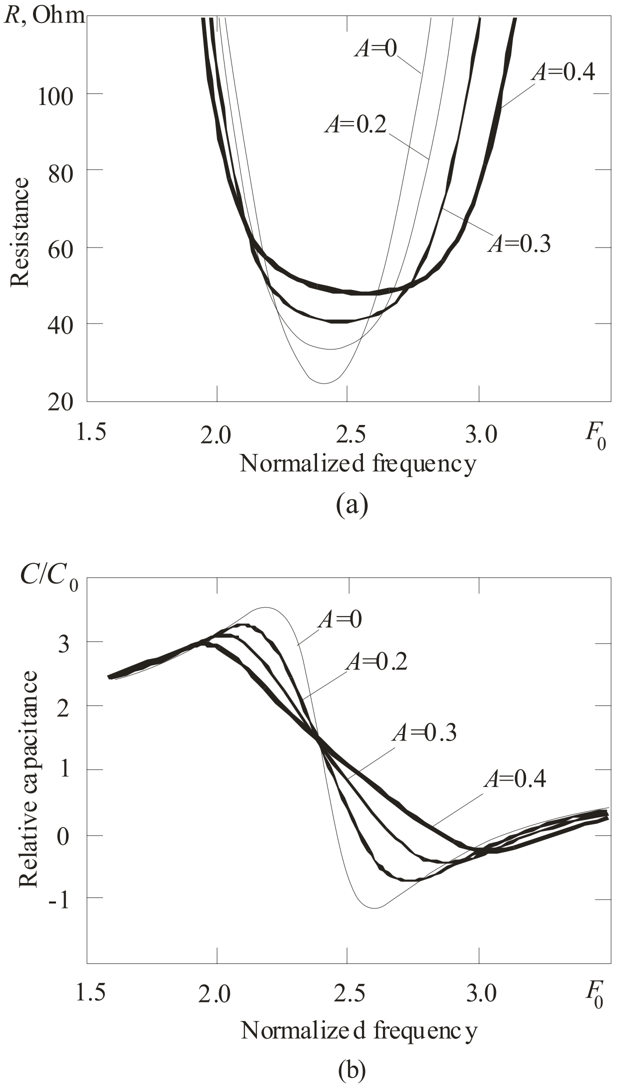
1) wedge-shaped transducers;
2) transducers composed of homogeneous piezoelectric plates with different thickness;
3) parabolic transducers with a quadratic dependence $h(x)$.

A part of these results relevant to the wedge-shaped transducers is presented below. The simulations have been carried out for an X-cut $LiNbO_3$ transducer attached to a 0^0-cut paratellurite cell. The dependence $h(x)$ in the form

$$h(x) = h_0(1 + \alpha x) \tag{7}$$

has been assumed where α is the wedge angle and h_0 is the thickness of the piezoelectric plate in its centre.

Figure 3 displays equivalent electrical parameters as a function of the acoustic frequency. Here the dimensionless parameters $A = \alpha l/h_0$ and $F_0 = \Omega h_0/V_0$ are introduced for convenience of numerical calculations. The case $A = 0$ corresponds to a homogeneous transducer. As is clear from the graphs, the tapering leads to smoothing of the



(a)

(b)

Figure 3. Resistance (a) and capacitance (b) of wedge-shaped transducer as a function of frequency

characteristics. It means weakening of the electrical impedance frequency dependence and, as a consequence, facilitation of the matching of the transducer with the generator. Therein lies one advantage of the wedge-shaped transducers.

Another advantage consists in the possibility of ultrasound excitation in a wider frequency range. This peculiarity is illustrated by Figure 4 where the frequency dependencies of the conversion coefficient are presented. The reason of the frequency band broadening is evident: different parts of the piezoelectric plate have different resonant frequencies. The operating range of the transducer increases with the wedge angle. As seen from the picture, at $A = 0.4$ the broadening of the band attains 50% without any deterioration in the conversion coefficient.

As noted above, for AO characteristics to be calculated, we have to know the distribution of amplitude and phase inside of the acoustic beam. Figure 5 presents amplitude (a) and phase (b) structure of the acoustic field at different frequencies.

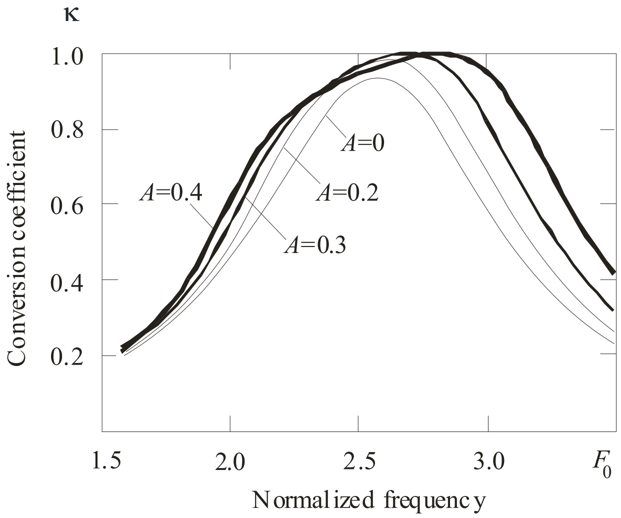
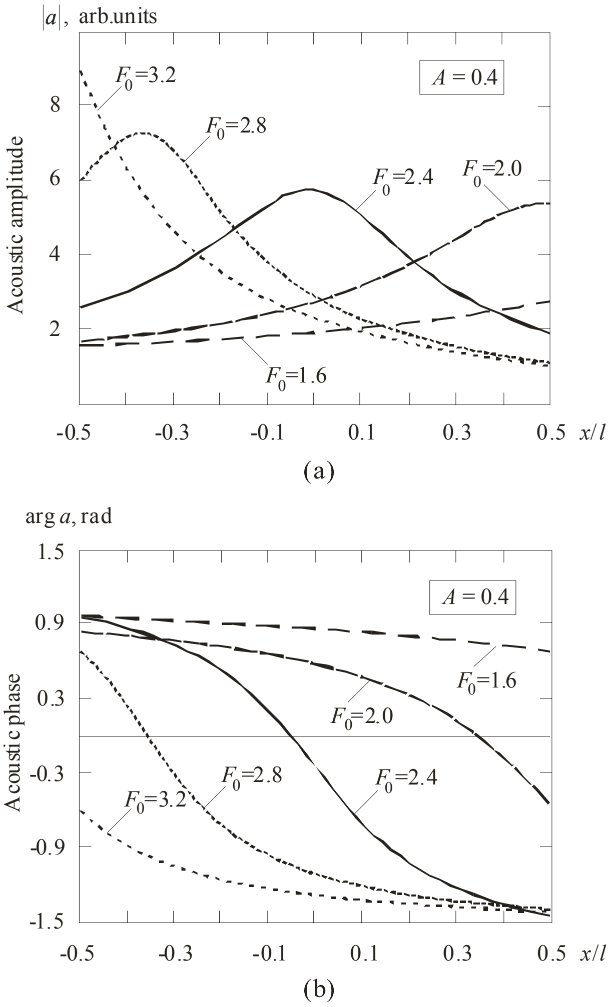


Figure 4. Conversion coefficient as a function of acoustic frequency

According to Equation (7), the thickness of the piezoelectric plate increases from the left to the right. As follows from Figure 3(a), the value $F_0 = 2.4$ corresponds to the resonant frequency of the central part of the piezoelectric plate. Therefore, it is here that the strain amplitude peaks and the phase shift passes



(a)

(b)

Figure 5. Distribution of acoustic amplitude (a) and phase (b) over wedge-shaped transducer surface

zero. With increasing the frequency the plots shift to the thinner end of the plate, and conversely. The phase graphs demonstrate in substance the shape of the wave front in the acoustic beam. As the front is curved, the conventional definition of the Bragg angle loses here its meaning.

One of the most important AO characteristics is the angular dependence of diffracted light intensity which determines the angular range of AO interaction [2]. In Figure 6, the intensity I_d is plotted against the normalized angular mismatch $\Delta\Theta = \left(\vartheta_0 - \vartheta_B\right) / \vartheta_B^{isotr}$. The calculations are carried out from Equation (6) for different acoustic frequencies. For comparison, the dashed line indicates sinc^2-function which is typical for a homogeneous acoustic beam. Two important peculiarities have to be remarked. First, the angular characteristics in case of the wedge-like transducer have a complicated asymmetrical view. Second, the maximum intensity of the diffracted light is attained at the angle ϑ_0^{opt} shifted from the Bragg angle ϑ_B.

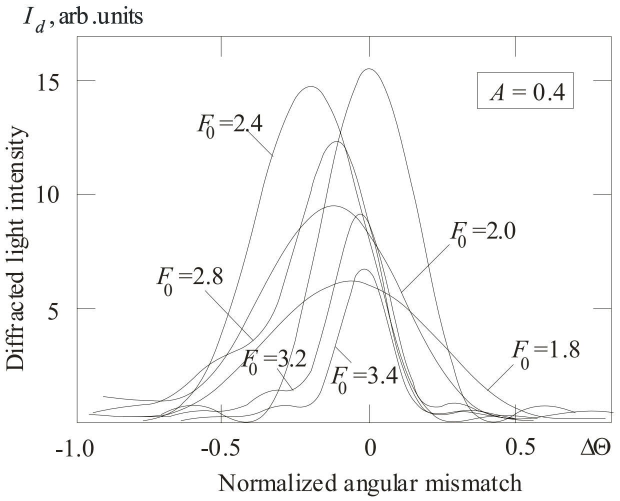


Figure 6. Angular characteristics of AO diffraction at different acoustic frequencies

The graphs presented in Figure 7 give additional information. Here the horizontal straight line $\Theta = 0$ corresponds to the conventional Bragg angle. The curves Θ_1 and Θ_2 show the boundaries of AO interaction defined by the acoustic beam divergence $\varphi = 2\pi V_1 / l\Omega$. The bold curve demonstrates the frequency dependence of the normalized angular shift $\Delta\Theta_B = \left(\vartheta_0^{opt} - \vartheta_B\right) / \vartheta_B^{isotr}$. It is seen that the shift reaches half of the angle φ in a good agreement with our experiment. The dependences $I_d(F_0)$ and $\kappa(F_0)$ also depicted in the figure indicate that the shift $\Delta\Theta_B$ is maximal just in the range where the intensity peaks, whereas the maximum of the conversion coefficient lies in quite another range. This distinction of the characteristics $I_d(F_0)$ and $\kappa(F_0)$ is another peculiarity unique to AO cells with inhomogeneous transducers.

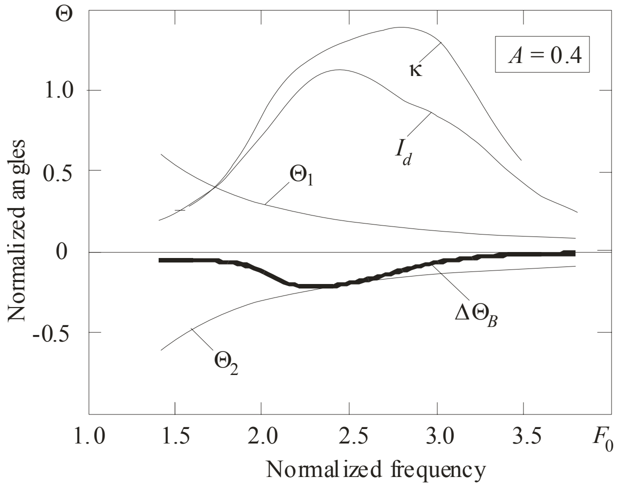


Figure 7. Frequency dependencies of angular shift $\Delta\Theta_B$, diffracted light intensity I_d and conversion coefficient κ

Conclusion

AO cells with inhomogeneous transducers of different types are studied in this work. It has been shown that variations of the thickness of piezoelectric plates lead to both amplitude and phase inhomogeneity of the acoustic field excited by these transducers. The inhomogeneity can essentially change characteristics of AO diffraction and parameters of AO devices. The phase inhomogeneity gives a more noticeable effect, showing itself in changing the Bragg angle and the diffraction efficiency. In those cases when such changes are not desirable, the formulas obtained in the work enable estimating a necessary precision of transducer making. On the other hand, the performed analysis opens up new facilities for improvement of AO devices.

Acknowledgement

This work has been supported in part by Russian Foundation for Basic Research, grant # 02-07-90448.

References

[1] V.I.Balakshy, V.N.Parygin, and B.P.Philippov, "Angular displacement of an acoustic beam excited by a wedge-shaped transducer", Sov. Acoust. J., vol. 22, pp. 596-598, 1976.

[2] V.I.Balakshy, V.N.Parygin, and L.E.Chirkov, Physical principles of acousto-optics, Radio I sviaz, Moscow, 1985.

[3] N.F.Foster, G.A.Coquin, G.A.Rozgonyi, and F.A.Vannatta, "Cadmium sulphide and zinc oxide thin-film transducers", IEEE Trans. Son. Ultrason., vol. SU-15, pp.28-35, 1968.