PROFILED PLANE WAVES AS THE FRAMEWORK FOR THE INTERACTION OF 3D BOUNDED BEAMS WITH ANISOTROPIC MATERIALS

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Abstract

Recently a new theory has been developed which is based on a local approximation of a bounded beam by means of inhomogeneous waves. The resulting waves are called profiled plane waves or simply beams formed by the local inhomogeneity approach. The main difference between profiled plane waves and the classical superposition of inhomogeneous waves to form bounded beams [J. Acoust. Soc. Am. 72(2), 585-590] is that exponentially growing tails in the beam profile do not appear and that there is no uncertainty as to what inhomogeneous waves are involved in the formation. Since a 2D approximation of the interaction of ultrasound with materials is not very realistic in the case of anisotropic materials due to the inherent 3D nature of anisotropic materials, it is necessary to extend profiled plane waves to a 3D description. Then, a study can be performed on the interaction of 3D bounded beams with such materials. Special attention is given to beam deformations at critical angles of incidence.

Introduction

The Schoch effect [1-8], where a reflected beam is considerably transformed due to incidence at the Rayleigh angle, has been studied before. From those studies it is found that the Fourier method describes the phenomenon relatively well and so does the inhomogeneous wave theory. The difference between both methods is mainly the understanding of the physics behind the phenomenon. In the Fourier method, the resulting bounded beam is formed as a superposition of pure homogeneous plane waves and especially their phase-amplitude interactions. So, even though the result corresponds more or less to experiments, it is hard to imagine what exactly causes the deformation. From experiments it is known that it is the generation of leaky Rayleigh waves that is the cause, but pure plane waves never correspond to the complex Rayleigh pole whence it is hard to see what really happens. The inhomogeneous wave method on the other hand describes the reflected sound as a superposition of inhomogeneous waves. If one of those inhomogeneous waves corresponds to the Rayleigh pole, it stimulates a leaky Rayleigh wave very efficiently and is shifted considerably along the interface. Hence it is more straithforward to understand what is causing the Shoch displacement and the Schoch deformations. There are however some problems with that inhomogeneous wave approach. The current work applies the shift-properties of inhomogeneous waves and describes sound as a local approximation by means of inhomogeneous waves. Hence local parts of the sound beam can be shifted in the vicinity of the Rayleigh angle.

Some problems concerning the Fourier method

It is hard to introduce a new model if the old Fourier method seems to be perfect. Hence it is interesting to note that it is not perfect. In the Fourier method, a bounded beam is formed as a superposition of pure homogeneous plane waves all travelling in different directions and having an amplitude that is determined by means of the Fourier transform of the beam profile at a chosen position in space. Then, typically a beam is formed as in Figure 1

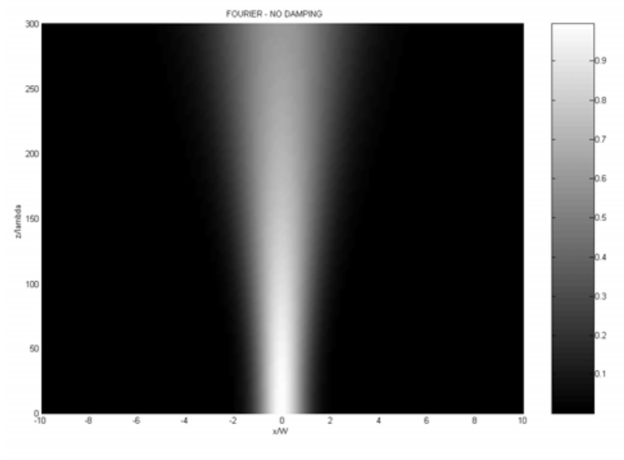

Figure 1 : A bounded gaussian beam formed with the Fourier method. Beam spreading is noticed along the propagation direction Z.

This beam looks perfect. However, due to the fact that for numerical reasons the bounded beam consists of a limited number of pure homogeneous plane waves, there are neighbours both along the positive axis and along the negative axis. This can be seen in Figure 2

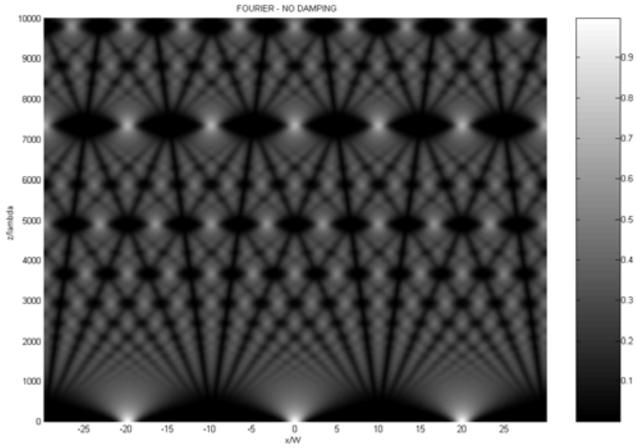

Figure 2 : If a larger area is taken into account compared with Figure 1, then it is seen that 'neighbors' appear on the left and on the right and that those additional sound fields interact with each other along the direction of propagation

The neighboring beams interfere with the actual beam under consideration whence a sound pattern appears that does not correspond to what one considers to be a gaussian beam. In most applications this is no big problem, but it shows that the method is not perfect and therefore cannot forbid the introduction of new models. Furthermore it has been shown in [9] that in the case of diffraction on rough surfaces there are situations where the Fourier method predicts results that do not agree with experiments, whereas the inhomogeneous wave method does not contain this anomaly.

Some problems concerning the inhomogeneous wave theory for bounded beams

The inhomogeneous wave theory builds bounded beams by superposing inhomogeneous waves in a best-fit approximation. The result is that a bounded beam contains the necessary inhomogeneities to stimulate leaky Rayleigh waves. Nevertheless there are some problems that struggle from the lack of determinism. This is because there is freedom of choosing what inhomogeneities are part of the superposition set. Since nature at this level is deterministic, this causes interpretation problems. Also, the numerical optimizations that are used to form a bounded beam cannot avoid the appearance of exponentially growing tails of amplitude at considerable distances measured form the center of the beam. This can be seen in Figure 3.

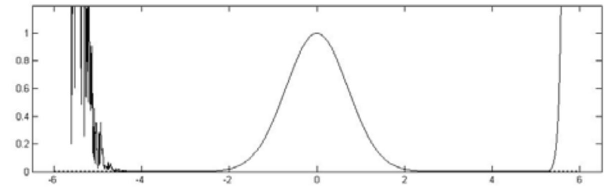

Figure 3 : A Gaussian beam profile formed by means of a superposition of inhomogeneous waves. The appearance of exponentially growing tails is visible at more than 4 beamwidths.

The basics of the local inhomogeneity approach

The local inhomogeneity approach is based on the fact that amplitude changes for exponential functions can be interpreted as parallel position shifts.

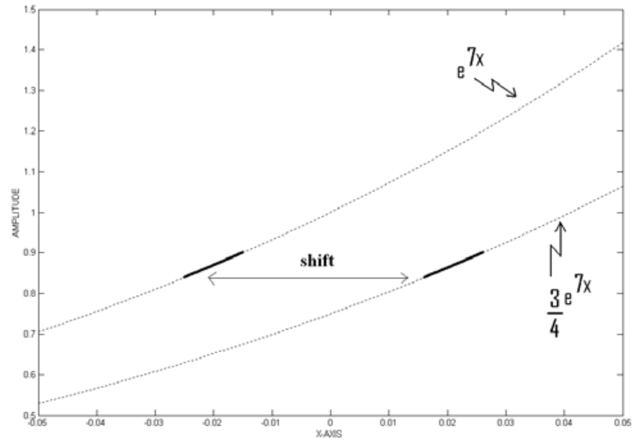

Figure 4 : The principle of a spatial shift if the amplitude of an exponential function changes.

The local inhomogeneity approach describes the profile of a bounded beam locally as an inhomogeneous wave. This wave then interacts with a considered interface and gets reflected having a different amplitude, whence it is shifted in space.

The formation of a realistic beam

It is possible to rewrite the wave equation in a form where no big changes are to be expected for sound waves in the propagation direction. The resulting wave equation is called the paraxial wave equation. A spreading gaussian beam is a solution of this equation. We have extended the description to a 3D description of bounded beams travelling in an arbitrary chosen propagation direction in 3D space. The mathematical description is as follows:

$$u(x,y,z) \approx \frac{1}{S} e^{-ih} e^{i\psi} e^{-\frac{p^2}{S^2}} e^{-ik\frac{p^2}{2R}}$$

with

$$h = \mathbf{k} \bullet \left(\mathbf{r} - \mathbf{r_0} \right)$$

$$S = S_0 \sqrt{1 + \frac{\left(\mathbf{k} \bullet \left(\mathbf{r} - \mathbf{r_0} \right) \right)^2}{z_R^2 \left(\mathbf{k} \bullet \mathbf{k} \right)}}$$

$$R = \frac{h}{|\mathbf{k}|} + z_R^2 \frac{|\mathbf{k}|}{h}$$

$$\psi = \arctan\left(\frac{K \left| \mathbf{r_0} + \mathbf{k} \right|}{z_R} \right)$$

$$K\mathbf{k} \bullet \left(\mathbf{r} - \mathbf{r}_0 - K\mathbf{k} \right) = 0$$

An example is shown in Figure 5 for the amplitude distribution of such a beam, originating on the left side of the figure and propagating in the XZ-plane at an angle of 20 degrees measured form the Z-axis.

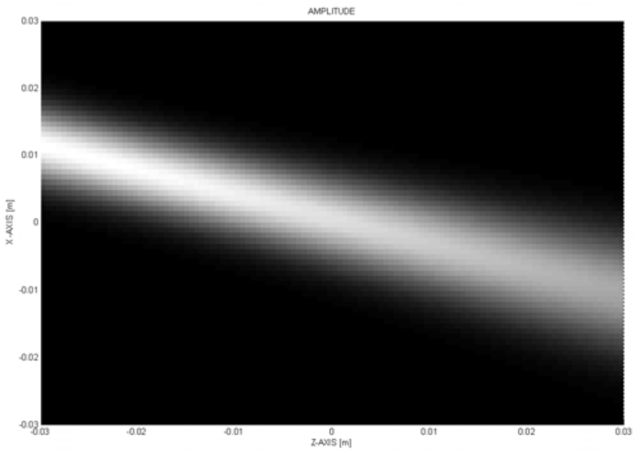

Figure 5 : The propagation of a bounded gaussian beam in the paraxial wave description.

Now, if we need to approximate sound near an interface, we must not solely reckon with the amplitude distribution but also with the local propagation direction that differs from spot to spot if compared to the direction of propagation at the center of the beam. The reason is of course beam spreading. This can be better seen in Figure 6 where the real part of the complex amplitude is given for the same beam.

Beam deformations at the Rayleigh angle

Here we concentrate on what happens at the Rayleigh angle of incidence. The interface between brass and water is considered and is given by the XY-plane. In Figure 7 the new position in the X-plane after reflection is plotted as a function of the old position in the XY-plane. The considered bounded beam has a physical beam width of 1 cm. It is noticed that spots far from the center (0,0) of the incident beam do not get displaced at all. Spots closer to the origin get displaced.

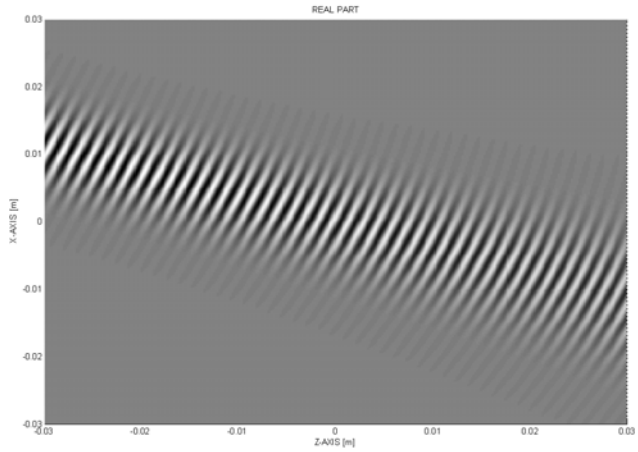

Figure 6 : The real part of Figure 5. It is seen that the propagation direction differs slightly at the sides if compared with the center of the beam.

Some spots move to different places, others move to the same place, where they can interfere constructively or destructively. This is the reason for beam deformations like the Schoch effect.

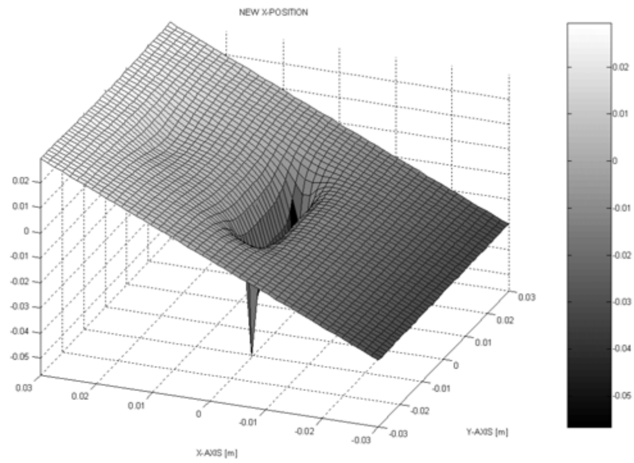

Figure 7 : The new position in the X-direction as a function of the original position on the XY-plane at the Rayleigh angle of incidence. It is seen that strong beam deformations occur inside the beam that cause the Schoch displacement.

The resulting bounded beam is then formed by all these new spots with corresponding amplitude and phase. Our computer program that numerically deals with the formation of the reflected beam appears to work perfectly except for some small anomalies that need to be resolved in the near future.

Conclusions

It is shown that it is possible to describe a bounded beam by means of a local approach in terms of inhomogeneous waves. This method does not suffer from indeterminism as in the classical inhomogeneous wave approach and does not result in exponentially growing tails some distance away from the center of the bounded beam. Furthermore the method results in a better intuitive understanding of what happens to a bounded beam if incidence occurs at the Rayleigh angle. It appears that some parts of the bounded beam shift to such positions that there is a null zone and a second (nonspecular) lobe. Because a description has been performed in 3D, calculations can be expected in the near future that describe the interaction of 3D bounded beams with anisotropic materials. It is expected that deformations will occur also in out of incidence plane directions.

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