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Spectral Analysis of a PM Space in the Simulation of Nonlinear Ultrasonic Wave Propagation

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Abstract

Various models have been proposed for the simulation of classical and nonclassical effects on the propagation of ultrasonic waves in nonlinear mesoscopic elastic materials. They usually assume the presence of a large number of soft interstices, which are responsible for the nonlinear and hysteretic behavior of the material. In order to simplify the treatment, a so-called "PM space" of pairs of preassigned interstice strain states and corresponding pressure values, at which transitions from one state to the other are assumed to take place, is often considered. The relationship between the choice of the PM space and the consequent nonlinearity is, however, inferred only phenomenologically. Starting with the case of only one interface, the interdependence among the parameters of the model, the input excitation, and the spectral contents of the specimen's response is derived analytically. The results are related to the strains and restoring forces as present in thin bonded interfaces.

Introduction

Experiments on e.g. rocks, soil, cement, concrete, and damaged elastic materials have revealed evidence for nonlinearity, hysteresis, and discrete memory in their elastic behavior. These discoveries suggest the existence of a nonlinear mesoscopic elasticity (NME) universality class, to which all the aforementioned materials and others belong [1]. Hence the appearence of a variety of nonlinear effects in both quasi-static and dynamic experiments, such as the resonance frequency downwards shift with increasing excitation amplitude, the generation of higher harmonics, and the so-called slow dynamics. Various models have been proposed for the simulation of these classical and nonclassical effects on the propagation of ultrasonic waves in nonlinear mesoscopic elastic materials [1-7], based on a statistical Preisach-Mayergoitz space [8-10]. They assume the presence of a large number of soft interstitial regions, which are taken to be responsible for the nonlinear and hysteretic behavior of the material specimen [2-5]. In order to simplify the treatment, the so-called "PM space" of pairs of preassigned interstice strain states and corresponding pressure values, at which transitions from one state to the other are assumed to take place, is considered. The relationship between the choice of the PM space and the consequent nonlinearity is, however, inferred only phenomenologically. The investigation of the binding forces in adherent joints, in which the interface between bonded elements is the primary source of nonlinearity [11-13] and the general theoretical analysis of the background [14] may allow more detailed and realistic conclusions about the originating forces of nonclassical nonlinear (NCNL) effects.

Starting with the case of only one interstice, the interdependence among the parameters of the PM model, the input excitation, and the spectral contents of the specimen's response is derived analytically. The results are related to the strains and restoring forces as present in a thin bonded interface.

Spectral analysis of a single rectangular hysteretic mesoscopic elastic unit (HMEU)

A sample of a linear elastic substrate material containing one adherent joint, which is the source of nonlinearity, is considered. In contrary to [11-13], the bonded interface is described by a rectangular hysteretic meso-scopic elastic unit (HMEU) as defined e.g. in reference [9] (Fig. 1). The interface distance may have only two stable values l_o and l_c, $l_o > l_c$, which correspond to a so-called open or closed state. If the HMEU is initially in its open state l_o and an increasing external pressure is applied, the interface remains in the open state till the pressure P_c is reached, where the interface distance changes abruptly to its closed state l_c. A further increase of the applied pressure does not change the interface distance anymore. If now the pressure is decreased again, the HMEU remains closed till the pressure $P_o \leq P_c$ is reached, where it jumps into its open state again and remains there even if the pressure is further decreased. The HMEU shows hysteresis if $P_o < P_c$, no hysteresis occurs if $P_o = P_c$. The external pressure has, because of the balance of forces, the same amount as the restoring forces in the interface referred to in [11-13] but acts into the opposite direction.

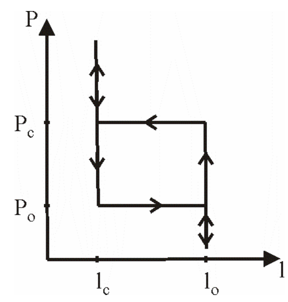

Figure 1 : Rectangular hysteretic mesoscopic elastic unit as used in the PM space model [9]

Figure 2 shows a one-dimensional model of the HMEU joined to a linear elastic substrate. The HMEU is excited by an external sinusoidal force, the pressure

$$P = f(\tau) = -f_0 \sin\tau \ . \quad (1)$$

Here, $\tau = \omega t$ is the normalized time, and ω is the excitation frequency. The phase is chosen in agreement with the description in references [11-14], i.e. the excitation starts with a tension force on the interface to increase its distance. The resulting restoring force in the interface is $F = -P$. The force amplitude f_0 has to be larger than the opening and closing pressures P_o and P_c, i.e., $f_0 \geq |P_o|$, $f_0 \geq |P_c|$, because otherwise, the interface would remain in its initial state (closed or open). The excitation of the HMEU generates elastic waves in the joined linear elastic material. As in reference [13, 14], the strain of the transmitted waves, the response, is represented as a Fourier series:

$$\varepsilon_t(X,\tau) = \varepsilon_0 + \sum_{n=1}^{\infty} \varepsilon_n \sin(n\tau - nX + \varphi_n) \ , \quad (2)$$

$X = kx$ is the normalized length coordinate, $k = \omega/v_L$ is the wave number, and v_L the sound velocity in the linear elastic substrate. The HMEU is at X = 0.

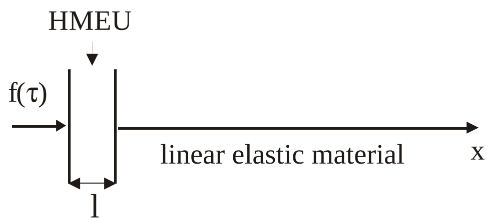

Figure 2 : One-dimensional model of a HMEU joined to a linear elastic material

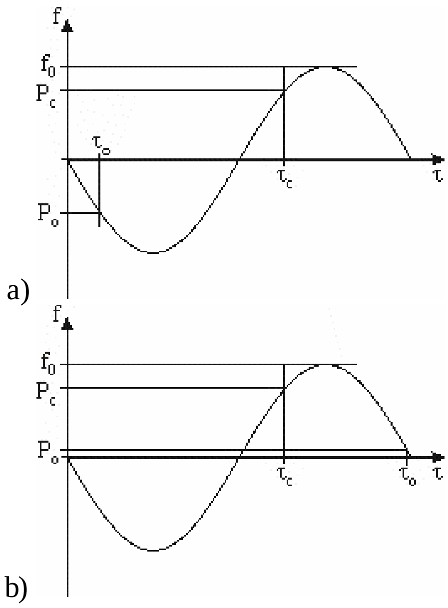

Figure 3 : External sinusoidal excitation $f(\tau)$, opening and closing pressures P_o and P_c and times τ_o and τ_c within a cycle are indicated a) starting with a closed and b) starting with an open interface, respectively

Figure 3 shows a qualitative plot of one cycle of the sinusoidal excitation $f(\tau)$, in which the opening and closing pressures P_o and P_c as well the corresponding opening and closing times τ_o and τ_c are indicated starting with a closed (Fig. 3a) and with an open (Fig. 3b) interstice, respectively. The opening and closing times τ_o and τ_c are related to the pressures P_o and P_c by

$$P_o = -f_0 \sin\tau_o \ , \quad \left.\frac{df(\tau)}{d\tau}\right|_{\tau=\tau_o} = -f_0 \cos\tau_o < 0 \ , \quad (3a)$$

$$P_c = -f_0 \sin\tau_c \ , \quad \left.\frac{df(\tau)}{d\tau}\right|_{\tau=\tau_c} = -f_0 \cos\tau_c > 0 \ , \quad (3b)$$

i.e., $0 \leq \tau_o < \pi/2$, $3\pi/2 \leq \tau_o < 2\pi$; $\pi/2 \leq \tau_c < 3\pi/2$. The conditions concerning the derivatives of the force ensure that opening occurs for decreasing and closure for increasing pressure. Within a cycle, $\tau_o \leq \tau_c$ for an initially closed, and $\tau_c \leq \tau_o$ for an initially open state.

As long as the interface distance is rigid, the sinusoidal force is directly transferred into the elastic material. At the opening and at the closing pressure jumps occur corresponding to the change in strain, $\Delta l/l_c$ and $\Delta l/l_o$, respectively, depending on if the initial state is closed or open. $\Delta l = l_o - l_c$, is the difference in the interface distance between the open and the closed state. That is, in case of a closed state as starting position, the strain boundary condition

$$\varepsilon_t(X{=}0,\tau) = \frac{f_0}{c_{11}} \sin\tau - \frac{\Delta l}{l_c} \sum_{\nu} \{ \theta(\tau-(\tau_o+2\pi\nu)) - ?(\tau-(\tau_c+2\pi\nu)) \} \quad (4)$$

has to be fulfilled. ν is an integer, $\theta(x)$ is the step function defined by $\theta(x) = 0$ if $x < 0$ and $\theta(x) = 1$ if $x \geq 0$, $c_{11} = \rho v_L^2$ is the elastic constant, ρ the density and v_L the sound velocity in the linear elastic material. If we start with an open state the boundary condition is

$$\varepsilon_t(X{=}0,\tau) = \frac{f_0}{c_{11}} \sin\tau + \frac{\Delta l}{l_o} \sum_{\nu} \{ \theta(\tau-(\tau_c+2\pi\nu)) - \theta(\tau-(\tau_o+2\pi\nu)) \} \ . \quad (5)$$

With the strain boundary conditions (4) and (5), the spectral representation (2) of the strain at the HMEU

$$\varepsilon_t(X{=}0,\tau) = \varepsilon_0 + \sum_{n=1}^{\infty} \varepsilon_n \sin(n\tau + \varphi_n) \ , \quad (6)$$

and the orthogonality relations of the trigonometric functions $\sin(n\tau)$ and $\cos(n\tau)$ the amplitudes ε_n and the phases φ_n of the response can be calculated. In case of an initially closed state we get

$$\varepsilon_0 = -\frac{\Delta l}{l_c} \frac{\tau_c - \tau_o}{2\pi} \ , \quad (7a)$$

$$\varepsilon_n \sin\varphi_n = -\frac{\Delta l}{n\pi l_c} \{\sin(n\tau_c) - \sin(n\tau_o)\} \quad (7b)$$

$$= -\frac{2\,\Delta l}{n\pi l_c} \cos\frac{n(\tau_c + \tau_o)}{2} \sin\frac{n(\tau_c - \tau_o)}{2} \ ,$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} + \frac{\Delta l}{n\pi l_c}\{\cos(n\tau_c) - \cos(n\tau_o)\} \qquad (7c)$$

$$= \frac{f_0}{c_{11}}\delta_{n1} - \frac{2\Delta l}{n\pi l_c}\sin\frac{n(\tau_c+\tau_o)}{2}\sin\frac{n(\tau_c-\tau_o)}{2} \ .$$

The results for an open state as starting position are

$$\varepsilon_0 = \frac{\Delta l}{l_o}\frac{\tau_o-\tau_c}{2\pi}, \qquad (8a)$$

$$\varepsilon_n \sin\varphi_n = \frac{\Delta l}{n\pi l_o}\{\sin(n\tau_o) - \sin(n\tau_c)\} \qquad (8b)$$

$$= \frac{2\Delta l}{n\pi l_o}\cos\frac{n(\tau_o+\tau_c)}{2}\sin\frac{n(\tau_o-\tau_c)}{2} \ ,$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} - \frac{\Delta l}{n\pi l_o}\{\cos(n\tau_o) - \cos(n\tau_c)\} \qquad (8c)$$

$$= \frac{f_0}{c_{11}}\delta_{n1} + \frac{2\Delta l}{n\pi l_o}\sin\frac{n(\tau_o+\tau_c)}{2}\sin\frac{n(\tau_o-\tau_c)}{2} \ .$$

Eqs. (7) and (8) show, that in general the response of a sinusoidally excited rectangular HMEU contains the incident frequency, all of its higher harmonics, and a static part. The amplitudes ε_n and phases φ_n of the transmitted waves contain the amplitude of the excitation indirectly via the opening and closure times τ_o and τ_c as a parameter. The parameters of the transmitted fundamental frequency additionally depend on the ratio of the excitation amplitude and the elastic constant c_{11} in the substrate. Its strain follows linearly the excitation without a delay (i.e. $\varphi_1 = 0$) if the first term in Eqs. (7c) and (8c) for n=1 dominates.

The static strain ε_0 during insonification is negative if the closed state of the interface is the starting position, i.e., the static force pushes apart the surfaces forming the HMEU, the mean interface distance increases during insonification. Vice versa, if initially the HMEU is in its open state, the static force tightens together the surfaces, the mean interface distance decreases during insonification.

The strain amplitudes and phases of the higher harmonics ($n \geq 2$) are given by

$$(\varepsilon_n)^2 = \{\frac{2\Delta l}{n\pi l_{c/o}}\sin\frac{n(\tau_c-\tau_o)}{2}\}^2, \qquad (9a)$$

$$\tan\varphi_n = \cot\frac{n(\tau_c+\tau_o)}{2} \ , \qquad (9b)$$

$$\varphi_n = \frac{\pi - n(\tau_c+\tau_o)}{2} \quad (\pm\, \nu\pi \ , \ \nu \text{ is an integer}).$$

The amplitudes ε_n decrease only with 1 over its order n, i.e., like the elements of a harmonic series. That is, for the response of a rectangular HMEU, a cut-off of the Fourier series (2) with only a finite number of higher harmonics cannot be a good approximation.

For increasing excitation amplitude $f_0 \rightarrow \infty$ the closure time τ_c moves towards π and the opening time τ_o towards 0 or 2π, depending on the initial state (see Fig. 3). This leads to approximate Eqs. (7) and (8):

$$\varepsilon_0 = -\frac{\Delta l}{2 l_c}, \qquad (10a)$$

$$\varepsilon_n \sin\varphi_n = -\frac{2\Delta l}{n\pi l_c}\cos\frac{n\pi}{2}\sin\frac{n\pi}{2} \ , \qquad (10b)$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} - \frac{2\Delta l}{n\pi l_c}\sin\frac{n\pi}{2}\sin\frac{n\pi}{2} \ , \qquad (10c)$$

and $\quad \varepsilon_0 = \frac{\Delta l}{2 l_o}, \qquad (11a)$

$$\varepsilon_n \sin\varphi_n = \frac{2\Delta l}{n\pi l_o}\cos 3\frac{n\pi}{2}\sin\frac{n\pi}{2} \ , \qquad (11b)$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} + \frac{2\Delta l}{n\pi l_o}\sin 3\frac{n\pi}{2}\sin\frac{n\pi}{2} \ , \qquad (11c)$$

respectively. The results show, that for large excitation amplitudes only odd harmonics are generated. The limiting values of the amplitudes and phases of the the odd harmonics are

$$\varepsilon_1 = \frac{f_0}{c_{11}} - \frac{2\Delta l}{\pi l_{c/o}} \ , \ \cos\varphi_1 = 1, \text{ i.e. } \varphi_1 = 0, \qquad (12a)$$

$$\varepsilon_{2n+1} = \frac{2\Delta l}{(2n+1)\pi l_{c/o}}, \ \cos\varphi_{2n+1} = -1, \text{ i.e. } \varphi_{2n+1} = \pi. \qquad (12b)$$

No hysteresis occurs if $P_c = P_o$. In this case, Eqs. (3) and Fig. 3 yield $\tau_c + \tau_o = \pi$ and 3π and $0 \leq \tau_c - \tau_o \leq \pi$ and $0 \leq \tau_o - \tau_c \leq \pi$ for the starting position in the closed and in the open state, respectively. This simplifies Eqs. (7) and (8), and we get

$$\varepsilon_n \sin\varphi_n = -\frac{2\Delta l}{n\pi l_c}\cos\frac{n\pi}{2}\sin\frac{n(\tau_c-\tau_o)}{2} \ , \qquad (13a)$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} - \frac{2\Delta l}{n\pi l_c}\sin\frac{n\pi}{2}\sin\frac{n(\tau_c-\tau_o)}{2} \ , \qquad (13b)$$

if the initial state is closed and

$$\varepsilon_n \sin\varphi_n = \frac{2\Delta l}{n\pi l_o}\cos 3\frac{n\pi}{2}\sin\frac{n(\tau_c-\tau_o)}{2} \ , \qquad (14a)$$

$$\varepsilon_n \cos\varphi_n = \frac{f_0}{c_{11}}\delta_{n1} + \frac{2\Delta l}{n\pi l_o}\sin 3\frac{n\pi}{2}\sin\frac{n(\tau_c-\tau_o)}{2} \qquad (14b)$$

otherwise. In agreement with references [11-14], these equations yield two possibilities for the phase of each transmitted wave in the non hysteretical case:

$\varepsilon_{2n+1} \sin\varphi_{2n+1} = 0$, n = 0, 1, 2, ... ; (15a)

i.e. $\cos\varphi_{2n+1} = \pm 1$, $\varphi_{2n+1} = 0, \pi$;

$\varepsilon_{2n} \cos\varphi_{2n} = 0$, n = 1, 2, 3, ... ; (15b)

i.e. $\sin\varphi_{2n} = \pm 1$, $\varphi_{2n} = \pi/2, 3\pi/2$.

The actual values depend on the ratio of the amplitude f_0 of the excitation force and the opening and closing pressure $P_o = P_c$, which determines the difference $\tau_o - \tau_c$. The phase of the transmitted fundamental frequency also depends on the ratio of f_0 to the elastic constant c_{11} in the substrate. As already pointed out in the general case, it is zero ($\varphi_1 = 0$) if the first term in Eqs. (13b) and (14b) for n=1 dominates (in agreement with [11-14]). The difference of the normalized opening and closure time t_o and t_c is within the region from 0 to π, which leads to the phase of the second harmonic $\varphi_2 = \pi/2$ for an initially closed interface (again in agreement with [11-14]) and to $\varphi_2 = 3\pi/2$ if the initial state is open. For large excitation amplitudes, $f_0 \rightarrow \infty$, the amplitudes and phases in the response obtain the limiting values given in Eqs. (12), i.e. in this limit, the influence of hysteresis disappears.

Strains and restoring forces in thin bonded interfaces

A thin bonded interface in a component can be described by two surfaces of the substrate material joined together by interaction forces. In general, the binding forces are nonlinear and cause a nonlinear transmission of ultrasonic waves. The response to a monochromatic excitation contains not only the incident frequency but also its higher harmonics. Measured amplitudes and phases of the transmitted waves may be used to determine the force - distance curve in the interface [11-13], or the equivalent, its stress - strain relation.

In the preceding chapter, the opposite was carried out. In contrary to [11-13], the force - distance relation of a thin bonded interface in a linear elastic substrate material was described by a rectangular HMEU as defined e.g. in [9]. For an incident monochromatic compressional wave the amplitudes and phases of the waves generated in transmission were calculated. The general results are given in Eqs. (7) to (9)).

The rough description of the force - distance relation in the interface as a rectangular HMEU yields the general behavior of a nonlinear interface as the generation of higher harmonics and a constant contribution, which changes the mean interface distance during insonification. But, as can be seen from Eq. (9a) and discussions there, the higher harmonics of high order are overestimated. To confirm this, we like to mention that in our experiments of nonlinear ultrasonic transmission through thin bonded interfaces we have measured amplitudes of higher harmonics above noise only up to the third order [11-13]. The overestimation of the higher harmonics is probably due to the "unphysical" edges in the force - distance relation of a PM space unit (Fig. 1). The effect might be reduced by smoothing the force - distance curve. In particular, if the amplitude of the excitation force reaches the maximum of the restoring force in the interface [11-13] the description by a rectangular HMEU can no longer be used, even though it yields good results in the simulation of wave propagation in materials with a large number of nonlinear HMEUs and a convenient distribution of different pairs of opening and closure pressures and interface distances [e.g. 7]. The integration over a large number of different HMEUs has a smoothing effect. Additionally, for low excitation amplitudes which are not capable to cause a jump out of the initial state, a rectangular HMEU does not reproduce the reflection of waves at interfaces with a linear elastic behavior different from that of the substrate, but behaves like a perfect bond.

Summary

The spectral contents of the response of a sinusoidally excited rectangular HMEU in a linear elastic material is derived analytically. The results are related to strains and restoring forces in thin bonded interfaces. A generalization to the case of many hysteretic interstices is planned, either analytically or by means of numerical simulations.

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