LAMB WAVE PROPAGATION IN ISOTROPIC MEDIA WITH ROUGH INTERFACE : COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

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Abstract
A small perturbation method is presented in order to study the propagation of Lamb waves in isotropic media with a roughly interface in vacuum, assuming that the thickness and its slope have small arbitrary variations. The theoretical results are compared to experimental results on rough glass plates.

Introduction

The goal of this paper is to analyze a plate rough on one side, the other side being considered as the reference side. As a matter of fact, for improving the wetting of the glue in a bonded structure, for instance composed by the gathering of two plates, a part of the structure is previously sanded. Guided waves are useful in order to inspect such bonded structure [1] and future works will concern the ultrasonic analysis of the influence of the roughness on the bonding [2,3]. Rough surfaces have been the subject of many studies involving the propagation of Rayleigh type waves (see e.g. [4]). For future applications guided waves, which are sensitive to internal interfaces, will be used whereas Rayleigh type wave propagates along the surface of the structure and therefore is not an efficient tool for investigating an internal interface.

In this paper, both theoretical and experimental studies are performed. The experimental part will mainly concern ultrasonic guided wave propagation (Lamb type wave) and optical measurement. Optical measurement will allow us to measure the classical parameters describing the roughness. The sensitivity of the ultrasonic guided waves is experimentally investigated and a theoretical modelisation is proposed.

Experimental set-up

The experimental set-up is reported in Fig. 1. A pulse generator delivers a very short pulse voltage (about 300 V during 300 ns) to an emitting piezo-composite transducer. The receiver transducer is an air-coupling piezo-electric transducer. The emitter transducer remains unmoved whereas the receiver is translated along the propagation direction of Lamb waves. The two transducers are set on the non-rough side. The displacement amplitudes are collected from x = 10 mm to x = 90 mm by 0.1 mm step (origin x = 0 corresponds to the wedge position). For each position of the air-coupling transducer, a 200 µs long signal is acquired on 10000 points. In order to improve the signal to noise ratio, we performed an average of 1000 successive shots.

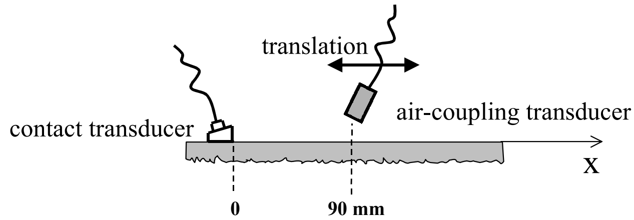

Figure 1 : Experimental set-up

The studied samples are glass plates with 200 mm sides and 5 mm thick. These plates are treated on one side only in order to obtain a rough surface except one that is considered as a reference plate. The other surface is roughless and is exactly the same for all samples. The plate surface topographies are obtained by means of an optical surface profiler. To evaluate the roughness amplitude, we use the usual statistic parameters R_a (roughness average) and R_q (root mean square roughness). Their mathematical expression is :

$$R_a = \frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}\left|Z_{ij}\right| \; ; \; R_q = \sqrt{\frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}Z_{ij}^2} \,, \quad (1)$$

where height deviations Z_{ij} are measured from the mean surface and M, N are the number of data points in each direction of the array. These parameters are given in Table 1 for each plate.

Table 1 : Roughness statistic parameters for the studied glass plates

sample	R_a (µm)	R_q (µm)
non treated	< 0.01	< 0.01
sanded	4.8	6.0
shot blasted	23.3	29.8
strong shot blasted	52.4	67.4

To evaluate the spatial wavelengths, we use the PSD (Power Spectrum Density). Generally speaking, a maximum corresponds to a spatial wavelength of the surface profile. It can be seen in Fig. 2 that there are several maxima, and thus, several possible spatial wavelengths Λ .

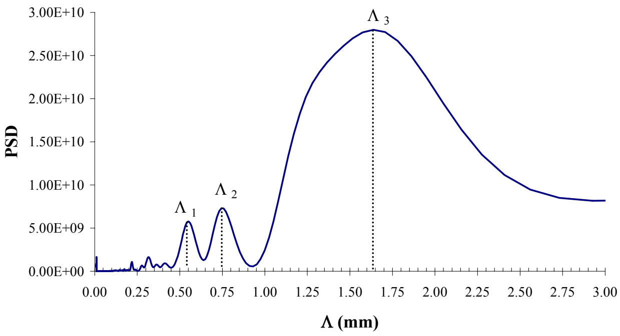

Figure 2 : PSD of a shot blasted plate

Let us note k_x the projection of the wave number vector on the x-axis, its real and imaginary parts being respectively noted k'_x and k''_x :

$$k_x = k'_x + i\, k''_x \quad . \tag{2}$$

Signal analysis allows us to extract Lamb waves characteristics for each Lamb mode : phase velocities (related to k'_x) and attenuation (related to k''_x). The experimental attenuation of a given Lamb mode can be qualified as "apparent" because it is related to different phenomena : the roughness, the coupling with the surrounding media and the diffraction of the bounded beam on the sample. As the theoretical model exposed in the next Section does not take into account neither the surrounding fluid nor the bounded nature of the ultrasonic beam, the theoretical attenuation for a given Lamb mode will be only linked to the roughness.

Modeling : perturbation method

A 2D model is developed for an isotropic plate with a rough interface in vacuum, characterized by its thickness d, and the velocities V_L and V_T of the longitudinal and transversal waves. The boundary surface $z = H(x)$ has a weak variation h(x) about the plane $z = -d/2$ (see Fig. 3). The slope dh/dx is also assumed to be small. A perturbation method is presented in order to express the dispersion equation of the rough plate as a sum of the dispersion equation of the plate with roughless surfaces and of a perturbation.

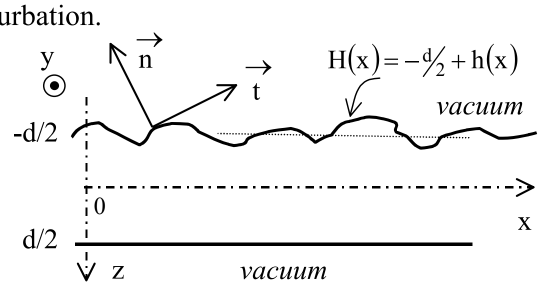

Figure 3 : Geometry of the problem

The dispersion equation is given by the writing of the boundary conditions. As the plate is in vacuum, the stress vector linked to the normal of the interfaces has to be zero, which can be written as following :

$$\tilde{\sigma}_{nt} = 0,\ \tilde{\sigma}_{nn} = 0 \quad \text{at } z = H(x), \tag{3}$$

and

$$\sigma_{xz} = 0,\ \sigma_{zz} = 0 \quad \text{at } z = d/2, \tag{4}$$

where $\vec{z}$, $\vec{x}$ are respectively the normal and tangential vectors to the lower interface and $\vec{n}$, $\vec{t}$ are respectively the normal and tangential vectors to the upper roughed interface.

$\tilde{\sigma}_{nt}$ and $\tilde{\sigma}_{nn}$ can be easily expressed as a function of σ_{xz}, σ_{zz} and σ_{xx}, by means of a change of basis. A first-order expansion of the normal stress vector permits to obtain the following equations :

$$\tilde{\sigma}_{nt} \approx \tilde{\sigma}_{xz} + \frac{dh}{dx}\left(\sigma_{zz} - \sigma_{xx}\right), \tag{5}$$

and

$$\tilde{\sigma}_{nn} \approx \sigma_{zz} - 2\frac{dh}{dx}\sigma_{xz} \quad . \tag{6}$$

Finally, $\tilde{\sigma}_{nt}$ and $\tilde{\sigma}_{nn}$ are expanded at a second-order expansion about $z = -d/2$.

The boundary conditions (3) and (4) lead to a 4-th order homogeneous system of equations of the form :

$$MX = 0 \quad . \tag{7}$$

M is a (4x4) matrix and X is a column vector containing the amplitudes of the longitudinal and transversal waves. M can be expressed as follows :

$$M \approx M_0 + \delta M = M_0\left(I + M_0^{-1}\delta M\right), \tag{8}$$

where M_0^{-1} is the inverse of M_0 and I is the identity matrix. The matrix M_0 corresponds to the homogeneous system of equations, written for roughless surfaces, and δM is given by :

$$\delta M = \frac{dh}{dx}M_1 + h(x)M_2 + \frac{h^2(x)}{2}M_3 \quad . \tag{9}$$

M_0, M_1, M_2 and M_3 are (4x4) matrices, which depend on V_L, V_T, d, k_x and ω (pulsation).

A first-order expansion of Eq. (8) permits the determinant of M to be expressed as follows :

$$\det M \approx \det M_0 + \mathrm{Tr}\left(\tilde{M}_0 \delta M\right), \tag{10}$$

where Tr (A) designates the trace of A and $\tilde{A} = \mathrm{Adj}(A)$. Eq. (10) can thus be written in the form :

$$F\left(k_x,\omega\right) \approx F_0\left(k_x,\omega\right) + \delta F\left(k_x,\omega\right), \tag{11}$$

which is the sum of the characteristic function for the plate with roughless surfaces and of a perturbation. Eq. (7) has non zero solutions if the determinant of M is equal to zero. Substituting Eq. (9) into Eq. (10) leads to the following dispersion equation :

$$\det M_0 + \frac{dh}{dx}\,\mathrm{Tr}\left(\tilde{M}_0 M_1\right) + h(x)\,\mathrm{Tr}\left(\tilde{M}_0 M_2\right) + \frac{h^2(x)}{2}\,\mathrm{Tr}\left(\tilde{M}_0 M_3\right) = 0 \quad , \tag{12}$$

which can be expressed of the following form, using Eq. (11) :

$$F_0(k_x,\omega) + \delta F(k_x,\omega) = 0 \ . \tag{13}$$

It can be noticed that the cancellation of the function $F_0(k_x,\omega) = \det M_0$ corresponds to the dispersion equation for Lamb modes in a plate with plane surfaces, which are given by the following Equation :

$$F_0(k_{x0},\omega_0) = 0 \ , \tag{14}$$

and Lamb modes in a plate with a rough surface are given by :

$$F(k_x,\omega) = 0 \ . \tag{15}$$

Thus, it can be assumed that, for a given pulsation ω, the solution k_x of Eq. (13) will be of the form :

$$k_x = k_{x0} + \delta k_x \ . \tag{16}$$

Modeling : numerical results

Experimental and numerical studies have been done on each rough sample, but subsequently, all the results are illustrated by Tables or Figures related to the shot blasted plate.

For a random profile H(x), the Lamb wave "sees" a kind of average parameters (<< >>) of the surface, involved in the dispersion equation (12) or (13) :

$$\alpha = << \frac{dh}{dx} >> ,\ \beta = << h(x) >> \ \text{and}\ \gamma = << \frac{h^2(x)}{2} >> .$$

These parameters depend on the mode and on the spatial wavelength Λ (given by the PSD) corresponding to the roughness profile. As a first approach, α and γ can be identified to $\alpha = 4R_a/\Lambda$ and $\gamma = R_q^2/2$.

An example of the dispersion curves corresponding to the solutions of Eq. (13) for a shot blasted glass plate is given in Figs. 4 and 5, using the following parameters : $d = 5$ mm, $V_L = 5825$ m/s, $V_T = 3485$ m/s, $R_a = 23.3$ µm, $R_q = 29.8$ µm, $\Lambda = \Lambda_1 = 0.546$ mm, $\beta = 0$.

The roughness has no effect on the phase velocities (related to k'_x). It has a influence only on the attenuation (related to k''_x).

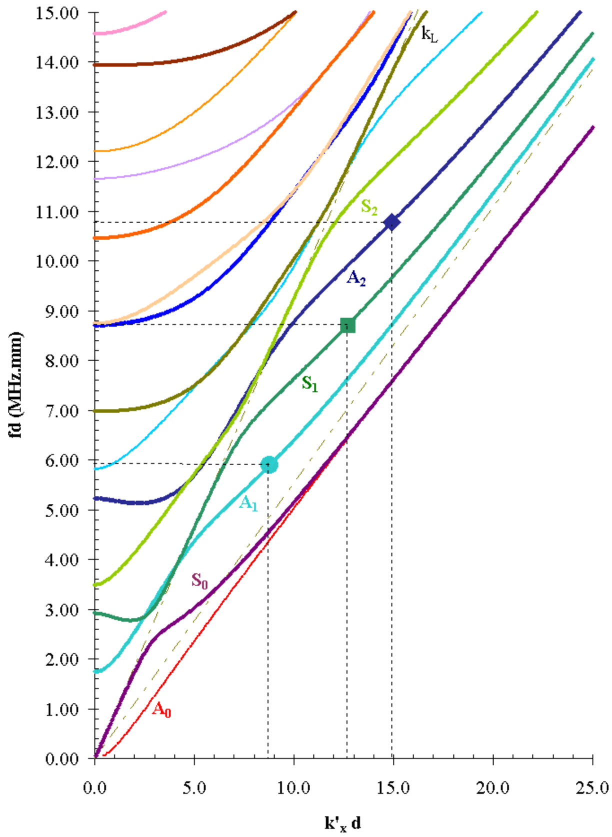

Figure 4 : Dispersion curves for Lamb modes in a glass plate, in the plane $(k'_x d, f d)$; rough shot blasted or roughless plate

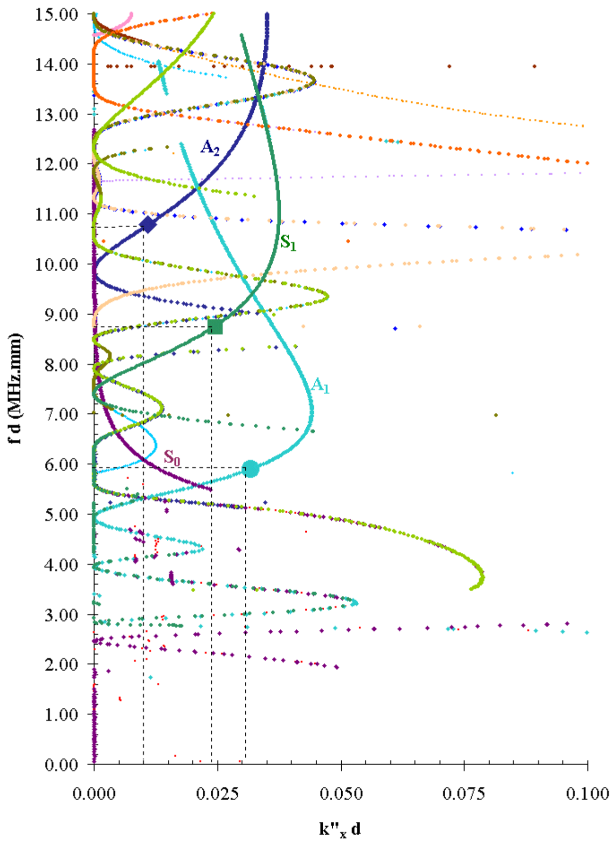

Figure 5 : Dispersion curves for Lamb modes in a roughed shot basted glass plate, in the plane $(k''_x d, f d)$

Experimental results

The phase velocities (related to k'_x) are found to be very close for rough and roughless plates [4]. The displacement amplitude of each Lamb mode as a function of x is experimentally obtained by means of a Short Spatial Fourier Transform (see Fig. 6 for the shot blasted plate). A fit with the logarithm of these experimental curves permits the "apparent" k''_{x_a}, linked to the attenuation, to be obtained. Assuming that, for a given Lamb mode, the effects of the surrounding fluid, of the bounded nature of the ultrasonic beam, and of the roughness are added the ones to the others, the attenuation linked to the roughness can be deduced through the following formula :

$$k''_{x_a} = k''_x + k''_{x_{ref}} , \qquad (19)$$

where $k''_{x_{ref}}$ corresponds to the attenuation of the Lamb mode for the roughless plate, which takes into account the first two effects. Table 2 presents all these results for the shot blasted plate. For all the plates, k''_x increases with R_q.

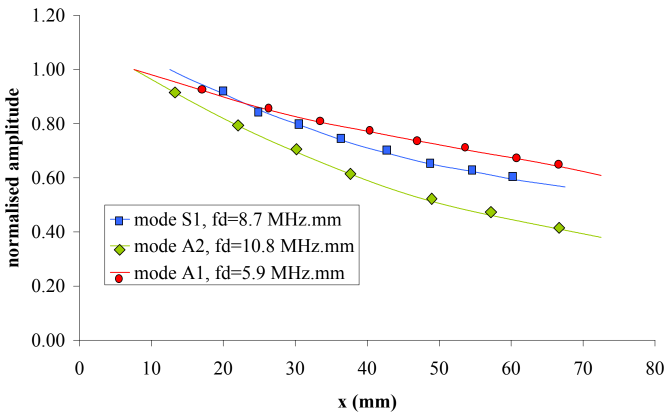

Figure 6 : Experimental normalized displacement amplitudes for Lamb modes in a roughed shot blasted glass plate.

Table 2 : Experimental (exp.) and theoretical (th.) characteristics of Lamb modes for the shot blasted plate. $\Lambda = \Lambda_1 = 0.546$ mm

Lamb mode	A1	S1	A2
f d (MHz.mm)	5.9	8.7	10.8
$k'_x d$ (th.)	8.8	14.7	14.9
$k''_{x_a} d$ (exp.)	0.0365	0.0525	0.0755
$k''_x d$ (exp.)	0.0075	0.025	0.0052
$k''_x d$ (th.)	0.032	0.024	0.011

Comparison between experimental and theoretical results - discussion

The results given in Table 2 show that experimental and theoretical k''_x perfectly match for the mode A_1, but not for the other modes. There are at least two major explanations :
- First, the model is a 2D-model, whereas a plate with a shot blasted roughness needs a 3D-model.

- Secondly, as above-mentioned, the PDS of Fig. 2 shows several maxima, which correspond to several possible spatial wavelengths Λ (involved in the parameter α). The results presented in Table 2 corresponds to $\Lambda = \Lambda_1 = 0.546$ mm, but it is likely that Λ depends on the Lamb mode. For instance, taking $\Lambda = \Lambda_3 = 1.64$ mm (see Fig. 2) leads to $k''_x d = 0.011$ for the mode A_2, which is close to the experimental value.

For all the plates, both experimental and theoretical phase velocities (related to k'_x) are found to be the same, for rough and roughless plates. In the same way, k''_x increases with R_q.

Prospects

The model has to be extended to a 3D-model. The influence of the spatial wavelengths, through the PDS, has to be taken into account, notably, the relation between the possible spatial wavelength Λ, the propagation of one Lamb mode and its wavelength.

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