DIFFRACTION OF LIGHT BY TWO SUPERPOSED ULTRASONIC BEAMS OF FREQUENCY RATIO 1:2

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Abstract

In previous papers it was shown both theoretically and experimentally that light diffracted by adjacent or superposed ultrasonic beams produce equal diffraction patterns provided that weak acousto-optical interaction (Raman-Nath regime) is considered. Beyond this regime, theory predicts totally different diffraction patterns for the situations mentioned. The light-sound interaction for adjacent ultrasonic beams was experimentally investigated in essential details whilst those for superposed beams are still lacking. In other words, the experimental verification of the theoretical predictions is still an unsolved problem.

In the present paper, light diffraction by two superposed ultrasonic waves of the frequency ratio 1:2 is experimentally investigated in the intermediate range between weak and strong acousto-optical interaction. The theoretical predictions established by Mertens are confirmed. Besides the broadening of the basic knowledge, the findings are of particular interest as regards the investigation of nonlinear sound propagation by means of acousto-optical techniques.

Introduction

For many years the diffraction of light by two parallel superposed or adjacent ultrasonic waves has been the subject of intense research work, both theoretical and experimental. In the 1950s and at the beginning of the 1960s the work was mainly confined to the range of weak acousto-optical interaction, i.e. to the Raman-Nath regime [1-4]. The light-sound interaction in this regime is now well understood and it has been verified that the diffraction patterns are equal for both sets of experimental conditions. Later on, in the 1970s, the theoretical research work was extended beyond the Raman-Nath regime [5-8] and essential differences of the diffraction patterns for superposed and adjacent ultrasonic beams were predicted. Only in 2000 [9] were the theoretical findings for adjacent ultrasonic beams verified experimentally. However, the respective confirmation for superposed ultrasonic beams is yet to come.

In the present paper, light diffraction by two superposed ultrasonic waves of the frequency ratio 1:2 is experimentally investigated as a function of the phase shift δ between the sound beams in the range beyond the Raman-Nath regime. The aim is twofold, 1) to examine the theoretical predictions established by Mertens [5] and 2) to gain the basic tools for the investigation of nonlinear sound propagation by means of acousto-optical techniques.

Theory

In his paper [5], Mertens has shown that the method of the *N*th order approximation (*N*OA method) leads to more acceptable results beyond the Raman-Nath regime than does that of successive approximations (SA method). This is why the present paper is confined to the *N*OA method. It considers the approximation N=2 which means that the light intensities in the diffraction spectrum for orders N>2 are negligibly small. The following parameters will be used: Klein-Cook parameter

$$Q_{1,2} = 2\pi\lambda L / \mu_0 \Lambda_{1,2}^2 \quad (1)$$

and Raman-Nath parameter

$$v_{1,2} = 2\pi\mu_{1,2} L / \lambda , \quad (2)$$

where the indices point to the first and the second ultrasonic beam. $\Lambda_{1,2}$ and λ are the wavelengths of the ultrasonic waves and the light wave respectively. L is the sound field depth which is the same for the two sound beams. μ_0 is the refractive index of the undisturbed medium, and $\mu_{1,2}$ are the maximum variations of the refractive indices caused by the sound beams. For the frequency ratio of $f_1 : f_2$=1:2, the Klein-Cook parameter of the second beam is $Q_2 = 4Q_1$.

Fig. 1 shows the normalized light intensity as a function of the Raman-Nath parameter. In contrast to Fig. 15 in [5], parameter Q has been kept constant and thus yields a representation which is more suitable

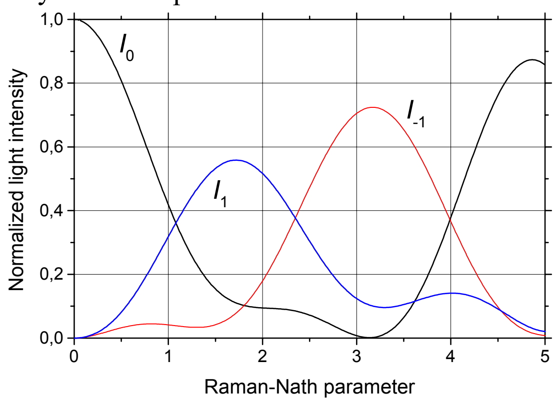

Fig. 1: Normalized light intensity of two superposed ultrasonic beams vs. Raman-Nath parameter, v_1, for $v_1 / v_2 = 1$, $f_1 : f_2$=1:2, $Q_1 = 1$ and $\delta = 0$ according to [5].

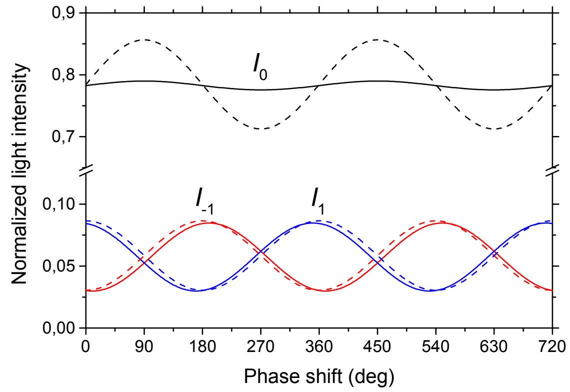

Fig. 2: Normalized light intensity of two superposed (solid lines) and adjacent (dashed lines) ultrasonic waves of frequency ratio $f_1 : f_2 = 1{:}2$ vs. the phase shift δ between the sound beams. $Q_1 = 0.5$; $v_1 = v_2 = 0.5$.

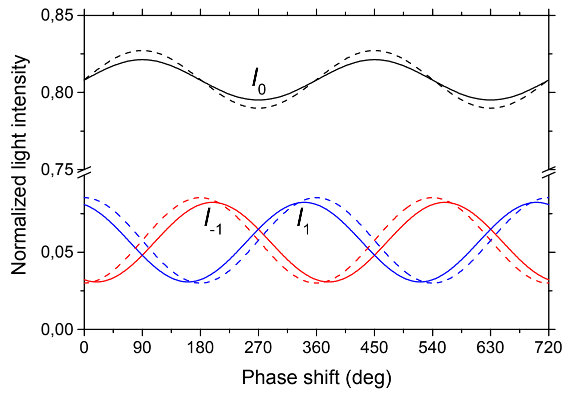

Fig. 3: Normalized light intensity of two superposed (solid lines) and adjacent (dashed lines) ultrasonic waves of frequency ratio $f_1 : f_2 = 1{:}2$ vs. the phase shift δ between the sound beams. $Q_1 = 1.0$; $v_1 = v_2 = 0.5$.

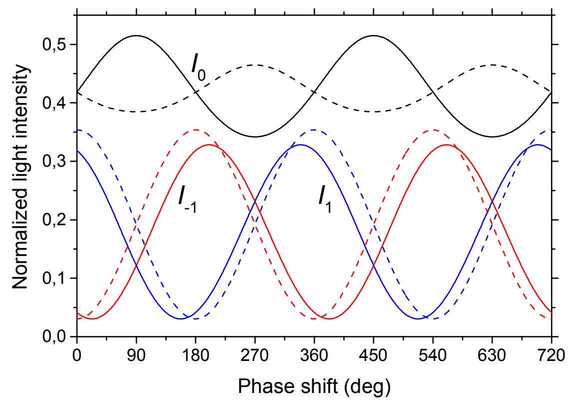

Fig. 4: Normalized light intensity of two superposed (solid lines) and adjacent (dashed lines) ultrasonic waves of frequency ratio $f_1 : f_2 = 1{:}2$ vs. the phase shift δ between the sound beams. $Q_1 = 1.0$; $v_1 = v_2 = 1.0$.

from the experimental point of view. For the calculations, an in-phase condition ($\delta = 0$) of the two superposed beams was assumed. In the experimental part this aspect, i.e. the phase shift dependence of the light intensities, will be treated in detail.

For completeness, Figs. 2 to 4 compare the theoretical predictions for the normalized light intensities, I_0, I_{+1}, I_{-1}, versus the phase shift, δ, between the ultrasonic beams for both superposed and adjacent beams and for different values of the Klein-Cook parameter, $Q_{1,2}$, and the Raman-Nath parameter, $v_{1,2}$. The numerical calculations for adjacent beams are carried out in accordance with the theoretical model established by Leroy [7]. The figures clearly reveal the essential differences in the diffraction behaviour for both experimental conditions beyond the Raman-Nath regime.

In the $Q - v$ range considered, the intensity modulation of the 0th diffraction order increases steadily with increasing $Q_{1,2}$ and $v_{1,2}$ in the case of superposed waves, whereas for adjacent waves the intensity modulation first decreases and then increases again with opposite phase. The particular behaviour for adjacent beams can be understood from the existence of the so-called additional optical phase shift [10] caused by the first ultrasonic beam. In both cases, the mean values of the light intensities are, however, equal.

Considering the ±1st diffraction order, the light intensity for adjacent beams follows the simple formula [7, eq. (31)]

$$I_{\pm 1} = C_1 (1 \mp C_2 \cos\delta) \qquad (3)$$

where C_1 and C_2 are functions of both, $Q_{1,2}$ and $v_{1,2}$. Consequently, the intensity shapes intersect at the same intensity levels. On the other hand, Mertens shows in his paper [5, eqs. 26 and 32b-c] that for superposed beams the light intensity is described by a combination of $\cos\delta$ and $\sin\delta$ terms. Thus, the intensity shapes alternately intersect at higher and lower levels, whereby the value depends on $Q_{1,2}$ and $v_{1,2}$.

Experimental investigations and results

In order to experimentally realize two superposed ultrasonic waves with adjustable phase shift, an arrangement is used as outlined in Fig. 5. Two circular lithium niobate ($LiNbO_3$) transducers (Tr1, Tr2) 25 mm in diameter are mounted in parallel and co-axially with a spacing of $l \approx 30$ mm. They are excited at their fundamental frequency or higher harmonics making sure that Tr2 operates at twice the frequency of Tr1. The sound wave radiated by Tr2 is superposed with that of Tr1 after passing through Tr1 (see Fig. 5). The thickness of Tr1 allows for maximum sound transmission. For the adjustment precise mechanical positioning systems are used which allowed the transducers to be independently adjusted. It is estimated that the uncertainty of the parallelism between the transducers is approximately 0.5 mrad. The transducer unit is mounted in the water tank and the angle of normal incidence (perpendicular to the direction of sound propagation) is adjusted with an accuracy better than 0.1 mrad. To avoid multiple reflections between the transducers and thus distortion of the superposed beams, the transducers are excited by tone bursts of $\tau < c/2l$ duration, where c stands for the sound velocity in the sound-transmitting medium (distilled water).

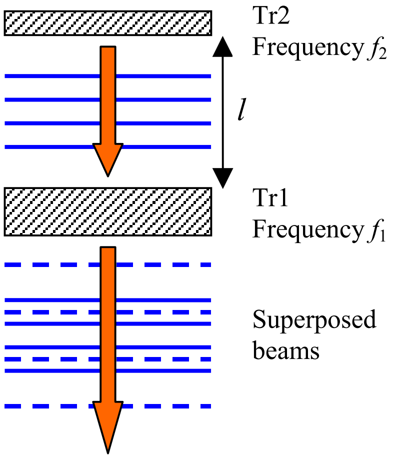

Fig. 5: Sketch of transducer arrangement.

Fig. 6 shows the complete experimental arrangement. Transducer Tr1 is excited via the power amplifier (9) by a f_1-frequency signal generated by the high-stability rf generator (8) and gated by the function generator (7). Transducer Tr2 is excited by (8) via the digital delay generator (10), the frequency doubler (11) and the power amplifier (12). The generator (8) also delivers a trigger for the light detection path (see below).

As light source a cw 4 mW semiconductor laser (1) ($\lambda = 397$ nm) is used. The laser beam is expanded to 95 mm in diameter and its angular divergence is of the order of 8×10^{-6} rad. By means of the spatial filter (3) positioned in the focal plane of the lens (2) the individual diffraction orders are examined. The light intensity is recorded by means of the photomultiplier (4) whose electric output signal is amplified using the broad-band preamplifier (5) and then measured using the multi-channel photon counter (6). Due to the pulse excitation of the transducers, the photon counter (6) is suitably synchronized with the function generator (7).

To precisely adjust for normal incidence of the light, the symmetry of the light intensities is controlled in the diffraction orders ±1 for the individual ultrasonic beams, i.e. when either Tr1 or Tr2 is working. In the following step, the sound field depth L, the Klein-Cook parameters $Q_{1,2}$ and the Raman-Nath parameters $v_{1,2}$ are determined for the individual ultrasonic beams. The detailed procedure is described elsewhere [9,11]. The estimated expanded uncertainty (95% confidence level) of the measured Klein-Cook parameter and the Raman-Nath parameter is ±5%.

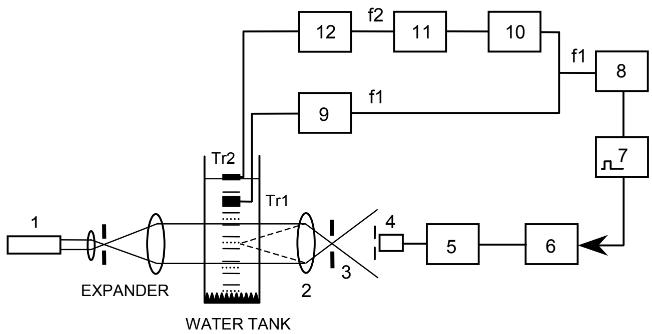

Fig. 6: Experimental arrangement
(1) laser, (2) lens, (3) spatial filter, (4) photomultiplier, (5) preamplifier, (6) photon counter, (7) function generator, (8) rf generator, (9) power amplifier, (10) delay generator, (11) frequency doubler, (12) power amplifier.

The experimental results together with the theoretical predictions according to Mertens [5] are plotted in Figs. 7 and 8. It can be seen that the theoretical model and the experimental findings are in acceptable agreement. Perfect agreement actually cannot be expected for several reasons:

- The theoretical model supposes plane, parallel superposed ultrasonic waves. This ideal condition cannot be realized in the experimental practice. The experiments are rather performed in the Fresnel zone (acoustical near field), which can be considered a reasonable approach.

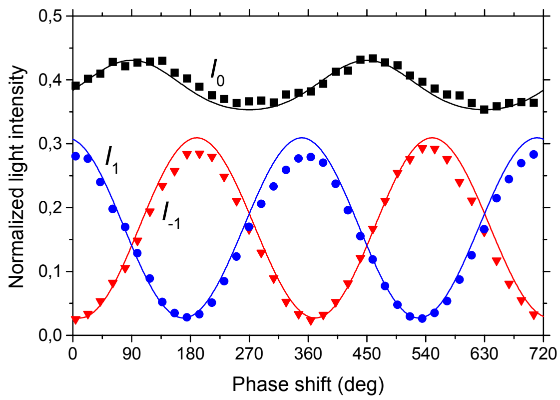

Fig. 7: Normalized light intensity of two superposed ultrasonic waves of frequency ratio $f_1 : f_2 = 1:2$ vs. the phase shift δ between the sound beams. $Q_1 = 0.46$; $v_1 = v_2 = 0.95$; $f_1 = 5.272$ MHz. Theory: solid lines; experiment: symbols.

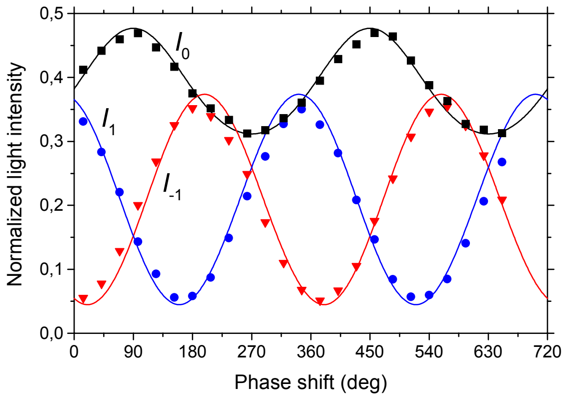

Fig. 8: Normalized light intensity of two superposed ultrasonic waves of frequency ratio $f_1 : f_2 = 1{:}2$ vs. the phase shift δ between the sound beams. $Q_1 = 0.9$; $v_1 = 1.1$; $v_2 = 0.9$; $f_1 = 7.402$ MHz. Theory: solid lines; experiment: symbols.

- The theoretical model proceeds from equal interaction lengths, $L = L_1 = L_2$, for both sound field components, which leads to $Q_2 = 4Q_1$. This condition cannot be fulfilled experimentally in the strict sense either. It turned out that the interaction length of the ultrasonic field radiated by transducer Tr1 (see Fig. 5) is slightly greater than that of Tr2.
- It also turned out that the adjustment of the two transducers shown in Fig. 5 is very critical because of the rather high frequencies used in the experiments.

Discussion

The paper deals with the experimental investigation of the diffraction of light by two parallel superposed ultrasonic waves of the frequency ratio 1:2. Whilst this phenomenon is well understood in the range of weak acousto-optical interaction, the theoretical model established by Mertens in 1962 [5] for the intermediate range between weak and strong acousto-optical interaction had to be verified experimentally. This particular task has been the subject of the present investigation. Rather good agreement could be found in spite of some deficiencies of the experimental arrangement compared with the idealized assumptions of the theoretical model. In other words, the validity of Mertens' predictions could be confirmed.

Besides the broadening of the basic knowledge of ultrasonically induced light-diffraction phenomena, the present findings are of particular interest as regards the investigation of nonlinear sound propagation by means of acousto-optical techniques.

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