REFLECTION OF TRANSVERSAL ULTRASOUND FROM A VISCOUS INTERFACE BETWEEN TWO SOLIDS

Guoxuan. Lian and Mingxuan. Li
Institute of Acoustics, Chinese Academy of Sciences, China
Ultra302cn@yahoo.com.cn

Abstract: Reflection of transversal ultrasound incident normally at a slip interface between solids is investigated. The exact formula is derived from complex acoustic viscosity impedance of Newtonian liquid while an approximate method is developed by interfacial stiffness of liquid interlayer’s viscosity. Numerical computation and experimental results show that, for a thin liquid layer with light viscosity, transversal wave reflection amplitude is suitable to evaluate the interlayer properties but for thick or high viscous interlayer, reflection phase has advantages to be used. The validity of value of the approximated method is also been demonstrated.

1. Introduction

Transversal wave has great advantages in liquid viscosity measurement\(^1\)\(^2\), adhesive bonds evaluation\(^3\)\(^4\) and composite monitoring\(^5\)\(^6\). In these works, transversal wave reflection and transmission are investigated extensively. For adhesive bonds, a weak joint named slip interface is also noticed by some authors\(^7\)\(^8\). It can be treated as a liquid interlayer filling in the gap between two solids. When the layer is an ideal liquid, the interface is lubricated and transversal wave will be reflected completely. If the liquid is viscous, there are two possible mechanisms relying on transversal wave module and viscosity of the liquid to transfer transversal stress through the interface. Greenwood\(^2\) demonstrates that viscosity of light viscous liquid determines the reflection of transversal wave at a solid-liquid interface while transversal wave module does that for highly viscous liquid. Rockhlin\(^7\) studied ultrasound reflection when the interlayer transits from liquid to solid by Maxwell model. We give a research on transversal wave reflection at a viscous liquid interface by using its complex acoustic impedance.

Because of the small thickness of interlayer, it is difficult to measure a tiny difference of attenuation or time-of-flight of the layer. Therefore, this paper’s topic focuses only on wave’s reflection.

Transversal wave reflection from a viscous liquid interlayer between two solids is different comparing with longitudinal wave’s action or reflection of transversal wave incident on a solid-liquid interface. For example, longitudinal wave reflection amplitude in frequency domain always increases monotonously as far as to the interlayer’s first resonance frequency, but transversal wave exhibits various tendencies including nearly frequency-independent characteristic. Advanced research is carried out theoretically and experimentally here, which provides some valuable clues to practical workers.

2. Theoretical analysis

![Figure 1](image-url)

As shown in Figure 1, a thin Newtonian liquid interlayer with light viscosity locates between two same solids. Transversal wave in the upper is incident normally at the thin viscous liquid layer. Some energy is rebounded and some penetrates into the lower solid. For such a model, an input impedance method is proposed to study the reflection:

\[
R = \frac{Z_s - Z_{in}}{Z_s + Z_{in}} \tag{1}
\]
where \( R \) is reflection coefficient, \( Z_s = \rho_s \nu_s \) and \( Z_{in} \) are the transversal wave impedance of solids and input impedance at liquid layer’s upper surface. \( \rho_s \), \( \nu_s \) denote density and transversal wave velocity in solids respectively. \( Z_{in} \) can be obtained by using continuous conditions of stress and displacement at two surfaces between liquid and solids:

\[
Z_{in} = \frac{Z_s + jZ_0 \tan(k_0 h)}{Z_0 + jZ_s \tan(k_0 h)} Z_0
\]

(2)

where \( Z_0 = \sqrt{j\omega \eta \rho_0} \) complex represents the transversal wave impedance for light viscous liquid ever given by Greenwood\(^2\). \( \eta \), \( \rho_0 \) are the viscosity factor and density of the liquid individually. \( k_0 = \frac{\omega}{Z_0 / \rho_0} = \sqrt{\frac{\rho_0 \omega}{j \eta}} \) is transversal wave number in the liquid. \( \omega \) and \( h \) denotes angular frequency and the liquid layer thickness.

Substituting \( Z_{in} \) into equation (1), the reflection results can be computed. When \( k_0 h \ll 1 \), \( \tan(k_0 h) \approx k_0 h \), replacing it into \( R \) and neglecting liquid’s mass (let \( \rho h = 0 \)), \( R \) can be expressed in a simple form:

\[
R = \left[ 1 + \frac{2\eta}{Z_s h} \right]^{-1}
\]

(3)

Equation (3) means the reflection is independent on frequency but only a function of liquid layer thickness, viscosity and solid’s impedance. This result can also be got by a spring boundary condition. For a thin viscous liquid layer, the interfacial stiffness is\(^9\)

\[
K = \frac{j \omega \eta}{h}
\]

(4)

Substituting it into the reflection formula by Tattersall\(^{10}\) or Baik\(^{11}\).

\[
R = \frac{j \omega Z_s / K}{2 + j \omega Z_s / K}
\]

(5)

equation (3) is obtained directly. The latter approximate method provides more insights in physics than equation (2). Similar equations for longitudinal wave can be got in terms of longitudinal wave impedance and normal interfacial stiffness.

3. Numerical and experimental results

Numerical computation and experiments use two block of polystyrene as the solids which have longitudinal and transversal wave velocities of 2350 m/s and 1120 m/s, density of 1060 kg/m\(^3\). The liquid’s longitudinal wave velocity is 1500 m/s and density 1000 kg/m\(^3\). For numerical analysis, viscosity factor \( \eta \) and thickness \( h \) are changing parameters as shown in figures and Table 1.
Table 1

<table>
<thead>
<tr>
<th>Curve</th>
<th>S1</th>
<th>S2</th>
<th>D1</th>
<th>D2</th>
<th>P1</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$(um)</td>
<td>1.5</td>
<td>150</td>
<td>1.5</td>
<td>150</td>
<td>1.5</td>
<td>150</td>
</tr>
<tr>
<td>$\eta$(Pa.s)</td>
<td>0.1</td>
<td>10</td>
<td>0.5</td>
<td>50</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>$\eta/h$ (Pa.s/um)</td>
<td>1/15</td>
<td>1/15</td>
<td>1/3</td>
<td>1/3</td>
<td>4/3</td>
<td>4/3</td>
</tr>
</tbody>
</table>

A transversal wave transducer made of 2/2 type piezo-composite is used in Fig.1. The received signals have been deconvoluted by reflected signal from a solid-air interface. Experimental results are shown in Fig.4.

Numerical results are depicted in Figure 2 and Fig.3. Table 1 lists different pairs of $\eta$ and $h$ used by curves in Fig.3. Capital letters S, D and P denote solid, dashed and dot line individually. It can be seen from Fig.2 that longitudinal wave reflection has resonance peaks in Fig.2a and sudden phase rise in Fig.2b, but for transversal wave reflection none of them exists as shown in Fig.3. At the same time, the approximated method shows a poor accuracy for longitudinal wave in Fig.2a and Fig.2b.

In Fig.3a, every pairs of curves in same type (S, D or P) have a common start point. The reason is that they have equal value of $\eta/h$ (Refer to Table 1).

Because these points stand near zero frequency, equation (3), the approximated method, can work well and predict those points correctly. Meanwhile, for very thin liquid layer ($h=1.5$um, three “1” curves), reflection amplitudes remain almost constants, which are coincident with Equation (3) again. Besides, a smaller value of $\eta/h$ is related to a bigger starting point. This can be explained physically that a small $\eta/h$ produces a small interfacial stiffness $K$ in equation (4), which then results in a strong reflection in equation (5). Generally, every curve in Fig.3a goes down at first until to a minimum, and then runs up with frequency increase. The higher viscosity of liquid, the faster changing of the amplitude (P1>D1>S1) and
vice versa. P1’s minimum is near zero so its minimum is invisible in Fig.3a. In the middle frequency band, all curves interweave when one is stepping up and another is going down such as at point A. At infinite frequency, it can be imaged that all amplitude value will be unit in Fig.3a and the phase in Fig.3b be $-\pi$, just as the incident wave meets an ideal hard interface.

In Fig.3b, the curves never meet each other but the start point; so much clear information on liquid’s thickness or viscosity can be obtained by it. Although, all “1” curves are so closed for their small $\eta$s and tiny thickness, which leads insensitive to measure them. Nevertheless, for higher viscous liquid shown as “2” curves in Fig.3b, phase technique has vivid advantage comparing to interweaving amplitude.

Fig.4 shows experimental reflection results from a water layer and two honey layers. According to the water layer response and transducer’s spectrum, the valid frequency window between 0.5-1.5MHz is got. I can be seen that light viscous honey’s reflection amplitude has weak dependence on frequency. Therefore, $\eta/h$ can be fitted by equations (4) and (5) simply and directly. In Fig. 4(b), the reflection phase curves of honey are close as predicted in Fig.3b.

4. Conclusion
(1) Transversal wave reflection amplitude is more sensitive to recognize light viscosity of liquid than the phase. The proposed approximated method is enough to do it.
(2) Comparing to longitudinal wave, transversal wave reflection has no resonance peaks and corresponding phase sudden changes.
(3) For thicker or higher viscous liquid, transversal wave reflection amplitude curves interweave each other while those of phase never meet. They decrease monotonously and could provide clear information on liquid’s thickness and viscosity.

Reference