## SIMULATION OF DISPERSION CHARACTERISTICS OF GUIDED-WAVE IN COMPOSITE PIPES BASED ON THE INNER-RADIUS-THICKNESS RATIO

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#### Abstract

The paper describes an investigation of the longitudinal guided waves propagation characteristics in composite pipes. The effects of the Inner-Radius-Thickness Ratio (IRTR) on the propagation characteristics of longitudinal guided waves in composite pipes have been analyzed by numerical simulation. For a particular material and frequency-thickness products (fd), the propagation characteristics of longitudinal guided waves are only related to IRTR. If only IRTR were fixed, the dispersion characteristics of the cylindrical guided waves don't change with the inner-radius and the thickness separately. While composite pipes of different IRTR were tested, following test conditions should be selected carefully, such as mode, fd and cycle numbers of the input signal. For detection of various distances, different modes, the cycle numbers and fd also need to be carefully chosen.

## Introduction

Because of its unique advantages, metal-plastic composite pipe has been widely used at industrial and household scales. Therefore, the detection of this kind composite structure has become a major problem in recent years. Ultrasonic guided wave is one of methods utilized for detection the composite structure. However, the major problem of guided waves nondestructive evaluation (NDE) techniques for composite pipes lies in the understanding of guided wave propagation in composite cylindrical structures [1-4].

A large amount of work has been contributed to the understanding of cylindrical wave propagation in elastic hollow cylinders [5-7]. In above analysis, when the complex dispersion curves were computed, they were assumed to be of a real wavenumber and a complex frequency.

This paper describes an investigation of the longitudinal guided waves propagation characteristics in composite pipes. When dispersion characteristics were computed, we modified the choice of the Bessel functions and assumed a real frequency and a complex wavenumber. Then the effects of the Inner-Radius-Thickness Ratio (IRTR) on the propagation characteristics of longitudinal guided waves in composite pipes were analysed by numerical simulation.

### **Dispersion equations**

The model investigated in the paper assumes that the composite pipe is axi-symmetric and infinitely long. The material properties may only vary in the radial direction and all of the variations occur as instantaneous changes at the boundaries of discrete layers. The boundaries between these layers are assumed to be perfect, the waves are assumed to be continuous, frequency is real and energy is finite. Based on the general motion equations, the expressions of displacements and equation, and the boundary conditions, producing a set of characteristic equations, and come into being a form of matrix with the unknowns amplitude  $A_m$ ,  $B_m$ ,  $C_m$  and  $D_m$  as follows[8]:

$$[M_{ij}] \cdot [N] = 0, \quad i, j = 1, 2, ..., 4m$$
(1)

where  $N = \begin{bmatrix} A_m & B_m & C_m & D_m \end{bmatrix}^T$ ,  $M_{ij}$  is a coefficient matrix. For an un-trivial solution of these simultaneous equations, the determinant of the coefficient matrix should vanish:

$$\left|\boldsymbol{M}_{ij}\right| = 0 \tag{2}$$

This equation is the characteristic dispersion equation of guided waves in composite pipes.

Here we consider the composite pipe as a two layers system, m=2. From equation (2), numerical results have been obtained for a polyethylene-brass composite pipe. Fig.1 shows the phase and group velocity dispersion curves over the fd ranging 0~2.0 MHz-mm for a composite pipe with 20 mm inner diameter, while either of the inner layer (polyethylene) thickness and outer layer thickness is 1 mm, where the density of brass is  $r = 8.56g/cm^3$ , longitudinal and wave velocity transverse of brass are  $c_1 = 4.28m/ms$  and  $c_t = 2.03m/ms$  respectively. The density of polyethylene is  $\mathbf{r}_0 = 2.21g/cm^3$  and longitudinal wave velocity is  $c_1 = 1.36m/ms$ , shear wave velocity is  $c_t = 0.54 m/ms$ . The modes only include a family of axially symmetric longitudinal guided waves.

From Fig.1, it can be seen that among these axisymmetric longitudinal guided wave modes, only the lowest-order mode, L(0,1), extends down to zero frequency. All other modes, such as L(0,2) and L(0,3), exhibit finite cut-off frequencies. At the near of such cut-off frequencies, the group velocities of these

modes are vanishingly small and they represent nonpropagating axial modes. Since this composite system represents a structure in vacuum, there are no eigenmode solutions with complex frequencies. The implication is that there are no leaky modes for this configuration.



Fig.1. Velocity dispersion curves of longitudinal guided waves in composite pipes (20 mm inner diameter with inner layer thickness of 1 mm and outer layer thickness of 1 mm).

The group velocity of L(0,2) mode reaches maximum at fd with 0.13 MHz.mm, while the group velocity of L(0,1) mode is comparatively lower than it, and L(0,3) et al. higher modes have not appeared yet. At 0.24MHz.mm, the group velocity of L(0,3) mode has a maximum, while L(0,1) and L(0,2) modes have lower values, and higher modes have not been formed. At about 0.36MHz.mm, the L(0,4) mode has a maximum of group velocity, while the group velocities of other lower modes are comparatively lower, and higher modes, such as L(0,5) and L(0,6), have not been appeared yet, and so on and so forth. The flatness range in phase velocity curves of each mode corresponds to the vicinity of maximum of group velocity of their modes respectively. That is to say, the smaller the dispersion is, the larger the group velocity will be.

#### Effects of IRTR on guided waves

The case of same IRTR

From equation (2), numerical results have been obtained for a polyethylene–brass composite pipe. Fig.2 is phase velocity dispersion curves of longitudinal guided waves in composite pipes with different radius and thickness. Where Fig.2(a) is the case of IRTR(r/d) with 2 (r/d=2/1, 4/2, 6/3, 8/4), and Fig.2(b) is the case of IRTR with 8 (r/d=8/1,16/2, 24/3,32/4).





It can be seen from Fig.2 that complete superposition of phase velocity of guided waves in composite pipes with different inner-radius and thickness (the ratio between inner layer and outer layer is fixed), but the same IRTR. Thus, for specific material and fd, the propagation characteristics of guided waves in composite pipes only relate to the Inner-Radius-Thickness Ratio (IRTR). If only IRTR were fixed, the dispersion characteristics of the cylindrical guided waves don't change with innerradius and thickness changes separately, so only IRTR was considered in following discussions.

In addition, an interesting phenomenon depicted is that the phase velocities of L(0,1) and L(0,2) modes be equal to each other at 0.5 MHz.mm and regardless the value of IRTR.

# *Relationship between IRTR and wave-packet amplitude*

The effects of decreasing wave-packet amplitude increasing duration and due to dispersion are both undesirable in long range guided wave testing[9]. First, the spreading of a wave-packet in space and time reduces the resolution that can be obtained. Second, the reduction in amplitude of a dispersive wavepacket reduces the sensitivity of the testing system.

In this work, a method [9] was used for predicting the spreading of a dispersive packet of guided waves as it propagates through composite pipes. The amplitude decrease can be estimated by energy conservation. Neglecting other losses, it can be assumed to a first approximation that the amplitude of a wave-packet will decrease in proportion to the square root of the duration increase:

$$\Delta A = C \cdot \sqrt{T_d} \tag{3}$$

where C is constant.

Fig.3 shows relationship of the wave packet amplitude of L(0,1) and L(0,2) modes vs. IRTR of composite pipes for propagation distance with 800mm and various fd (unit: MHz.mm). In all cases, the input signal is assumed to be a Hanning windowed toneburst and the number of cycles is 5. The maximum of each curve reveals optimum pipe size under fixed fd and cycle number with 5.



Fig.3 Relationship of IRTR and wave envelope amplitude for various fd (unit: MHz.mm).

It can be seen from Fig.3(a) that with IRTR increasing, the wave-packet amplitude of L(0,1) mode decreases at lower fd (e.g., 0.1MHz.mm), and then increases corresponding with IRTR. The IRTR corresponding to the minimum amplitude of each curve will decrease with fd increasing. At higher fd(e.g., 0.4MHz.mm), the wave-packet amplitude of L(0,1) mode will increase with increase of IRTR. On the other hand, the effects of IRTR on amplitude will decrease along with increase of fd; also, the effects of IRTR on amplitude decrease at higher IRTR values. Since the increase in amplitude of a dispersive wavepacket shows enhanced the sensitivity of the testing system. Therefore, using L(0,1) mode to testing defects in composite pipes of different IRTR, when IRTR is low, lower fd ought to be selected, and higher fd was chosen when IRTR is high.

From Fig.3(b), it can be seen that the wave-packet amplitude of L(0,2) mode will increase with increase of IRTR at lower fd (e.g., lower than 0.15MHz.mm). While fd is between 0.15MHz.mm and 0.4MHz.mm, with IRTR increasing, the amplitude increases at first, and then it will decrease. While fd is above 0.4MHz.mm, the amplitude will increase with increase of IRTR. The effects of IRTR on wave packet amplitude will be reducing for various fd at above large IRTR; otherwise, the effects will be increasing. Therefore, L(0,2) mode is utilized for defect detections in composite pipes of different IRTR, the chosen fd should be decreased with the increase of IRTR.

Fig.4 shows a graph of the wave packet amplitude of L(0,1) and L(0,2) modes vs. IRTR of composite pipes for propagation distance with 800mm and various cycle numbers in the input pulse. In all cases, the *fd* is assumed to be 0.50MHz.mm.

Fig.4(a) reveals that with IRTR increasing, the wave-packet amplitude of L(0,1) mode increases while the input pulse is with lower cycle numbers (e.g., 4), the ratio of increase reduces with futher IRTR increase. However, the wave-packet amplitude will decrease with increase of IRTR when the cycle numbers higher than 15. On the other hand, the effects of IRTR on amplitude will be lessened at higher values of IRTR (e.g., higher than 5), for various cycle numbers. Therefore, using L(0,1) mode to test defects in composite pipes of different IRTR, when IRTR is low, input pulse with higher cycle numbers ought be chosen. In this case, the dispersive wave-packet has enhanced amplitude. When IRTR is high, input pulse with lower cycle numbers should be chosen.

As for L(0,2) mode, at lower cycle numbers, the wave-packet amplitude will increase with the increase of IRTR. While at higher cycle numbers, the amplitude will firstly decrease and then increase with the increase of IRTR. On the other hand, as the number of cycles in the input signal is increased, the

wave-packet amplitude increases for various IRTR. So, if using L(0,2) mode to testing defects in composite pipes of different IRTR at 0.5MHz.mm, higher cycle numbers in input pulse should be selected. What need to be accentuated is that all the results here presented relate purely to the propagation of guided waves and having not taken any account of how a mode interacts with a particular feature or defect.



Fig.4 Relationship of IRTR and wave envelope amplitude for various cycle numbers in the input pulse.

## Conclusions

The paper describes an investigation of the longitudinal guided waves propagation characteristics in composite pipes. When computing the dispersion characteristics, we assumed a real frequency and complex wavenumber. Then the effects of IRTR on the propagation characteristics of longitudinal guided waves in composite pipes were analyzed by numerical simulation.

The results show that for specific material and fd, the propagation characteristics of the guided waves in composite pipes only related to IRTR. If only the IRTR were fixed, the dispersion characteristics of the cylindrical guided waves don't change with innerradius or thickness changes separately.

If using L(0,1) mode for defect detections in composite pipes of different IRTR, when IRTR is low, lower fd and higher number of cycles in the input signal ought to be chosen; when IRTR is high, higher

fd and lower cycle numbers ought to be chosen. When L(0,2) mode is utilized to test the composite pipes of different IRTR, the wave-packet amplitude will increase tardily with increase of IRTR at lower cycle numbers and higher fd.

The effects of IRTR on the propagation characteristics of longitudinal guided waves in composite pipes are notable while with lower IRTR and decreasing while with higher IRTR.

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