

## MEASURING METHOD FOR THE MECHANICAL ANISOTROPY OF SOLIDS BY WATER IMMERSION ULTRASONIC SING-AROUND METHOD

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### Abstract

Mechanical anisotropy of solids is derived with the ultrasonic velocity measurement. By applying the water immersion ultrasonic sing-around method, not only the ultrasonic dilatational wave velocity in the solids and the ultrasonic shear wave velocity in them, but also the ultrasonic pseudo-dilatational wave velocity in them and the ultrasonic pseudo-shear wave velocity in them can be obtained, where the ultrasonic wave from water into the solids intersects the solid sample as it measures the directionality of the solids. Unidirectional carbon fiber reinforced polymer laminates (CFRP) are chosen to investigate the mechanical anisotropy of solids. Using the ultrasonic velocity data of the CFRP, which are measured by the water immersion ultrasonic sing-around method, and the density data of them, which are measured by the hydro-weighing method, the elastic constants of them, namely,  $C_{11}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{33}$ ,  $C_{44}$  and  $C_{66}$  of them are derived and are tabulated.

### Introduction

Elastic constants of solids are one of the important mechanical properties of the solid materials. They are measured by the water immersion ultrasonic sing-around method, and they will be used effectively in the various designs which use the solids.

Water immersion ultrasonic sing-around method is a skillful method to measure both the ultrasonic dilatational wave velocity in the solids and the ultrasonic shear wave velocity in them [1]. Using a same one system, both the ultrasonic dilatational wave velocity in the solids and the ultrasonic shear wave velocity in them can be obtained only by selecting appropriately the incidence angle of the ultrasonic wave from water into the solids.

When the incidence angle of the ultrasonic wave from water into the solid specimen is below the critical angle for the ultrasonic dilatational wave in the solid specimen, the ultrasonic dilatational wave in it determines the sing-around period of the system. So, the ultrasonic dilatational wave velocity in the solid specimen can be obtained in this condition, where the incidence angle from water into the solid specimen is below the critical angle for the ultrasonic dilatational wave in it. On the other hand, when the incidence angle of the ultrasonic wave from water into the solid specimen is above the critical angle for the ultrasonic dilatational wave in the solid specimen, the ultrasonic dilatational wave does not propagate in it. But, in this

condition, the ultrasonic shear wave propagates into it large enough for the purpose to measure the ultrasonic shear wave velocity in the solid specimen. The ultrasonic shear wave determines the sing-around period of the system, then, the ultrasonic shear wave velocity in the solid specimen can be obtained by the system, same as the ultrasonic dilatational wave velocity in the solid specimen can be obtained.

The similar system is applied to measure the mechanical anisotropy of the solid materials [2]. Then, the similar system is extended to measure both the ultrasonic pseudo-dilatational wave velocity in the unidirectional carbon fiber reinforced polymer laminates (CFRP) and the ultrasonic pseudo-shear wave velocity in the CFRP [3]. Using these measurement data for the ultrasonic velocity in the CFRP, which are obtained by the water immersion ultrasonic sing-around method, and the density data of the CFRP, which are obtained by the hydro-weighing method, the elastic constants of the CFRP are derived [3,4].

### Water immersion ultrasonic sing-around method

Figure 1 shows the principle of the ultrasonic refraction, which is to be used in the water immersion ultrasonic sing-around method. The figure 1 indicates the top view of the measurement system. In the figure, the direction of the ultrasonic propagation is always on the face of the paper.

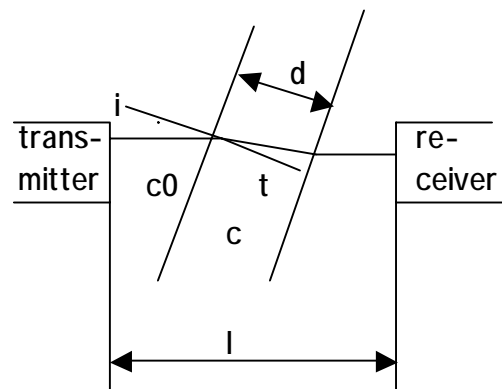


Figure 1: Principle of water immersion method

In the figure 1, (i) indicates the incidence angle of the ultrasonic wave from water into the solid specimen, (t) the refraction angle of the ultrasonic wave from water into the solid specimen, (d) the thickness of the solid specimen and (l) the distance between the transmitter and the receiver. The ultrasonic velocity in the solid specimen being set as (c), and the ultrasonic velocity in water to be (c<sub>0</sub>), the sing-around period (τ), when the solid specimen is immersed in water, is expressed as follows.

$$\tau = \frac{l - d \cos(t - i) / \cos(t)}{c_0} + \frac{d / \cos(t)}{c} \quad (1)$$

When the ultrasonic transmission system is composed only with water, the sing-around period (τ<sub>0</sub>) of this system is expressed as follows.

$$\tau_0 = \frac{l}{c_0} \quad (2)$$

Using the equations (1) and (2), the following equation (3) is derived.

$$c = \frac{1}{\frac{(\tau - \tau_0) \cos(t)}{d} + \frac{\cos(t - i)}{c_0}} \quad (3)$$

From the Snell's law of this ultrasonic system, the following equation (4) is derived.

$$t = \sin^{-1} \left( \frac{c}{c_0} \sin(i) \right) \quad (4)$$

By applying successive approximation simultaneously on equations (3) and (4), the ultrasonic velocity in the solid specimen (c), and the refraction angle of the ultrasonic wave from water into the solid specimen (t) can be computed precise iteratively. These equations from (1) to (4) are valid both for the ultrasonic dilatational wave in the solid specimen and for the ultrasonic shear wave in it.

### The apparatus used in the experiment for the measurement of the ultrasonic velocity in the solid specimen

The central frequency of the ultrasonic wave used in the sing-around system is 2MHz. With the ultrasonic sing-around method, the ultrasonic signal detected by the receiver is transformed into the electric signal, and it is amplified and the electric signal is investigated its feature for the following procedure. Detecting the typical feature of the receiving electric signal, the electric pulse is generated again, and it is transmitted to the transmitter, then, the ultrasonic wave is to be generated by the transmitter, and it propagates into water. As is shown here, the signal sings around the system, changing its feature as ultrasonic wave energy or as electric energy. The sing-around period is to be measured by a universal counter.

The temperature of the water tank is set to be 30°C by controlling proportionally the electric current, which is travelling in the heating wire that is immersed in the water tank. For the purpose to control the electric current arbitrary, the height of the mercury in the thermometer with coil glass tube, which is immersed in the water tank, is detected by a photo-sensor. According to the quantity of the luminous intensity, which is to be measured by the photo sensor through the mercury column in the glass tube, the quantity of the electric current in the heating wire is being controlled so as to get the desired stable temperature for the water, which is in the water tank.

The range of the incidence angles from water into the solid specimen is between 0° and 50° with the resolution of 0.1°.

### Elastic constants of isotropic solids

As a preliminary experiment, the elastic constants of the isotropic solids are investigated. In this case, the elastic constants can be derived with the following equations from (5) to (8). In these equations, c<sub>p</sub>, c<sub>s</sub> and ρ indicate the ultrasonic dilatational wave velocity in the solid, the ultrasonic shear wave velocity in it and the density of it, respectively. And, E, G, ν and K denote the Young's modulus of the solid, the rigidity of it, the Poisson's ratio of it and the bulk modulus of it, respectively.

$$E = \frac{\rho c_s^2 (3c_p^2 - 4c_s^2)}{c_p^2 - c_s^2} \quad (5)$$

$$G = \rho c_s^2 \quad (6)$$

$$\nu = \frac{c_p^2 - 2c_s^2}{2(c_p^2 - c_s^2)} \quad (7)$$

$$K = \rho \left( c_p^2 - \frac{4}{3} c_s^2 \right) \quad (8)$$

### Results of experiment for the isotropic solid

As examples for the isotropic solids, polymethyl methacrylate (PMMA) discs and polystyrene (PS) discs are selected, and the experimental data of them are obtained and are tabulated. The diameter of the sample disc is 37mm and the thickness of it is 2mm, 4mm or 6mm. Table 1 shows the result of the measurement for the ultrasonic dilatational wave velocity in the PMMA and for that in the PS by the water immersion ultrasonic sing-around method.

Table 1 : Ultrasonic dilatational wave velocity (km/s)

Thickness	2mm	4mm	6mm
PMMA	2.713	2.708	2.720
PS	2.678	2.677	2.677

Table 2 shows the result of the measurement for the ultrasonic shear wave velocity in the PMMA and for that in the PS by the water immersion ultrasonic sing-around method.

Table 2 : Ultrasonic shear wave velocity (km/s)

Thickness	2mm	4mm	6mm
PMMA	1.357	1.359	1.367
PS	1.331	1.334	1.335

Together both with the measurement data of the ultrasonic dilatational wave velocity in the solid specimen and with those of the ultrasonic shear wave velocity in them, the density of the solid specimen is measured by a hydro-weighing method. With these measurement data of PMMA and PS, Young's modulus of them, rigidity of them, Poisson's ratio of them and bulk modulus of them are derived by the equations from (5) to (8).

The derived results for the Young's modulus of the PMMA and the PS, those for the rigidity, those for the Poisson's ratio and those for the bulk modulus are shown in tables from 3 to 6.

Table 3 : Young's modulus (GPa)

Thickness	2mm	4mm	6mm
PMMA	5.84	5.85	5.92
PS	5.63	5.65	5.66

Table 4 : Rigidity (GPa)

Thickness	2mm	4mm	6mm
PMMA	2.19	2.20	2.22
PS	2.11	2.12	2.12

Table 5 : Poisson's ratio

Thickness	2mm	4mm	6mm
PMMA	0.333	0.332	0.331
PS	0.336	0.335	0.334

Table 6 : Bulk modulus (GPa)

Thickness	2mm	4mm	6mm
PMMA	5.84	5.80	5.84
PS	5.72	5.70	5.70

### Unidirectional carbon fiber reinforced polymer laminates

As example materials for the mechanical anisotropy of solids, unidirectional carbon fiber reinforced polymer laminates (CFRP) are selected, and the elastic constants of them are investigated by the water immersion ultrasonic sing-around method.

For the purpose to analyze the mechanical properties of the CFRP discs, two series of the measurements are performed for the ultrasonic wave velocities in the CFRP discs. The one is done, where the direction of the fiber is always perpendicular to the direction of the ultrasonic wave propagation. The other is done, where the direction of the fiber is contained in the incidence-refraction plane of the ultrasonic wave.

*The arrangement for the fiber of the CFRP discs, where the direction of the fiber is always perpendicular to the direction of the ultrasonic wave propagation*

At first, the CFRP discs are arranged so as similar to the figure 1, where the direction of the fiber is always set to be perpendicular to the face of the paper. In this condition, the mechanical properties of the CFRP discs are similar to those of the isotropic solids, such as the PMMA disks and the PS disks. Namely, both the ultrasonic dilatational wave velocity in the CFRP discs and the ultrasonic shear wave velocity in them are measured, where the direction of the ultrasonic wave propagation is always perpendicular to the fiber direction of the CFRP discs.

In this condition, the analysis for the elastic constants of the CFRP discs can be performed same as the analysis for the elastic constants of the isotropic solids such as the PMMA discs and the PS discs. Then, the three elastic constants of the CFRP discs, namely,  $C_{11}=\lambda+2\mu$ ,  $C_{66}=\mu$ ,  $C_{12}=\lambda$ , where  $\lambda$  and  $\mu$  are the Lamé's constants, are derived from both the ultrasonic dilatational wave velocity in the CFRP discs and the ultrasonic shear wave velocity in them, together with the density of the CFRP discs.

*Results of experiment for the CFRP discs, where the direction of the fiber is always perpendicular to the direction of the ultrasonic wave propagation*

The results of the measurement for the  $C_{11}$ ,  $C_{66}$  and  $C_{12}$  of the CFRP discs are tabulated in table 7. The investigated CFRP discs are two grades of the commercial CFRP products, which are shown in the table 7 as T700 and M40J.

Table 7 : Elastic constants of CFRP (GPa)

	$C_{11}$	$C_{66}$	$C_{12}$
T700	14.1	3.5	7.1
M40J	12.2	2.8	6.7

The arrangement for the fiber of the CFRP discs, where the direction of the fiber is contained in the incidence-refraction plane of the ultrasonic wave

Next, the CFRP discs are arranged so as the direction of the fiber is set always to be on the face of the paper. And the measurements for the sing-around periods are performed as the previous measurements mentioned before, using the same ultrasonic sing-around system. In this condition, the ultrasonic wave velocity changes its magnitude always according to the refraction angle from water into the CFRP discs. Then, the ultrasonic pseudo-dilatational wave in the CFRP discs and the ultrasonic pseudo-shear wave in them propagate in this system, because the ultrasonic wave in the CFRP discs intersects the fiber differently according to the increase of the incidence angle of the ultrasonic wave from water into the CFRP discs.

By the least square method [5], the other elastic constants of the CFRP discs, namely,  $C_{13}$ ,  $C_{33}$  and  $C_{44}$  of the CFRP discs are computed with successive approximation, using the measurement data of the ultrasonic pseudo-dilatational wave velocity in the CFRP discs and those of the ultrasonic pseudo-shear wave velocity in them, together with the measurement data of the density for them.

*Results of experiment for the CFRP discs, where the direction of the fiber is contained in the incidence-refraction plane of the ultrasonic wave*

Table 8 shows the results of the computation for the elastic constants, namely  $C_{13}$ ,  $C_{33}$  and  $C_{44}$  of the CFRP discs, with the successive approximation based on the data of the ultrasonic pseudo-dilatational wave velocity in the CFRP discs, and on the ultrasonic pseudo-shear wave velocity in them, together based on the data of the density for them. In the table 8, the results for the CFRP discs are shown, as in the table 7.

Table 8 : Elastic constants of CFRP (GPa)

	$C_{13}$	$C_{33}$	$C_{44}$
T700	9.4	164.1	6.5
M40J	13.7	327.2	5.5

### Conclusion

The measuring method for the mechanical anisotropy of solids by water immersion ultrasonic sing-around method is devised. By this method, the elastic constants of the unidirectional solids can be derived. This is an application of the water immersion ultrasonic sing-around method, which provides a skillful measure to determine the elastic constants for the isotropic solid materials.

Two grades of the products for the unidirectional carbon fiber reinforced polymer laminates (CFRP) are selected, and the various elastic constants of them are

derived and they are tabulated with the water immersion ultrasonic sing-around method.

At first, one series of the measurement data for the sing-around periods are obtained, where the fiber direction is always perpendicular to the ultrasonic wave propagation. From these measurement data,  $C_{11}$ ,  $C_{66}$  and  $C_{12}$  of the CFRP discs are obtained explicitly, which use the measurement data of the ultrasonic dilatational wave velocity in the CFRP discs and those of the ultrasonic shear wave velocity in them, together with the measurement data of the density for them. These analyses are similar to those for the mechanically isotropic solids.

Next, the other series of the measurement data for the ultrasonic sing-around periods are obtained, where the fiber direction is contained in the incidence-refraction plane. In this case, the ultrasonic pseudo-dilatational wave velocity in the CFRP discs and the ultrasonic pseudo-shear wave velocity in them are obtained by the ultrasonic sing-around method, using the same ultrasonic system as before, only by arranging differently the CFRP discs, where the direction of the fiber is turning right angles within the face of the CFRP discs. Using the successive approximation, together with the density data of the CFRP discs,  $C_{13}$ ,  $C_{33}$  and  $C_{44}$  of the CFRP discs are computed for the two grades of the CFRP specimens. In conclusion, the whole results of the computation for the CFRP specimens are tabulated in two tables.

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