VISUALIZATION OF THE EFFECTS OF STATIC STRESS ON THE CELERITY OF ULTRASONIC WAVES BY THREE-DIMENSIONAL REPRESENTATIONS

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Abstract

Ultrasonic wave propagation in prestressed materials has been extensively studied during last decades. From a practical point of view, materials are submitted to static stresses that are directly applied or residual. The knowledge of these stresses permits both to evaluate materials lifetime and to improve its intrinsic qualities. In order to point out these stresses ,we make use of acoustoelasticity theory.

The present work is concentrated on the effect of static stresses on the velocities of ultrasonic waves in cubic materials with low and high anisotropy. A rigorous quantitative description of acoustoelastic effects is performed for silicon and aluminium, it involves tridimensional representation of slowness surfaces for a better understanding of acoustoelastic effect distribution. Even though these two materials have the same symmetry, the acoustoelasticity reveals a significant difference in their behaviour.

The computational procedure includes acoustoelastic (AE) effects for compression and shear residual stresses respectively σ_{33} and σ_{12}. The illustration is achieved by a colour code, the colour varies from white (no acoustoelastic effects) to black (strong effects) .

Introduction and theoretical basis

The study of elastic wave propagation in prestressed materials, attracted the attention of a number of researchers in the past years. The basic theory that relates the change of wave speeds to initial stresses is the acoustoelasticity theory, a modern approach has been then presented by introducing third order elastic constants in the constitutive equations [1].

Eagle and Bray [2] have been the first to introduce the acoustoelastic coefficients; these coefficients translate the effect of a static stress field to the ultrasonic wave velocities. The theoretical determination of these coefficients is more or less complex, it depends on the degree of the material symmetry.

Therefore, the existing calculations and experiences evoke the acoustoelasticity following axes [3,4,5,6] or inside symmetry planes [7]. In this paper, acoustoelasticity theory has been applied to calculate the distribution of acoustoelastic coefficients and acoustic birefringence following all propagation directions of ultrasonic waves.

In order to examine effects related to anisotropy and residual stress direction, we consider two cubic materials (Si, Al) and two kinds of stress (σ_{33}, σ_{12}) (figures 1-a and 1-b).In the natural state Aluminium slowness sheets exhibit slight deviations from isotropic slowness surfaces. For silicon these deviations are more pronounced ,and a large acoustic birefringence is observed. In 3D geometry the representation of acoustoelastic effects distribution is performed on the basis of a colour code varying from white (no acoustoelastic effect) to black (strong effect).

To describe wave propagation in a prestressed medium the approach proposed by Man and Lu [8] is used. The prestressed configuration is the only reference configuration in this approach and the initial static stress is included in the constitutive equation by:

$$\sigma_{ij}=\sigma^{o}_{ij}+C_{ijkl}\varepsilon_{kl}+u_{i,k}\sigma^{o}_{kj} \quad \text{(eq 1)}$$

Where σ_{ij} is the first Piola-Kirchoff stress tensor, σ^{o}_{ij} is the initial static stress, ε_{kl} is the elastic strain due to wave propagation, $u_{i,k}$ is the displacement gradient and C_{ijkl} is the fourth rank tensor of stress dependent elastic constants[7]. Assuming that the material and local stresses are homogeneous and using a plane wave solution for the displacement u_i, the equation of motion leads to Christoffel equation for an anisotropic material under stress:

$$[G_{jk} - \rho V^2 \delta_{jk}]\, p_k = 0 \quad \text{(eq 2)}$$

Where $\vec{p}$ is the unit displacement vector, $\vec{k}=k.\vec{n}=(\frac{\omega}{V}).\vec{n}$, the wave number vector, $\vec{V} = V.\vec{n}$ the wave phase velocity, $\vec{n}$ the unit wave normal and G_{jk} is the generalised Christoffel tensor with components:

$$G_{jk} = (C_{ijkl} + \sigma^{o}_{il}\delta_{jk})n_i n_l \quad \text{(eq 3)}$$

Equation 2 has non trivial solution when the determinant is equal to zero:

$$\left| G_{jk} - \rho V^2 \delta_{jk} \right| = 0 \quad \text{(eq 4)}$$

The G matrix is still symmetric, then the eigenvalue problem has three real solutions as for an unstressed medium. In the following we note by $\mathbf{V}^0_{L,T1,T2}$ and $\mathbf{V}_{L,T1,T2}$ the wave velocity of ultrasound respectively in free and prestressed material, for longitudinal (L) and shear waves (T_1, T_2).

The expressions of acoustoelastic coefficients $A^{(k)}_{ij}$ [2, 4] and acoustoelastic birefringence $B^{(k)}_i$ [1] are definite by:

$$A^{(k)}_{ij}=\left(\frac{V_{ij}-V^0_{ij}}{V^0_{ij}}\right)/\sigma_k \qquad \text{(eq 5)}$$

and $$B^{(k)}_i=\frac{V_{iT_1}-V_{iT_2}}{V_{iT}} \qquad \text{(eq 6)}$$

Where the superscript k denotes the direction of the static stress σ_k, the first subscript of V_{ij} indicates the propagation direction, the second one the polarisation direction of the wave, and V_{iT} represents the smallest value of shear waves speeds.

For cubic materials following the high symmetry axes, acoustic birefringence B_i vanishes; however following diagonal binary axes as [110] this parameter is maximal (fig7). When material is prestressed the $B^{(k)}_i$ deviations are investigated .

Results and discussions

Among cubic materials currently used, aluminium and silicon have been selected to illustrate acoustoelastic effects, they are characterized by two different anisotropy factors defined by: $A_n=\frac{2C_{44}}{C_{11}-C_{12}}$, their mechanical characteristics are grouped in table1 [9,10,11].

To determine slowness vectors for every directions angular parameters related to crystallographic Cartesian frame $\{\vec{u}_a,\vec{u}_b,\vec{u}_c\}$ are introduced, hence the space is displayed in a suitable way.

Basically the first computational step leads to acoustoelastic coefficient values $A^{(k)}_{ij}$,the obtained change is of the order of 10^{-4}/Mpa, 10^{-6}/MPa according to wave polarisation, direction and static stress. These values are in concordance with experimental results[4,5]. Secondly 3D slowness surfaces are drawn, particular attention is given to view angles (figure 1-c) .We notice that distribution of acoustoelastic effect is compatible with the symmetry of the prestressed medium, in the case of σ_{33} stress the symmetry becomes quadratic, while σ_{12} stress gives monoclinic symmetry.(figure 1–a,b). The contrast parameter for a given cell is calculated according Matlab programs.

3D representations of acoustoelastic effects related to Si submitted to compression stress σ_{33}, show local blackening around symmetry axes. These AE effects are concentrated around fourfold axes for longitudinal waves, for the shear wave T_2 AE effects are strong in the vicinity of diagonal binary axes. The second shear T_1 wave whose slowness sheet is not presented, reveals blackening around threefold axes(figure 2). σ_{12} residual stress give rise to local acoustoelastic effect differently distributed in space(figure3). The analysis of aluminium response, that submitted to the same stress σ_{33}, shows that blackening is more spread and differently located in comparison with silicon (figure 4-5), aluminium behaviour is close to isotropic materials. Contrarily to previous cases σ_{12} reveals limited acoustoelastic effects in space (figure6).

For both cubic samples speeds of longitudinal waves are less sensitive to residual stress, ie the longitudinal velocities changes are weak in comparison with transverse waves velocities.

Acoustoelastic effects related to a shear static stress σ_{12} look different for both tested materials, fourfold symmetry seems slightly affected.

The AE effects associated to compression and shear static stress show pronounced differences, further experiments have to be accomplished to elucidate the relationship between velocity change, residual stresses and anisotropy.

The greatest birefringence observed in unstressed silicon reaches $8{,}84.10^{-2}$ (figure 7), with axial stress birefringence varies from zero to $1{,}9.10^{-3}$ in the vicinity of threefold axes(figure 8). This parameter reveals an acoustoelastic effect quite strong, which can be exploited to perform best measurements of residual stress.

Conclusion

we have examined the acoustoelastic effects for two cubic materials with different anisotropy, those submitted to homogeneous compression stress σ_{33} or shear stress σ_{12}. The acoustoelastic responses show remarkable differences in extent and localization. The introduction of a colour code have permitted a visualisation of acoustoelastic effects by means of 3D slowness plots. It has been shown that birefringence can be a precious parameter in residual stress investigations. Experimental studies are to be planned to valid the obtained numerical results.

Table 1 : mechanical characteristics

	Aluminium	Silicon
ρ (Kg.m^{-3})	2702	2329
An	1.23	1.56
C^0_{11} (G Pa)	108	165.6
C^0_{12} (G Pa)	62	63.9
C^0_{44} (G Pa)	28.3	79.5
C111 (GPa)	-819	-795
C112 (G Pa)	-324	-445
C123 (G Pa)	101	-75
C144 (G Pa)	35	15
C155 (G Pa)	-422	-310
C456 (G Pa)	-1	-86

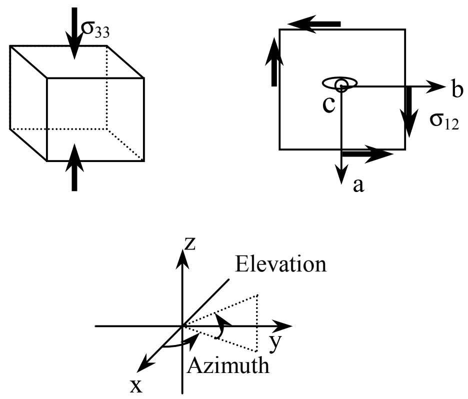

Figure 1: residual stresses and view angles

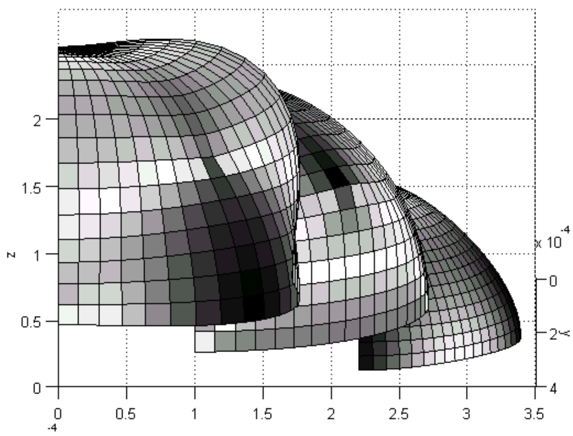

Figure 2: Silicon slowness surfaces with acoustoelastic effect corresponding to σ_{33}.

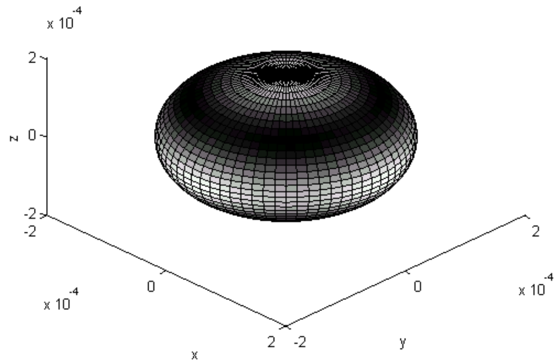

Figure 3 : Aluminium longitudinal slowness surface with acoustoelastic effect corresponding to σ_{33}.

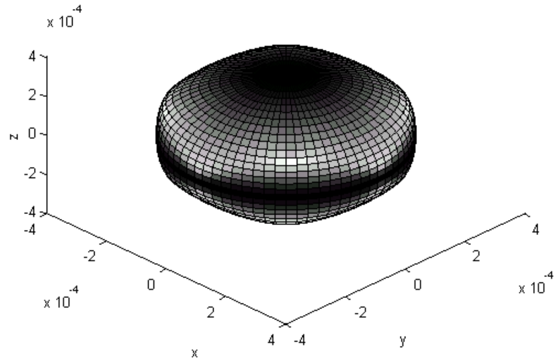

Figure 4 : Aluminium transverse slowness surface (T_2) with acoustoelastic effect corresponding to σ_{33}.

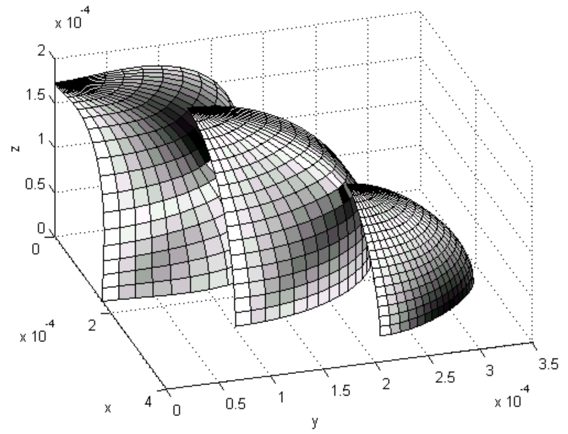

Figure 5: Silicon slowness surfaces with acoustoelastic effect corresponding to σ_{12}.

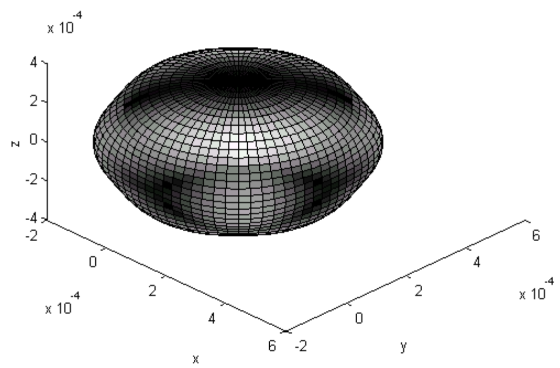

Figure 6 : Aluminium transverse slowness surface (T_1) with acoustoelastic effect corresponding to σ_{12}.

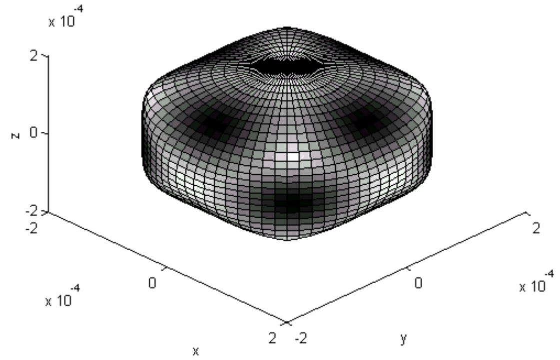

Figure 7: Slowness surfaces (T_2) with acoustoelastic birefringence for natural silicon.

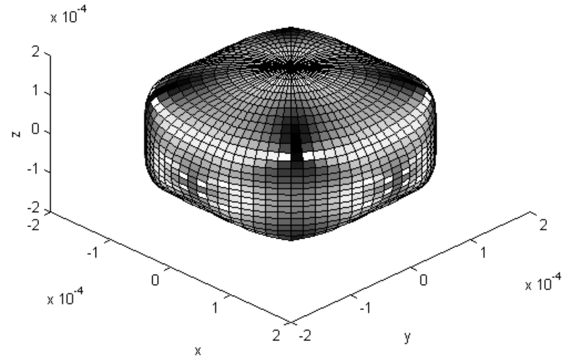

Figure 8: acoustoelastic birefringence deviation for silicon submitted to σ_{33}
(Slowness surfaces (T_2))

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