ACOUSTIC REMOTE PALPATION: THE INFLUENCE OF TISSUE VISCOSITY ON THE EXCITATION AND RELAXATION OF LOCAL IMPULSE SHEAR STRAIN

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Abstract

The influence of viscosity is studied for points with arbitrary radial coordinate respectively to the axis of radiation force. The magnitude and space distribution of induced strain have been found for different ratio between involved parameters. At strain relaxation the positive additive to displacement caused by viscosity decreases with time and does not depend on radial coordinate. At the presence of viscosity the magnitude of initial shear strain (before the shear waves escape the excitation area) is less for tissue particles inside the excitation area and greater out it. The developed theory correctly describes the functional dependencies of shear strain found experimentally and enables the assessment of tissue viscoelastic parameters using radiation force based methods.

Introduction

Several experimental methods can be used to research the elastic properties of soft tissues. They include Shear Wave Elasticity Imaging (SWEI) [1-3] and Acoustic Remote Palpation (ARP) [1,4-6] which are based on the excitation of strain by radiation force of focused acoustic beam, as it is shown in Fig. 1.

The viscosity is one of the distinctive features of biological tissues so they exhibit the viscoelastic behavior. The viscosity can be evaluated only at the presence of nonuniform distribution of tissue motion because the viscous friction is conditioned by velocity gradients. Thus at research of viscoelasticity it should be used the dynamical source of displacement fields producing the velocity distribution localized enough to provide the proper level of viscous force. ARP and SWEI satisfy these requirements because in both methods the dynamical shear strain is localized initially mainly in the focal area of ultrasonic wave beam. The difference consists in the fact that at ARP the tissue response is analyzed for directly enforced points in the focal area. SWEI grounded on the analysis of dynamic response in more wide area due to the appearance of traveling shear waves. The shear wave speed $c_t = \mu/\rho$ is determined by shear modulus μ although this way of it estimation is not quite local as any wave method (principle of uncertainty).

The strain induced by radiation force of ultrasonic beam with amplitude modulation was discussed in details by Rudenko *et al.* [1]. This consideration gives a clear disclosure of tissue under local pulse loading. It establishes a linear dependence of displacement magnitude on the pulse duration τ and predicts the strain decreasing in the focal area with time due to the formation of shear waves propagating away from the beam axis. However the derived dependence of strain relaxation on viscosity is not so valid. There are unclear also the lateral distribution of strain fields, its dependence on viscosity and details of strain relaxation for distant from the beam axis points which motion is caused by passing waves. At that a numerical solution of motion equations [4] does not show clearly the functional dependencies on tissue viscosity and elasticity suitable for their estimation. The research of these problems is important for medical implementation of ARP and SWEI methods.

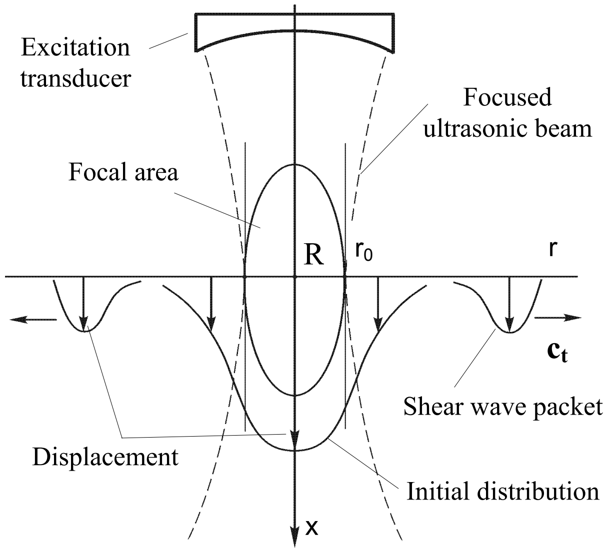

Figure 1 : Schematic of ARP and SWEI methods

Method and results

In the general case the integral expression for axial displacement derived with the radiation force of pulse focused beam with Gaussian profile has the form [1]:

$$S(r,t) = \frac{\alpha(a\gamma^{-1})^2 I^{(t)}}{2\rho c} \int_0^\infty \frac{\exp(-\frac{a^2k^2}{8\gamma^2})J_0(kr)}{\sqrt{k^2c_t^2 - \frac{k^4\nu^2}{4}}} I(k,t)kdk \ , (1)$$

$$I(k,t) = \int_{-\infty}^{t} b^2(t')\exp[\frac{k^2\nu}{2}(t'-t)]\sin[(t-t')\sqrt{k^2c_t^2 - \frac{k^4\nu^2}{4}}]dt'$$

Here, $J_0(x)$ is the Bessel function of the first kind, zeroth order; r is the radial coordinate; c is the speed of ultrasound; α is the attenuation coefficient of ultrasound; $\nu = \eta/\rho$ is the kinematic shear viscosity; a is the radius of beam at level e^{-1} at the transducer's surface; $2a\gamma^{-1}$ is the beamwidth in the focal plane so the value $2r_0 \equiv \sqrt{2}a\gamma^{-1}$ is the beamwidth at level e^{-1} of ultrasound intensity; $\gamma = \pi a^2/\lambda R$ is the focusing degree; R is the geometrical focus; $I^{(t)}$ is the space peak and pulse average intensity. The Eq. (1) is true at a small radial displacement, i.e. for planes not far from the focal depth. In order to simplify the analytical derivation we will assume the excitation pulse having Gaussian form: $b^2(t) = \exp\left(-4t^2/\tau^2\right)$. For the convenience we introduce also the dimensionless variables: $y = a\gamma^{-1}k/2$, $y_\nu = c_t a\gamma^{-1}/\nu$.

In the focal plane the excited strain is represented by some packet of cylindrical shear waves (see Fig.1). The range of variation $y > y_\nu$ corresponding to the negative radicands in Eq. (1) describes the domain of wave components attenuating aperiodically due to the viscous damping so the parameter y_ν indicates some dimensionless cutoff wave number. In the limit case $y_\nu \to \infty$ the negligible part of the total energy belongs to the waves damped aperiodically. Further we will assume $y_\nu \sim 1$. Taking into account that the minimum wave length of induced packet is defined by diameter of beamwidth, it means that contribution of aperiodic oscillations is not too much. At that viscosity is quite enough to reduce the displacement magnitude due to the absorption of low-frequency waves.

The relaxation of induced shear strain

For description of strain relaxation we will consider an extreme case of big time $t \gg \tau$, when integration with respect to time can be extended to $+\infty$ and integrand can be divided into even and odd functions. At that the desired value of $I(k,t)$ takes the form of tabulated integral. The relaxation in a given area starts just after shear waves have left it [1]. For the focal area the proper condition can be written as follows

$$t \gg a\gamma^{-1}/c_t \sim \tau . \qquad (2)$$

Taking into account (2) we find that the main contribution gives the range of integration:

$$y \le \sqrt{y_\nu a\gamma^{-1}/2c_t t} = y_\nu \sqrt{a\gamma^{-1}/2y_\nu c_t t} \ll y_\nu .$$

Therefore the radicals in Eq. (1) may be put equal to unity. For distant from the beam axis areas it is necessary to take into account a delay time caused by wave passing up to the point of view. The proper inequality $t \gg r/c_t$ together with (2) ensure that the argument of the Bessel function satisfies to inequality

$$2ry/a\gamma^{-1} = (2r/c_t t)(c_t t/a\gamma^{-1})y \ll c_t t/a\gamma^{-1} y ,$$

that allows to put equal to unity the Bessel function. As a result the displacement (1) takes the form:

$$S(r,t) = \frac{\sqrt{\pi}\alpha(a\gamma^{-1})\tau I^{(t)}}{2\rho c c_t}\int_0^\infty \exp\left(-\frac{2c_t t}{a\gamma^{-1}}\frac{y^2}{y_\nu}\right)\sin\left(\frac{2c_t t}{a\gamma^{-1}}y\right)dy ,$$

where integral can be expressed through the error functions $\Phi(\pm i\sqrt{c_t t y_\nu/2a\gamma^{-1}})$. Substituting the known asymptotic form for error functions at big argument we find the final representation for displacement:

$$S(r,t) = \frac{\sqrt{\pi}\alpha(a\gamma^{-1})^2\tau I^{(t)}}{4\rho c c_t^2 t}\left(1 + \frac{\nu}{c_t^2 t}\right) . \qquad (3)$$

Note that the model derivation [1] gives a formula with another sign of viscous term.

Spatial distribution and displacement magnitude

The influence of viscosity on displacement fields can be studied considering the case of small times and short pulses: $c_t t \sim c_t\tau \ll a\gamma^{-1}$. Then the integral on time in Eq. (1) can be written as a sum of two integrals $I = I_1 + I_2$ with limits of integration from $-\infty$ up to 0 and from 0 up to t, respectively. The dependence on the time can be neglected entirely in the limiting case

$$c_t t \ll c_t \tau \ll a\gamma^{-1} , \qquad (4)$$

that corresponds actually to the consideration of shear strain at $t = 0$, when $I_2 = 0$. For clearing up the role of viscosity it is enough to write down the first two terms in expansion of I_1 as series of $c_t\tau/2a\gamma^{-1}$ that gives

$$S(r,0) = \frac{\alpha I^{(t)}\tau^2}{4\rho c}\exp\left(-\frac{r^2}{r_0^2}\right)\left[1 - \sqrt{\frac{\pi}{2}}\frac{\nu\tau}{a\gamma^{-1}r}M_{3/2,0}\left(\frac{r^2}{r_0^2}\right)\right] .$$

Here, $M_{3/2,0}(x)$ is the Whittaker function. Using the known functional equality for Whittaker function we get the strain distribution induced by impulse force:

$$S(r,0) = \frac{\alpha I^{(t)}\tau^2}{4\rho c}\exp\left(-\frac{r^2}{r_0^2}\right)\left[1 - \frac{\sqrt{\pi}\nu\tau}{a^2\gamma^{-2}}\left(1 - \frac{r^2}{r_0^2}\right)\right] . \qquad (5)$$

The Eq. (5) describes the instantaneous lateral distribution at the moment of the radiation force extreme. At this moment the additional viscous term is greater as the squared beamwidth radius is less because the radius r_0 is a spatial range of induced velocity gradients. In this case the influence of the viscous force depends also on the pulse duration τ.

Another result can be derived for the time points:

$$c_t\tau \ll c_t t \ll a\gamma^{-1} . \qquad (6)$$

The direct series expansion of Eq. (1) accounting the square of small ratio $c_t t/a\gamma^{-1} \ll 1$ and integration give an expression similar to Eq. (5):

$$S(r,t) = \frac{\sqrt{\pi}\alpha I^{(t)}\tau t}{\rho c}\exp\left(-\frac{r^2}{r_0^2}\right)\left[1 - \frac{4\nu t}{a^2\gamma^{-2}}\left(1 - \frac{r^2}{r_0^2}\right)\right] . \qquad (7)$$

The relative variation of strain fields is linearly dependent on the time since the inertial particle motion results in prolongation of viscous force effect.

More complicated situation has a place in the case

$$c_t\tau \ll c_t t \sim a\gamma^{-1} , \qquad (8)$$

when displacement stops increasing and begins to decrease due to the formation of shear waves. Yet prediction of maximum value is possible since the turning point is about $t_0 = a\gamma^{-1}/c_t$. Substituting of t_0

into Eq. (7) gives following evaluation formula:

$$S_0(r) = \frac{\sqrt{\pi}\alpha a\gamma^{-1} I^{(t)}\tau}{\rho c c_t}\exp(-\frac{r^2}{r_0^2})[1-\frac{4\nu}{c_t a\gamma^{-1}}(1-\frac{r^2}{r_0^2})]. \quad (9)$$

The dependence of spatial distribution on viscosity computed using Eq. (5) is shown in Fig. 2a. Figure 2b depicts the increasing through the time of the viscous term. The displacement fields turn out to be slightly wider than that for ultrasonic intensity.

The short pulses best of all permit to reveal the tissue viscosity. In the case of the middle pulse length

$$c_t\tau \sim a\gamma^{-1} \le c_t t \quad (10)$$

the pulse termination contemporizes with the shear waves leaving. The maximum displacement should be about the pulse ending but its functional dependencies can be somewhat others compare to Eq. (9).

The tissue loading with very long force impulse can be disclosured by putting $t = 0$ into Eq. (1) and using the asymptotic form at great $c_t\tau / 2a\gamma^{-1} \gg 1$ to give:

$$S(r,0) = \sqrt{\frac{\pi}{2}}\frac{\alpha(a\gamma^{-1})^2 I^{(t)}}{4\rho c c_t^2}\exp(-\frac{r^2}{a^2\gamma^{-2}})I_0(\frac{r^2}{a^2\gamma^{-2}}), \quad (11)$$

where $I_0(x)$ is the hyperbolic Bessel function. The distribution (11) does not follow the radiation force profile independently on the viscosity and decreases much more slowly. Substituting the asymptotic form for Bessel function at great r into Eq. (11) we have no exponential, but power-behaved field decreasing:

$$S(r,0) = \frac{\alpha(a\gamma^{-1})^3 I^{(t)}}{8\rho c c_t^2 r}. \quad (12)$$

Such behavior is typical for localized static loading

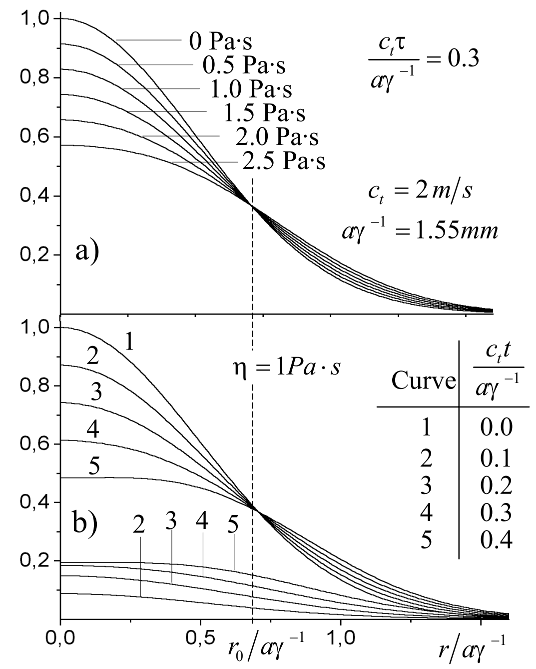

Figure 2 : Normalized distributions: a) – $S(r,0)$; b) - $S'(r,t) = t^{-1}S(r,t)$ (upper curves). Lower curves correspond to the distribution $(c_t t / a\gamma^{-1})S'(r,t)$

and has place in particular for strain fields associated with the infinitely small distributed force volume [7].

Discussion and conclusion

A specific feature of Eq. (3) for strain relaxation is its independence on coordinate r, so the relaxation in all excited areas passes in the similar way. The strain relaxation is sluggish due to the damping of high-frequency components. At that the viscous term has always a positive sign so irrespective of the effect of viscosity on the initial shear strain the displacement at relaxation is more as greater viscosity. This fact is confirmed by numerical computation at different values of viscosity [4]. Figure 3 shows clearly that corresponding graphs cross. The relative viscosity contribution is determined by the ratio η/μ and at the given value of this ratio the displacement magnitude turns out to be greater in tissue with smaller hardness.

It follows from Eqs. (5), (7) and (9) that in the absence of viscosity the lateral distribution in the focal plane duplicate the radiation force profile. The tissue viscosity leads to the field distortion differing for points inside the focal area and outside it. For inside points the displacement is less than that in the absence of viscosity, and for outer points is greater. This result can be understood accounting that quickly moving particles inside the focal area are braking by surroundings contrary to the particles in peripheral region for which the viscous force is accelerating.

The tissue displacement along the entire propagation path from the transducer to the focus was discussed minutely in several papers [1,5,6] whereas the lateral profile of strain fields has not been studied in details. The usually observed displacement magnitude at points aside the focal area shows actually the peak value in propagating shear wave. Such kind lateral distributions in tissue phantom were detected by Andreev *et al.* [2] and were roughly coincided with the transverse size of the beam except for the peripheral area, as it is shown in Fig. 4

The principal feature of the strain field (11) is its independence on the pulse length. Thus the distribution formed to the middle of the force action is

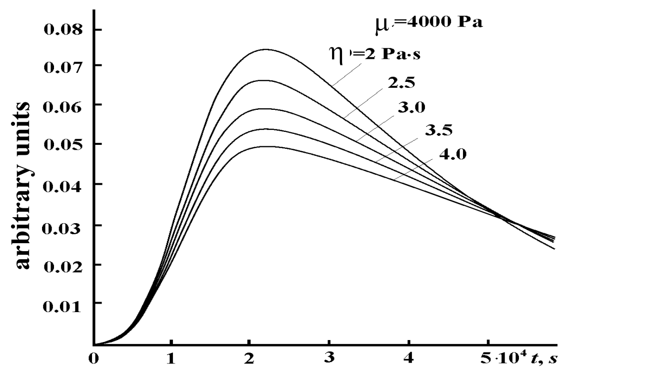

Figure 3 : Displacement through the time [4]

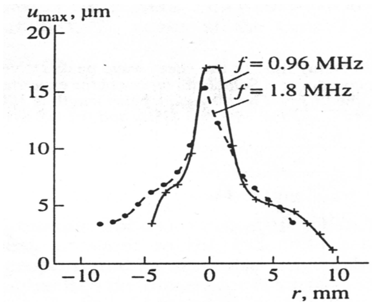

Figure 4 : Peak displacement in a shear wave [2]

constant and further effect of radiation force will not result in strain increase. In other words the long force impulse in its stationary phase effects similar to the static loading. The viscosity contribution can be neglected in this case whereas the dependence on the shear modulus is more strong compare to Eq. (9). Figure 5 shows the magnitude increasing and bringing to some saturation level with the growth of the pulse duration [8]. The same by its physical nature result was obtained by Nightingale *et al.* using long sequences of very short pushing pulses [6].

The theoretically predicted by Eq. (9) value of displacement magnitude is in a good agreement with the experimental one. Substituting into (9) numerical values of parameters $a\gamma^{-1} = 1.55mm$, $I^{(t)} = 145 W/cm^2$, $R = 7.0cm$, $\alpha = 0.041cm^{-1}$, $\tau = 2.18ms$, $\rho = 1g/cm^3$, $c_t = 2.3 m/s$, and $c = 1.81 \cdot 10^3 m/s$, which correspond to [9], we get for $\eta = 0.25 Pa \cdot s$, and r=0, the value $S_0(0) \approx 6\mu m$ that is very close to the measured one.

The present theory is developed in approach, which does not guess a smallness of parameter y_v^{-1}. In general the physical parameters of tissues and beams used in ARP and SWEI are in a good agreement with this assumption. The found features of shear strain can be observed experimentally [8,9], but the most important question to be settled for medical applications is the proper algorithm for quantitative assessment of tissue elasticity and viscosity. The usage of Eq. (3) gives such possibility as well as solution of combined Eqs. (3) and (9) after fitting of some numerical coefficients which depend on the parameters of used excitation transducer and pulses.

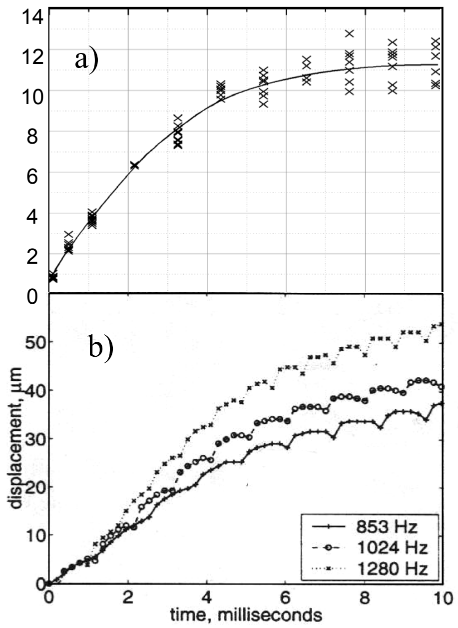

Figure 5 : Displacement: a) – as a function of pulse length [8]; (b) – induced by sequence of pulses [6]

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