ACOUSTIC WAVE SCATTERING BY A POROUS SPHERE EMBEDDED IN SOLID MATRIX

Tiehai Wang and Yaojun Wang
State Key Laboratory of Modern Acoustics, Nanjing University, P.R.China
yjwang@nju.edu.cn

Abstract

Acoustic scattering (acoustic resonance scattering in particular) are described for a porous sphere embedded in a solid matrix. A three phase composite model for evaluating the effective elastic moduli of porous spheres is introduced. Numerical computations of the acoustic resonance scattering spectra by a porous graphite sphere with changing porosity in iron matrix are made by using the acoustic resonance scattering theory. Possible applications of acoustic resonance scattering by solid and porous spheres are discussed.

Introduction

In 1978, Flax *et al* proposed an acoustic resonance scattering theory[1], showing that the scattering spectrum of a spherical or cylindrical scatterer may be decomposed into two contributions—a background and the resonance of scatterer. Whenever the resonance appears the surface waves (or interface waves) circumnavigating the scatterer match each other in phase. The resonance scattering theory also predicts that measurements of resonance scattering spectra could be used to inversely deduce the density and elastic constant of a fluid sphere embedded in elastic solid[2]. Excellent experimental investigations of ultrasonic scattering by elastic spheres in solid matrix have been made to verify the resonance scattering phenomenon[3]. In view of academic importance in acoustics and possible applications in nondestructive evaluation (NDE) of composite materials, acoustic scattering of spherical scatterers in solid matrix has been a subject which attracts much attention of investigators[4,5]. It is noticed that in some particle-reinforced composites, the spherical fillers such as carbon black and metal powers belong to porous media. Take nodular cast iron as an example, the fillers in the composite are porous graphite, the porosity of which may change obviously with manufacturing processes. To assess the porosity in such a composite, ultrasonic method needs to be consulted. Therefore, it is one of our motivations to carry out the study on acoustic scattering by porous spheres.

Acoustic resonance scattering of a sphere

To deal with acoustic scattering problem of an elastic sphere, it is the most convenient to use a spherical polar coordinate system with the origin at the center of the sphere. The incident wave is supposed to be a plane longitudinal wave propagating along the polar axis (z axis). Thus the scattered waves in the matrix and the excited waves in the elastic sphere are all symmetric with respect to the z axis, depending only upon the coordinate variables r and θ. In this instance, the displacement vectors of the incident, scattered and excited waves can be represented respectively in terms of different potential functions Φ and Π [6]

$$\vec{U} = -\nabla\Phi + \nabla\times\left[\nabla\times\left(r\hat{r}\Pi\right)\right] \tag{1}$$

where Φ and Π correspond respectively to the displacement potential functions for longitudinal and transverse waves, $\hat{r}$ is the unit vector in the radial direction, both Φ and Π satisfy the scalar Helmholtz equation, and can be expanded by spherical normal functions ($\Pi_i = 0$ for the incident longitudinal wave). By using the boundary conditions at the interface between the matrix and scatterer, the coefficients A_m and B_m respectively for the scattered longitudinal and transverse waves can be obtained.

The potential functions Φ_s and Π_s for the far-field scattered waves may be approximated as[1]

$$\begin{cases} \Phi_s = \dfrac{\Phi_0}{r}\exp(-ik_1 r) f^{pp}(\theta) \\ \Pi_s = \dfrac{\Phi_0}{r}\exp(-iK_1 r) f^{ps}(\theta) \end{cases} \tag{2}$$

where Φ_0 is the amplitude of the potential function of incident wave, k_1 and K_1 are the wave numbers for longitudinal and transverse waves in the matrix, $f^{pp}(\theta)$ and $f^{ps}(\theta)$ are usually termed as the form functions for the scattered waves in which superscript pp is for scattered longitudinal waves and ps for scattered transverse waves. The normalized form function amplitudes for $\theta=\pi$, which are called the backscattering spectra, have the following expressions

$$\begin{aligned} \frac{1}{a} f^{pp}(\pi) &= \left|\sum_{m=0}^{\infty}(2m+1)(-1)^m A_m\right| \\ \frac{1}{a} f^{ps}(\pi) &= \frac{k_1}{K_1}\left|\sum_{m=0}^{\infty}(2m+1)(-1)^m B_m\right| \end{aligned} \tag{3}$$

where a is the radius of scatterer. According to the acoustic resonance scattering theory[1], the resonance scattering spectra for the m-th mode partial wave may be finally written as

$$\begin{aligned} \frac{1}{a} f^{pp}_{m(res)}(\pi) &= (2m+1)\left|\left(A_m - A_m'\right)\right| \\ \frac{1}{a} f^{ps}_{m(res)}(\pi) &= \frac{k_1}{K_1}(2m+1)\left|\left(B_m - B_m'\right)\right| \end{aligned} \tag{4}$$

where A_m' and B_m' are the scattered wave coefficients from a rigid sphere or cavity with the same dimension as that of the scatterer.

Effective moduli of porous medium

The graphite particles in nodular cast iron composites are porous in structure. Each particle is assumed to have solid frame and air-filled pores. Here we use a so-called three phase composite model to compute the effective elastic moduli K_e and μ_e of a void- containing solid[7].

Fig.1 illustrates the concept of the model, in which the outer part with dilute dots represents the effective medium and the spherical shell with outer radius b is the solid frame (Its elastic moduli and density are K_m, μ_m and ρ_m respectively). The inner surface with radius c is bounded by air with bulk modulus K_i and density ρ_i. The volume concentration of voids

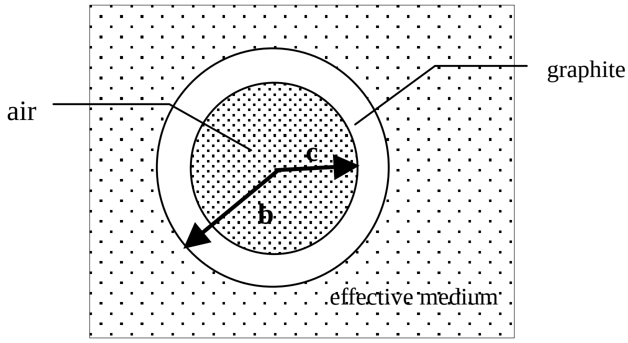

Fig.1 The three phase composite model for evaluating the effective elastic moduli of a void-containing solid

(i.e. porosity) is denoted by $\beta=(c/b)^3$. One may obtain the effective bulk modulus based on the composite model[7]

$$K_e = K_m + \frac{\beta(K_i - K_m)}{1+(1-\beta)(K_i-K_m)\Big/\left(K_m+\frac{4}{3}\mu_m\right)} \tag{5}$$

The effective shear modulus μ_e may be found from the following equation

$$A\left(\frac{\mu_e}{\mu_m}\right)^2 + 2B\left(\frac{\mu_e}{\mu_m}\right) + C = 0 \tag{6}$$

where A,B,C are the functions of the Poisson ratio of the matrix. According to the mixture rule the effective density $\rho_e=\rho_m(1-\beta)+\rho_i\beta$.

The wave numbers in the porous medium can be calculated from K_e, μ_e and ρ_e. Following the preceding procedures, the backscattering spectra and resonance modes of a porous sphere in solid matrix can be evaluated. In addition, the effective elastic moduli K_e and μ_e are varied remarkably versus the porosity of the porous scatterer, which may provide us a convenient way to examine the scattering characteristics of scatterers with different elastic moduli and densities embedded in the same matrix.

Numerical results

Numerical computations have been made for nodular cast iron composite as an example to analyze acoustic scattering by porous spheres in solid matrix. Figs. 2 (a) and 2(b) present the backscattering spectra $a^{-1}f^{pp}(\pi)$ and $a^{-1}f^{ps}(\pi)$ of a porous graphite sphere

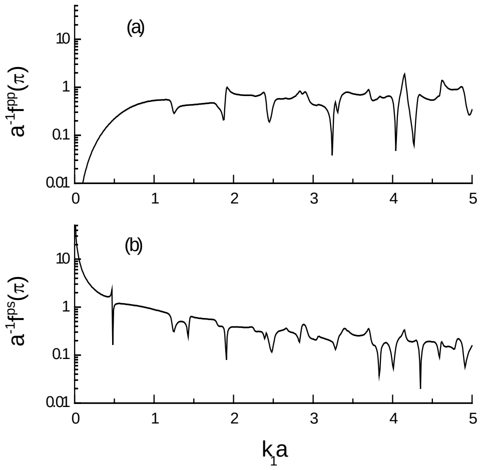

Fig.2 Backscattering spectra $a^{-1}f^{pp}(\pi)$ and $a^{-1}f^{ps}(\pi)$ by a porous graphite sphere in iron matrix ($\beta = 0.2$)

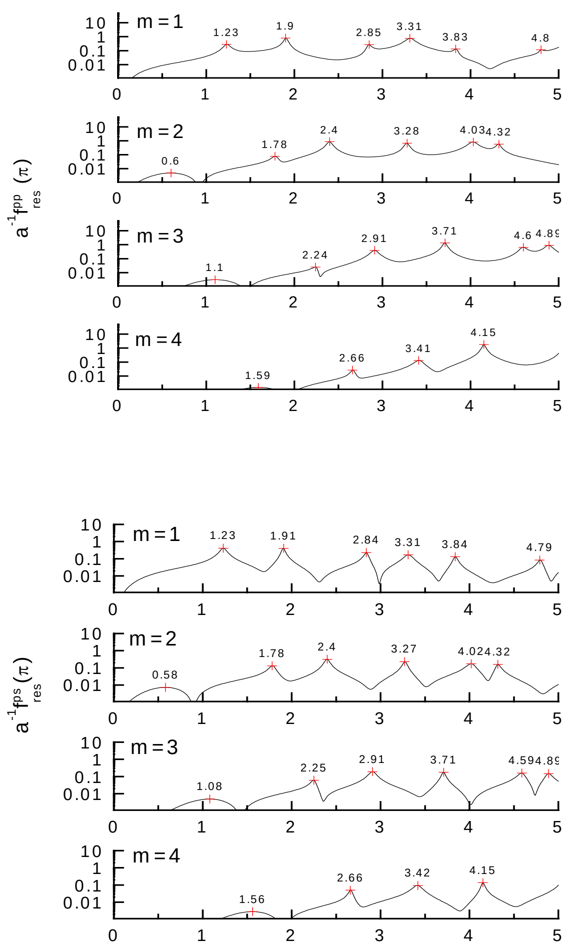

Fig.3 Resonance backscattering spectra $a^{-1}f^{pp}_{m(res)}(\pi)$ and $a^{-1}f^{ps}_{m(res)}(\pi)$ of partial waves with order $m = 1,2,3,4$ by a porous graphite sphere in iron matrix at frequency range $k_1a = 0-5$ ($\beta = 0.2$)

in iron matrix. It is clear from these plots that the shapes of spectra $a^{-1}f^{pp}(\pi)$ and $a^{-1}f^{ps}(\pi)$ differ obviously from each other. Since a backscattering spectrum is a linear combination of the scattering spectra of all the partial waves, precise discernment of the resonance properties of the porous sphere is difficult. To better understand the resonance of porous spheres, we have calculated the resonance scattering spectra $a^{-1}f^{pp}_{m(res)}(\pi)$ and $a^{-1}f^{ps}_{m(res)}(\pi)$ of some partial waves by using Eq. (4), as shown in Fig.3

In the calculation the backscattering spectrum of a cavity is taken as the background. As seen in Fig.3, two kinds of spectra are very similar and reflect very clearly the resonance phenomenon. Physically, the resonance frequencies in the resonance scattering spectrum are the same as those in the resonance vibration spectrum, because resonance scattering occurs only when the frequency of incident wave is identical with one of the resonance frequencies of scatterer. It is shown in Fig.3 that there are many peaks in a partial wave scattering spectrum of any order, say order m, so two labels (namely m and l) are needed to define a resonance mode of a spherical scatterer.

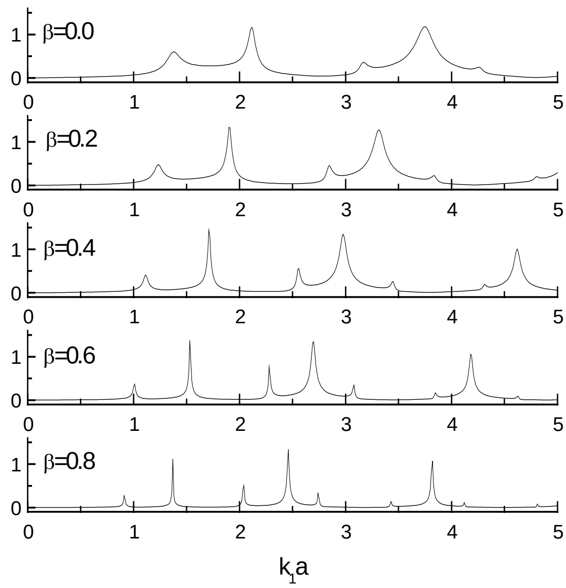

Fig.4 Variation of spectrum $a^{-1}f^{pp}_{1(res)}(\pi)$ by a spherical graphite scatterer in iron matrix against porosity β

Using Eq.(4), we have calculated the backscattering spectra for the partial wave of order 1 ($m = 1$)by a spherical graphite scatterer with different porosities in iron matrix at frequency range up to $k_1 a = 5.0$ (see Fig.4). The backscattering spectra are found to vary greatly with changing porosity and the resonance frequencies can be determined. To get more information about porous spheres in composite, we have computed the normalized resonance frequencies $\omega_{res} a / v_{L1}$ of some different modes ($m = 1, l = 1, 2, 3, 4, 5$), which are gathered in Fig.5 (v_{L1} is the longitudinal wave speed in the matrix). It is found that the values of $\omega_{res} a / v_{L1}$ decrease linearly as β increases for the examined vibration modes. These results may provide us a physical foundation for properly using ultrasonic scattering technique to evaluate porous particles in composite materials based on the positions and shapes of resonance modes.

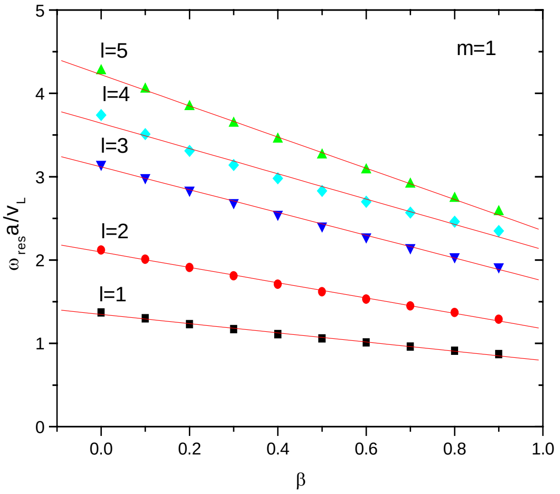

Fig.5 Normalized resonance frequencies of some different modes by a spherical graphite scatterer in iron matrix

Conclusion

In this paper a physical model is briefly described for studying the acoustic scattering by spherical scatterers in elastic solid. A special attention is paid to the resonance scattering of a porous sphere with changing porosity. The numerical results show that the resonance peaks in the scattered partial waves of each order are closely related to the porosity of spherical graphite scatterer. The resonance frequencies move to the lower values with increasing porosity. Therefore experimental measurements of the backscattering spectra and resonance frequencies may be used in evaluation of the porosity of spherical graphite in solid matrix.

The resonance frequencies of a layered sphere in free state were calculated by Chen and Ding[8]. If a vibrating sphere (or shell) is loaded with liquid or solid, the calculations of resonance frequencies are complicated because solution of complex roots of a characteristic equation should be dealt with. The resonance scattering method discussed in the paper is simple for determining the resonance frequencies of a loaded sphere.

Acknowledgement

This work is supported by the National Natural Science Foundation of China under Grant No. 10174037.

References

[1] L. Flax, G. C. Gaunaurd and H. Überall, Physical Acoustics XV, Ed. W. P. Mason and R. N. Thurston, Academic Press, New York, 1980.

[2] G.C.Gaunaurd, H.Überall, "Identification of cavity fillers in elastic solids using the resonance scattering theory," Ultrasonics, vol.18, pp. 261-269 1980.

[3] J.P.Sessarego, J. Sageloli, Régine Guillermin, "Scattering by an elastic sphere embedded in an elastic isotropic medium," J. Acoust. Soc. Am., vol. 104, pp. 2836-2844, 1998.

[4] E. Ruffino, P. P. Delsanto, "Scattering of ultrasonic waves by void inclusions," J. Acoust. Soc. Am., vol.108, pp. 1941-1945, 2002.

[5] Zhenyao Liu, Xixiang Zhang, Yiwei Mao *et al*, "Locally resonant sonic materials," Science, vol. 289, pp. 1734-1736, 2002.

[6] C.F.Ying , R.Truell, "Scattering of a plane longitudinal wave by a spherical obstacle in an isotropic elastic solid," J. Appl. Phys., vol. 27, pp. 1086-1097, 1956.

[7] R.M.Christensen, Mechanics of Composite Materials, John willey & Sons Inc., 1979.

[8] W.Q.Chen, H.J.Ding, "Natural frequencies of a fluid-filled anisotropic shell," J. Acoust. Soc. Am., vol. 105, pp. 174-182, 1999.