EXPERIMENTAL RESEARCH OF AMPLITUDE DEPENDENT INTERNAL FRICTION EFFECTS IN SANDSTONE BAR RESONATOR

V. Nazarov and A. Radostin
Institute of Applied Physics RAS, Nizhny Novgorod, RUSSIA
radostin@hydro.appl.sci-nnov.ru

Abstract

The results of the experimental research of amplitude dependent internal friction effects (nonlinear losses, shift of a resonance frequency and sound by sound damping) in sandstone bar resonator are presented. The measurements were conducted on the first five longitudinal modes of resonator. The analytical description of observed effects is carried out within the framework of phenomenological state equations containing hysteretic and dissipative nonlinearity. To describe the nonlinearity parameter dependencies on frequency the rheological model of medium containing two types of defects is proposed. The defects of the first type have hysteretic and relaxation properties and defects of second type have nonlinear dissipative and resonance properties.

From comparison of experimental and analytical dependencies of nonlinear effects the values of hysteretic and dissipative nonlinearity parameters of sandstone are determined and some characteristics of defects (such as ratios of nonlinearity parameters and concentrations to the relative elasticity) are estimated.

Introduction

The results of experiments conducted with rocks and polycrystalline metals show that the similar media have strong acoustic nonlinearity containing, as a rule, hysteretic nonlinearity, and some of them have dissipative nonlinearity [1,2]. One of the most relevant and distinctive features of these two nonlinearities are their different dependencies on acoustic wave frequency: the hysteretic nonlinearity with increase of frequency decreases, and dissipative nonlinearity on the contrary grows. This circumstance, together with amplitude difference of hysteretic and dissipative nonlinearities, allows to separate their contributions to a development of different effects and to conduct experiments so that the contribution of this or that nonlinearity becomes dominant.

Cubic Hysteretic Nonlinearity

The experiment used a bar resonator made of coarse sandstone (the size of grains is about 0.2-0.3 mm), which was taken from a core extracted at an oil and gas production site. The length L of the bar was 27 cm and its diameter was d = 2 cm. A block diagram of the experimental setup is analogous to [2]. The bar is resonator with one soft and other rigid boundaries, the eigenfrequencies of longitudinal vibrations of this can be calculated as $F_p = \frac{C_0}{4L}(2p-1)$, where C_0 is the velocity of the longitudinal wave in the bar and p is the index of the longitudinal mode. The eigenfrequencies and Q factors of the first five longitudinal modes of the resonator at small excitation amplitudes are presented in Table 1.

Table 1: The eigenfrequencies and Q factors

p	1	2	3	4	5
F_p, kHz	2.1	6.2	10.2	14.0	18.2
Q_p	73	56	64	69	74

In the first series of experiments, a continuous wave was excited at every five modes in turn and dependencies of resonance frequency nonlinear shift ΔF_{nl} and nonlinear decrement m_{nl} on wave amplitude e_m in resonance were measured. These dependencies are shown in Figures 1 and 2. As seen from these figures $\Delta F_{nl}/F_p$ and m_{nl}/m_p are in proportion to e_m^2 ($m_p = 1/\Omega_p Q_p$, $\Omega_p = 2p F_p$).

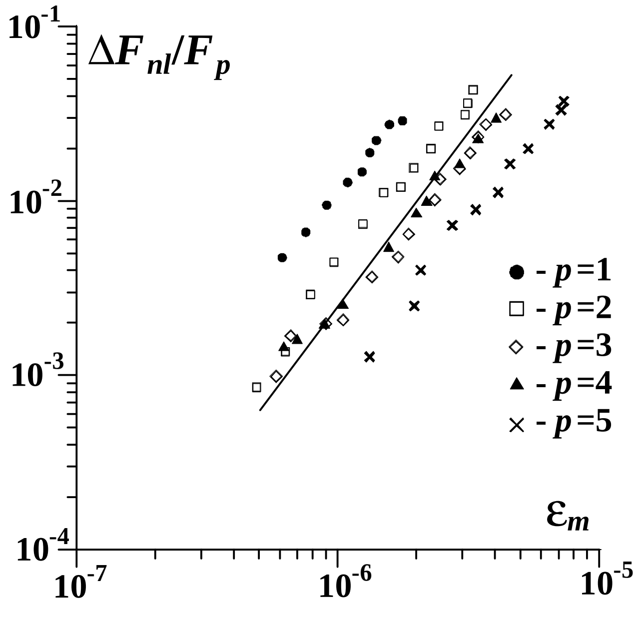

Figure 1 : Nonlinear resonance frequency shift $\Delta F_{nl}/F_p$ versus wave amplitude e_m for the five modes of the resonator

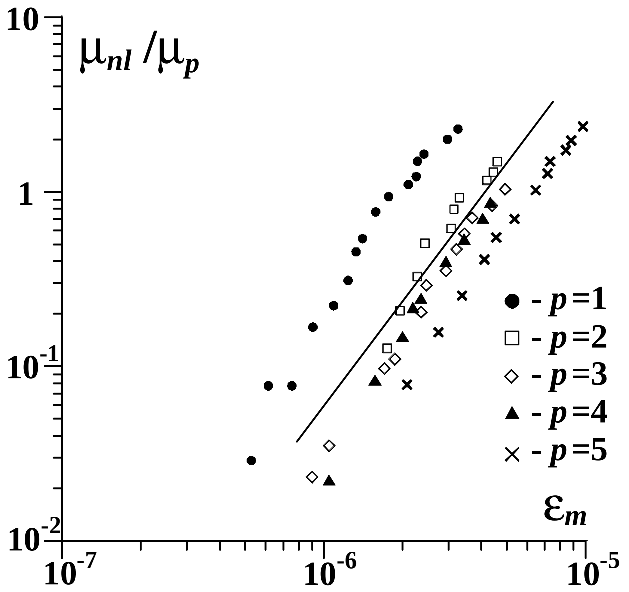

Figure 2 : Nonlinear decrement m_{nl} versus wave amplitude e_m

Such dependencies can be described in the frames of hysteretic state equation [2]:

$$s(e) = E[e - f(e,\dot{e})] + a r \dot{e}, \qquad (1)$$

$$f(e,\dot{e}) = \frac{1}{3}\begin{cases} g_1 e^3, & e > 0, \dot{e} > 0; \\ -g_2 e^3 + (g_1 + g_2) e_m^2 e, & e > 0, \dot{e} < 0; \\ -g_3 e^3, & e < 0, \dot{e} < 0; \\ g_4 e^3 - (g_3 + g_4) e_m^2 e, & e < 0, \dot{e} > 0, \end{cases}$$

where s, e, $\dot{e}$ are longitudinal stress, strain and rate of the strain, E is the Young modulus, a is the linear attenuation coefficient, r is the density. (Also the choice just of the cubic hysteretic function is conditioned by the experimentally revealed proportionality of second and third harmonics amplitudes to e_m^3). For such kind of resonators the expression for $\Delta F / F_p$ and m_{nl} / m_p have form [2]:

$$\Delta F_{nl} / F_p = -b_1 e_m^2, \quad m_{nl} / m_p = a_1 Q_p e_m^2, \qquad (3)$$

where $a_1 = \frac{1}{16p}(g_1 + g_2 - g_3 - g_4)$, $Q_p = C_0^2 / a\Omega_p$,

$$b_1 = \frac{1}{32}\left\{(g_1 + g_2 - g_3 - g_4) + \frac{3}{4}(g_1 - g_2 - g_3 + g_4)\right\}.$$

The comparison of the experimental results and analytical description (3) yields the values of the parameters a_1 and b_1.

Table 2:

p	1	2	3	4	5
$a_1 \cdot 10^{-9}$	13.4	4.8	2.9	2.7	1.4
$b_1 \cdot 10^{-9}$	70.4	29.6	15.2	14	5.3

As follows from Table 2, the nonlinearity parameters a_1 and b_1 become almost 10 and 13 times smaller with increasing of the wave frequency by a factor of 9.

To explain the experimental results we use the theory from [3], where the rheological model of microinhomogeneous medium, containing defects with relaxation, was proposed. Here we assume that defects have a relaxation frequency $W = V_1 E / h_1$, a non-dimensional (relative) compliance coefficient $V_1 << 1$ and cubic hysteretic nonlinearity and it state equation has form:

$$s(x) = V_1 E[x - f(x, \dot{x})] + h_1 \dot{x}, \qquad (4)$$

where x is the strain, h_1 is the viscous coefficient. In the case of small concentration of the defects, the state equation of the medium has form [3]:

$$s(e) = E\left[e - \int_0^\infty N_1(W) R[e] dW\right] - - EV_1 \int_0^\infty N_1(W) R\left[f\left(R[e]\right)\right] dW \qquad (5)$$

where $N_1 = N_1(W)$ is a distribution function of the defects under the parameter W, $R[...] = \frac{W}{V_1} \int_{-\infty}^{t} [...] e^{-W(t-t)} dt$. The state equation (5) yields expressions for $\Delta F_{nl} / F_p$ and m_{nl} / m_p, which have form (3), but parameters a_1 and b_1 depend on frequency:

$$a_1(\Omega_p) = \int_0^\infty \frac{N_1 \left\{ a_1 \left[1 - \left(\frac{\Omega_p}{W} \right)^2 \right] + 2b_1 \left(\frac{\Omega_p}{W} \right) \right\}}{\left[1 + \left(\frac{\Omega_p}{W} \right)^2 \right]^3} dW$$

$$b_1(\Omega_p) = \int_0^\infty \frac{N_1 \left\{ b_1 \left[1 - \left(\frac{\Omega_p}{W} \right)^2 \right] - 2a_1 \left(\frac{\Omega_p}{W} \right) \right\}}{\left[1 + \left(\frac{\Omega_p}{W} \right)^2 \right]^3} dW \qquad (6)$$

Figure 3 shows experimental and theoretical dependencies of parameters a_1 and b_1 on frequency F_p, when $N_1(W) = n_1 W^{-2} / \left(W_a^{-1} - W_b^{-1}\right)$, $W_a = 12.5 \cdot 10^3$ c^{-1}, $W_b = 12.5 \cdot 10^6$ c^{-1}. Figure 4 shows experimental and theoretical dependencies of Q_p^{-1} on frequency F_p. Comparison of these dependencies yields the estimation of the defect's parameters ratios: $n_1 / V_1 = 5 \cdot 10^{-2}, a_1 / V_1^2 = 2.6 \cdot 10^9, b_1 / V_1^2 = 2.6 \cdot 10^{11}$.

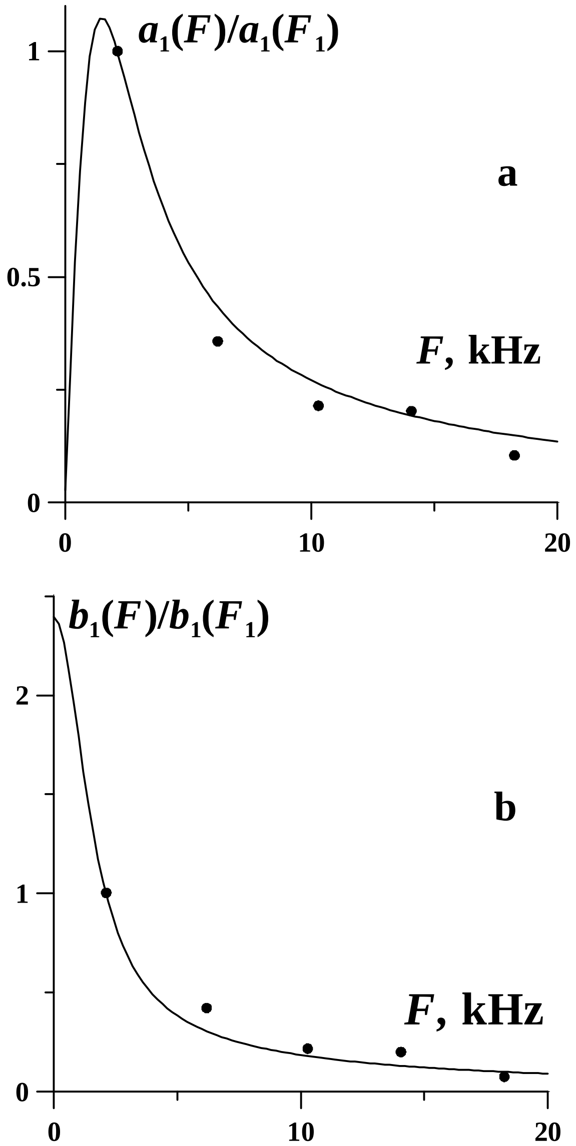

Figure 3 : Experimental (dots) and theoretical dependencies (curves) of parameters a_1 (a) and b_1 (b) on wave frequency F_p

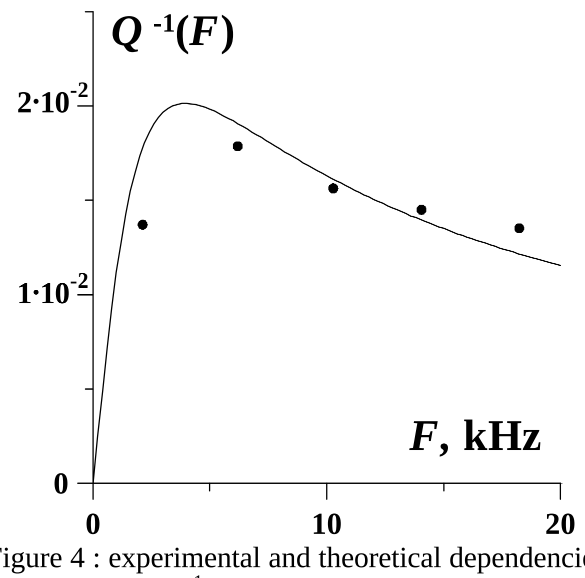

Figure 4 : experimental and theoretical dependencies of Q_p^{-1} on wave frequency F_p

Dissipative Nonlinearity

In the second series of experiments, a weak ultrasonic pulse was excited in the bar simultaneously with low-frequency pump wave at one of the five modes. When the strain amplitude e_m of the pump wave in the resonator increases, a decrease of the received pulse amplitude $U_2(e_m)$ was observed. The nonlinear damping coefficient $c(e_m) = \ln[U_0 / U_2(e_m)]$ (where U_0 is the amplitude of the pulse without the pump wave) was revealed to be independent on pump wave frequency F_p and in proportion to e_m^2 (Fig. 5).

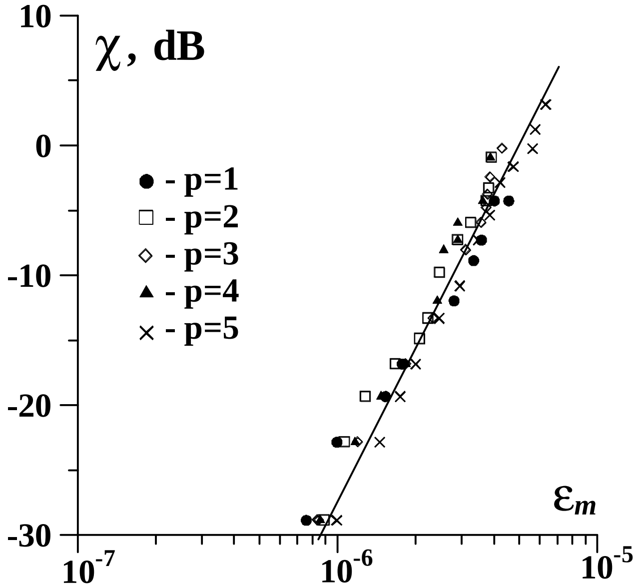

Figure 5 : The dependence of nonlinear damping parameter c on the pump wave amplitude e_m

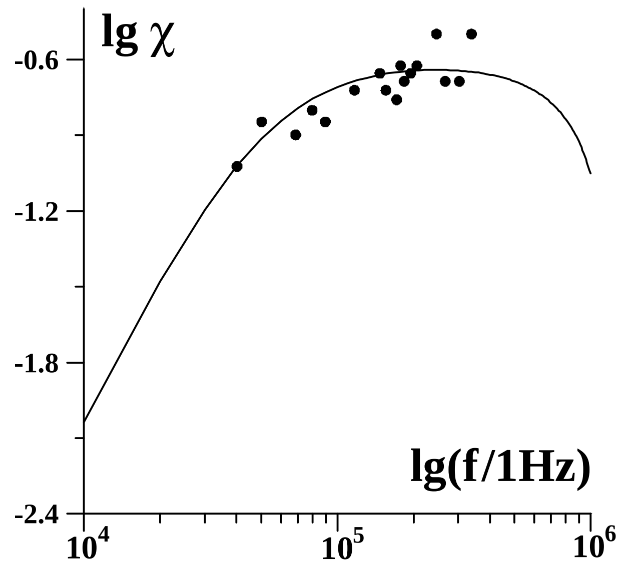

Figure 6 : The dependence of nonlinear damping parameter c on pulse frequency f

Figure 6 shows the dependence of the damping coefficient c on pulse frequency f (when pump wave amplitude was maintained at value $e_m = 3.3 \cdot 10^{-6}$ at first resonator mode).

As seen from this figure in the frequency range from 40 kHz to 400 kHz the dependence of nonlinear damping coefficient on frequency is approximately power, i.e. $c(e_m) \sim f^{1/2}$. Analogous dependencies were observed in [4].

Thus, our research have shown that hysteretic and dissipative nonlinearity of sandstone qualitatively differ from each other, when frequency of a pumping wave F_p increases the hysteretic nonlinearity decreases, but dissipative nonlinearity does not depend on this frequency, however, dissipative nonlinearity depend on the pulse frequency *f*. On the basis of this fact, it is possible to make the confirmation that the reasons of hysteretic and dissipative nonlinearities of sandstone are various and independent. In this connection, for the analytical description of sound by sound damping, generally speaking, other model and other equation of state may be used. However here again it is possible to use model of microinhonogneous medium from [3]. Follow [4] we consider a model of microinhomogeneous medium containing defects with dissipative nonlinearity and resonance properties. The state equation of these defects has form:

$$s(x) = V_2 E x + h_2 [1 + m |x|^n] \dot{x} + m \ddot{x}, \quad (6)$$

where V_2 is relative compliance coefficient ($V_2 << 1$), h_2 is the viscous coefficient, m is dissipative nonlinearity coefficient and m is reduced mass. In this case the state equation of medium has form (when $n = 2$):

$$s(e) = E\left[e - \frac{2}{V_2}\int_0^\infty \frac{w^2 N_2(w)}{l} S[e]dw\right] - \\ - \frac{8mdE}{V_2^3}\int_0^\infty \frac{w^6 N_2(w)}{l^4} S\left[S^2[e]\{dS[e] - lC[e]\}\right]dw \quad (7)$$

where $l^2 = 4w^2 - d^2$, $w = (V_2 E / m)^{1/2}$ is defect's resonance frequency, $d = h_2 / m$ is damping parameter, $N_2 = N_2(w)$ is distribution function,

$$S[...] = \int_{-\infty}^{t} [...] e^{-\frac{d}{2}(t-t)} \sin\left(\frac{l}{2}(t-t)\right) dt,$$

$$C[...] = \int_{-\infty}^{t} [...] e^{-\frac{d}{2}(t-t)} \cos\left(\frac{l}{2}(t-t)\right) dt.$$

The state equation (7) yields the dependence of nonlinear damping c parameter on pulse frequency f in case, when $\Omega_p << w$, $\Omega_p d << w^2$:

$$c = \frac{m d e_m^2 L w^2}{8 C_0 V_2^3} \int_0^\infty \frac{w^2\left[(w^2 - w^2)^2 - d^2 w^2\right] N_2(w) dw}{\left[(w^2 - w^2)^2 + d^2 w^2\right]^2}.$$

The dependence $c(f)$ is shown on the Figure 6, when $N_2(w) = n_2 w^{-1} / \ln(w_2 / w_1)$, $w_2 \le w \le w_1$, $w_1 = 4 \cdot 10^7$ c^{-1}, $w_2 = 10^9$ c^{-1}, $d = 5 \cdot 10^9$ c^{-1}, $e_m = 3.3 \cdot 10^{-6}$, $m n_2 / V_2^3 = 2.5 \cdot 10^{10}$. The estimation of the parameters n_2 and V_2 ratio yields: $n_2 / V_2 = 2,4 \cdot 10^{-1}$, $m / V_2^2 = 10^{11}$.

Conclusion

From outcomes of carried out experimental and theoretical research it is possible to draw a conclusion that a reasons of hysteretic and dissipative acoustic nonlinearity of sandstone are the defects of a different kind possessing applicable nonlinear relaxation and the resonance-frequency behaviour. In polycrystalline metals and rocks the role of such defects can be played, for example, by dislocations or their agglomerations, microcracks, boundary of grains. Generally speaking, for each particular medium these defects can be different and, certainly, one only acoustic measurements can not determine which defects respond for developments of this or that nonlinearity of medium. For this purpose engaging physical analogs of known defects and their nonlinear equations of state is necessary. Nevertheless, the described technique quite successfully allows to determine quality and quantitative characteristics and parameters of nonlinearity of these defects, that can be used for classification and diagnostic of different microinhomogeneous media.

Acknowledgements

The activity was supported by RFBR (project nos. 01-05-64417, 02-02-16237, 02-02-08021 INNO).

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