DETERMINATION OF THIRD ORDER ELASTIC CONSTANTS USING A SIMPLE ULTRASONIC APPARATUS

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Abstract:
The difficulty in measuring third-order elastic constants has been always in the design of the complex apparatus needed to generate both compressional and shear waves under uniaxial and hydrostatic pressures. Parameters, such as the angle of incidence have to be determined with accuracy using expensive devices. The apparatus described in this paper leaves just one variable to be determined with accuracy, namely, the time of flight of ultrasonic waves within the specimen under test. A variety of materials could be tested in this way, including rock specimens and concrete, thus enabling the evaluation of stresses in-situ within buildings and structures.

1-Introduction

Non-linear elastic moduli are important physical properties in conventional materials. They provide information about the inter-atomic bonding forces in crystalline solids.
Non-linear properties are also important in the non-destructive determination of applied and residual stress (strain) and several investigations have also established a possible relationship between non-linear elastic properties and ultimate strength in materials [1,2,3,4].

Changes in the velocity of ultrasonic waves brought about by the application of relatively high stresses are usually of the order of a few parts in ten thousand for most materials. Direct measurement of the transit-time of ultrasonic pulses may be made to about this level of accuracy with conventional techniques, but it is often more convenient and precise to use interferometric methods, or modern digital processing techniques.

In this research, an apparatus has been developed to determine the non-linear elastic moduli in a variety of engineering and building materials. This was accomplished by first determining the linear elastic moduli using a pulse-echo method. A special formula which relates wave velocities to transit-time measurements was derived.

2- Acousto-elastic theory

Murnaghan has shown that an elastic material initially in any state of stress other than a simple hydrostatic or compression cannot be elastically isotropic. Thus a body acted upon or containing a general stress system will exhibit anisotropic characteristics when ultrasonic waves are propagated through it [5].

By using the finite strain formulation of Murnaghan, Hughes and Kelly obtained five expressions for the stress dependence of the velocities of principal ultrasonic waves in initially isotropic materials related to simple cases of an applied uniaxial stress. Only three of the equations, which are of interest to us are shown below:

$$\rho_0 v_{lx}^2 = \lambda + 2\mu - \frac{C}{3K_0}\left(\frac{\lambda+\mu}{\mu}(4\lambda+10\mu+4m)+\lambda+2l\right)$$

$$\rho_0 v_{ly}^2 = \lambda + 2\mu + \frac{C}{3K_0}\left(\frac{2\lambda}{\mu}(\lambda+2\mu+m)-2l\right)$$

$$\rho_0 v_{sz}^2 = \mu + \frac{C}{3K_0}\left(2\lambda - m + \frac{n}{2} + \frac{\lambda n}{2\mu}\right)$$

In these relationships, ρ_0 is the density of the material in the initial unstrained state, v is the velocity of the ultrasonic wave, λ and μ are the familiar second-order Lamé's elastic constants for an isotropic material; l, m, n are the so-called Murnaghan third-order elastic constants for an isotropic material, C is the uniaxial compression acting on the material, and K_0 is the bulk modulus for an isotropic material in the unstrained state. The first subscript of velocity indicates the nature of the wave motion (longitudinal or shear), the second indicates the direction of applied stress (the x axis is parallel to the load C).

3- The apparatus

An immersion technique based on a pulse-echo method was used to design an ultrasonic apparatus for measuring elastic constants for a variety of materials. Both compressional and shear waves are generated by mode conversion in a prism-shaped specimen. The latter is fixed on a circular platform, whereas the transducer is made to rotate around the specimen. The whole set is put in a water tank (fig.1).

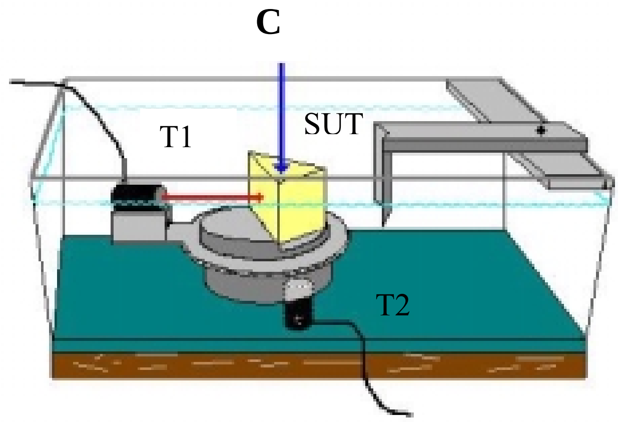

Figure1: The immersion unit.

T1: Immersion transducer; T2: Contact transducer; SUT: Specimen under test;
C: Compressive load. The angle arrangement allows for just one variable to be left in the equation that enables the computation of the velocity of ultrasonic waves:

$$v_{l,s} = \frac{\sqrt{2}R(t_2 - t_1)}{t_1(T_t - t_1)}$$

t_1 : Transit-time in water with specimen in place;
t_2 : Transit-time in water without specimen;
T_t : Total transit-time with the presence of specimen; R: Radius of circle made by the transducer around the specimen.

At the start of the experiment, the incident waves from the immersion transducer are made to impinge normally on the largest face of the prism. The cylindrically focused transducer helps to concentrate the energy of ultrasonic waves at the centre of the prism. Most of the energy is reflected from the face and the related echo gives the time-of-flight (t1) of the waves within the water interface, and also helps in measuring the thickness of the cube.

Next, the angle of incidence is increased slowly until the disappearance of the first echo and the emergence of a second echo related to compressional waves (Tt).

After that, the angle is increased further, a third echo appears, and this time it is related to shear waves within the specimen.

Finally, the velocity of compressional waves is cross-checked by using a contact-transducer put on top of the specimen (on one of the parallel faces). The whole apparatus is shown on fig.2 (left). In order to evaluate third-order elastic constants, the immersion cell is put in a compression unit, and the same measuring procedure is followed again but this time with the specimen under uniaxial pressure (fig.2, bottom).

The accuracy of measurements depends only on one parameter, namely the time-of-flight of ultrasonic waves. It could be determined with high precision using well-known signal processing techniques, such as the overlap, cross-correlation or cepstrum algorithms.

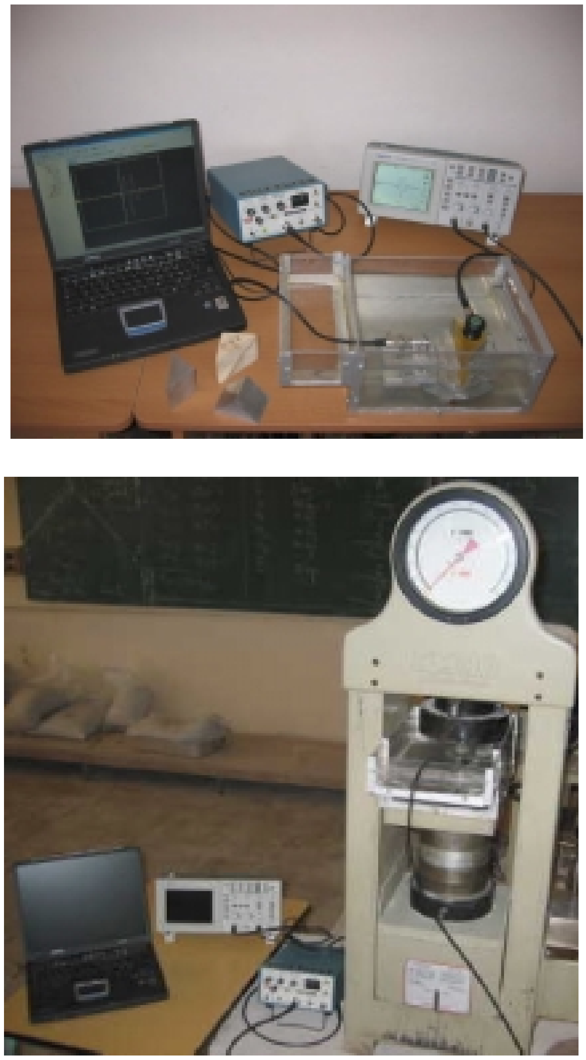

Figure2: General view of the ultrasonic apparatus for evaluating both second (top) and third order elastic constants (bottom).

Table1 shows some preliminary results related to second order elastic constants for a specimen of aluminum 2017A.

	ρ_0 (Kg/m^3)	V_{l0} (m/s)	V_{s0} (m/s)	E (Gpa)	K_0 (GPa)	σ
Aluminum (2017A)	2800	6305	3009	70	103	0.39

Table1: Second-Order elastic constants.

Measurements are being carried out in order to determine higher order elastic constants in a variety of materials (fig.3), before investigating stresses in-situ, which would require, in its simplest form, only the measurement of the velocity of compressional waves from a hammer impact on the part of the structure to be tested.

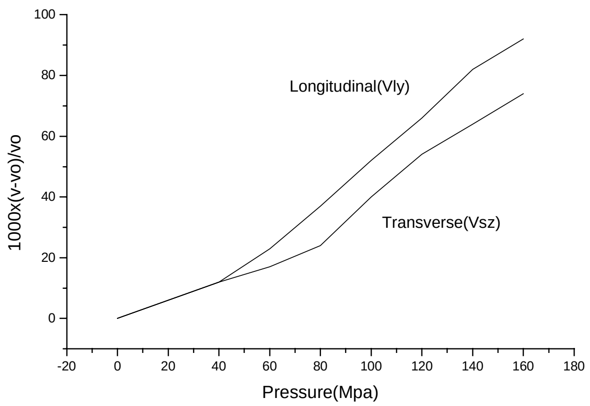

Figure3: Pressure dependence of transverse(Vsz) and longitudinal(Vly) waves.

Results are in good agreement with those obtained in [2]. Since signal echoes were strong enough, there was no need for signal processing techniques, and transit-times were measured by means of the cursors of the digital oscilloscope. A definition of 1nano-second is achieved by this technique, which is largely enough for detecting the small changes in wave velocity.

4- Conclusion

This research provides initial measurements related to the characterization of the non-linear elastic properties of materials using a simple ultrasonic apparatus. These will provide the basis for studying the relationships between non-linear properties and the more important engineering properties such as ultimate strength, residual strength after impact, and fatigue loading. The results related to second order elastic constants were obtained from the computation of both longitudinal and shear wave velocities using a single formula with just one variable: the transit-time of ultrasonic pulses. The results obtained are in good agreement with those found in the literature and there is the potential of making high precision transit-time measurements after calibrating the instrument and using digital signal processing techniques, such as cross-correlation.

5- References

[1] L.G. Hwa et al, "Pressure and temperature dependance of elastic properties of ZBLAN glass", J. Materials Chemistry and Physics, Vol.74, pp. 160-166, 2002.

[2] D.E. Bray, "Current directions of ultrasonic stress measurement techniques",15th WCNDT, Roma 2000.

[3] M. Landa, "Ultrasonic techniques for non-destructive evaluation of internal stresses", 15th WCNDT, Roma 2000.

[4] J.S. Popovics, "Determination of elastic constants of a concrete specimen using transient elastic waves", J. Acoust. Soc. Am., Vol.98, pp.2142-2148, 1995.

[5] RT Smith, "Stress-induced anisotropy in solids: The acousto-elastic effect", Ultrasonics, pp.135-147, July-September 1963.