Numerical Simulation of the Production of Pedal Notes in Brass Instruments

L. Velut[a], C. Vergez[a], J. Gilbert[b] and F. Silva[c]

[a]LMA - CNRS, 31, chemin Joseph Aiguier, 13009 Marseille, France

[b]LAUM - UMR CNRS 6613, Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France

[c]GIPSA-Lab, Domaine Universitaire, BP 46, 38402 St Martin D'Heres Cedex, France

velut@lma.cnrs-mrs.fr

Conical brass instruments, such as saxhorns, show an input impedance with almost harmonically distributed peaks, which allows to play notes whose frequencies approach a harmonic series. Conversely, all brass instruments with a long cylindrical section (notably trumpet and trombone) have their first impedance peak heavily shifted towards low frequencies (38 Hz for a trombone instead of 62 Hz if the first resonance frequency was belonging to the harmonic series formed by the upper peaks). However, trombonists can play the so-called "pedal note" despite its frequency close to 62 Hz, around a minimum of the input impedance.
Previous publication showed interesting numerical and experimental results when using alternatively a saxophone reed and a brass mouthpiece on both a trombone and a saxhorn.
In this paper, we try to reproduce numerically and extend the simulations results with a mouthpiece using Moreesc, a numerical tool based on the modal decomposition of the bore which allows changing parameters during the simulation. By simulating brass mouthpiece coupled with modal fits of measured trombone and saxhorn impedances, we examine this mysterious regime of oscillation. This allows to check the robustness of this phenomenon regardless of the numerical method. The influence of the number of modes taken into account is particularly investigated.
Keywords: Pedal note, Trombone, Saxhorn, Modal expansion, Moreesc

1 Introduction

When playing a brass instrument, most notes are played by exciting a specific acoustic mode of the bore of the instrument (corresponding to a peak of its input impedance). The musician selects the mode by adjusting the characteristics of the reed -his/her lips- and the static pressure in the mouth. However, this phenomenon does not seem to explain the possibility to play some notes, particularly the so-called "pedal note" of instruments whose bore profile is mostly cylindrical (e.g. trumpet, trombone). Due to this cylindricity, the lower mode of these instruments is shifted towards low frequencies : for the trombone, the resonance frequency of the first mode is around 38 Hz, instead of 62Hz if this mode was belonging to the harmonic series formed by the upper peaks (which occurs for more conical bores such as saxhorns); However, experienced trombone players can produce a note at a frequency close to the one which would match the "missing" peak in the impedance: this is called the pedal note. Figure 1 shows the difference between the impedances of the instruments used thereafter in the simulations.

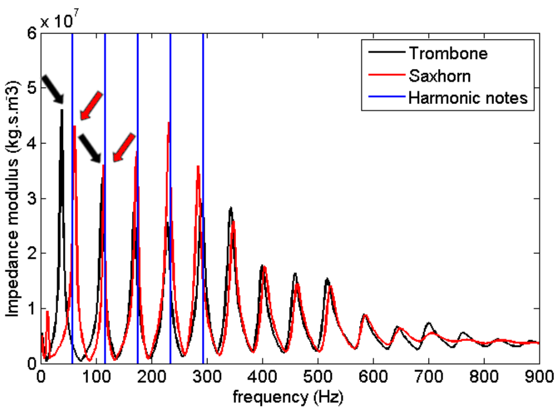

Figure 1: Module of the input impedance of a Saxhorn (red) and a Trombone (black). Blue vertical lines show the expected playing frequencies of first regimes. Arrows point the first and second modes of both instruments, the shift between first modes is easily noticeable.

Previous work about this particular regime of oscillation [1] carried out time-domain simulations of this regime with a quite simple model (reflection function of the instrument, coupled with a single DOF lips). On the contrary, in this paper, a modal model of the instrument impedance is used, with a variable number of modes taken into account. A single-DOF outward valve, non-linearly coupled with the bore, models the lips of the musician.

This paper is firstly about using a modal approach for brass simulation, different from Ref. [1] (which uses reflection function), validated by comparisons of oscillation thresholds and emerging frequencies for 1-mode resonators. Then, the flexibility of the modal expansion model is used to study the influence of the number of modes taken into account on the production of the pedal note.

After a description of the model and a presentation of the tools used in section 2, a set of trombone and saxhorn simulations are examined in section 3, with detailed discussion of the relations between the number of modes taken into account and the playing frequencies in the 4th section.

2 Model and tools

2.1 Modeling of the instrument

The model presented there have been widely used for woodwind [2] and brass instruments [3]. The musician's lips are represented by a single-degree of freedom oscillator, defined by the lips' resonance angular frequency ω_r (rad/s), the damping coefficient q_r, the lips stiffness K_r (N/m^4) and the opening area of the lips channel at rest S_0. This gives a relation between the opening area S (m^2) and the difference between the static pressure in the player's mouth p_{mouth} and the acoustic pressure at the inlet of the instrument $p(t)$ (Pa).

$$\frac{d^2S}{dt^2} + q_r\omega_r\frac{dS}{dt} + \omega_r^2(S - S_0) = \frac{\omega_r^2}{K_r}(p_{mouth} - p(t)) \qquad (1)$$

This represents a "striking outward" valve, which tends to open when the mouth pressure exceeds the mouthpiece pressure as described by Helmholtz [4].

As in Ref. [5], the input impedance of the instrument's bore is described as a sum of complex modes characterized by residues C_n and poles s_n:

$$Z(\omega) = \frac{P(\omega)}{U(\omega)} = \sum_{n=1}^{N} \frac{C_n}{j\omega - s_n} + \frac{C_n^*}{j\omega - s_n^*} \qquad (2)$$

A Courtois T149 and an old Couesnon saxhorn have been measured in Ref. [1] with the CTTM sensor [6], with the same brass mouthpiece. A sum of complex modes is fitted on these measurements to 4096 Hz with 15 modes for the saxhorn and 18 for the trombone (using a least-square optimization method). The parameters C_n, s_n for the five first

Modes	C_n	s_n	Q_n
	Saxhorn		
1	$5,5.10^8+j.2,96.10^6$	$-12,2+j.389,34$	$15,9$
2	$3,3.10^8+j.3,61.10^6$	$-10,3+j.715,90$	$34,9$
3	$5,8.10^8+j.5,51.10^6$	$-14,9+j.1092,4$	$36,6$
4	$9,5.10^8+j.7,35.10^6$	$-21,9+j.1457,2$	$33,3$
5	$8,6.10^8+j.8,99.10^6$	$-24,4+j.1783$	$36,6$
	Trombone		
1	$5,3.10^8+j.1,20.10^6$	$-12,9+j.238,62$	$9,3$
2	$5,8.10^8+j.3,54.10^6$	$-17,4+j.697,05$	$20,1$
3	$6,3.10^8+j.5,35.10^6$	$-22,2+j.1061,3$	$23,9$
4	$6,6.10^8+j.7,25.10^6$	$-26,1+j.1437,4$	$27,6$
5	$8,4.10^8+j.9,23.10^6$	$-28,8+j.1829$	31.8

Table 1: Values of the residues, poles and quality factor Q_n of the 5 first complex modes of each impedance

modes are given in Table 1, along with the corresponding quality factors $Q_n = |s_n|/(-2.Re(s_n))$ The number of modes considered for simulations can be chosen by the user.

The air flow $u(t)$ in the channel between the lips is supposed to follow Bernoulli's law [3]. Moreover, additional hypothesis on turbulent dissipation of the jet in the cup of the mouthpiece leads to :

$$u(t) = \sqrt{\frac{2}{\rho}} S(t) \sqrt{p_{mouth} - p(t)} \tag{3}$$

where $\rho = 1.1851 kg/m^3$ is the air density at 25°C.

2.2 Time-domain simulation with MoReeSC

All the time-domain simulations were carried out with a Python library previously developed in the L.M.A : MoReeSC (Modal Resonator - reed interaction Simulation Code) [7]. This freely distributed code was specifically developed for simulating reed or brass music instruments, allowing temporal evolution of most of the usually fixed control parameters (including acoustic and reed resonances, see Ref. [5]).

2.3 Linear stability analysis

A linear stability analysis of the model with a single acoustic mode has been conducted to estimate the oscillation pressure threshold and the corresponding emerging frequency (example in Fig. 3). Comparisons between this theoretical analysis and time-domain simulations with 1 mode showed matching oscillation thresholds and frequencies, and thus validated the simulation tool for both trombone and saxhorn.

3 Simulations

3.1 Methodology

The set of parameters has been taken from different sources, particularly [8]. As previously mentioned, the linear stability analysis guided the choice of a relevant blowing pressure. The following table presents the parameters used for all the simulations :

q_r	S_0	K_r	p_{mouth}
$4/\omega_r$	$8.12e-6m^2$	$\mu.\omega_r^2/L$	500 Pa

where $\mu = 4kg.m^{-2}$ is the surface mass of the lips, and $L = 14mm$ is the width of the lip channel.

Due to the lack of convincing values of lips resonance frequency in the literature (measurements seem particularly complicated), the following simulations were carried out using linear ramps of this parameter (slope = $4Hz/s$). For each simulation, the median value of the range of oscillation frequencies is supposed to be the "natural" playing frequency. As revealed by [9] the oscillation thresholds change whether the parameters are stationary or dynamic. We choose to simulate both increasing and decreasing resonance frequencies to consider the influence of the dynamic effects on the oscillation threshold. The main objective is to estimate the influence of the number of modes on the sounding of the trombone pedal note, so different simulations with a various number of modes taken into account are compared. They are also compared to saxhorn simulations, providing an instrument whose resonance frequencies are more harmonically distributed.

3.2 Description

Results of simulations of both a trombone and a saxhorn are presented, with a various number of modes : firstly, the maximum possible, which corresponds to the complete impedance (Figs. 4(a) & 5(a)); then 1 mode (Figs. 4(b) & 5(b)), 2 modes (Figs. 4(c) & 5(c)) and 3 modes (Figs. 4(d) & 5(d)) to see the influence of a growing number of modes.

Top plot of each figure represents the oscillation frequencies observed for increasing (blue) and decreasing (red) lip resonance frequency ($f_{lips} = \omega_r/2\pi$). Resonance frequency of 1^{st} and 2^{nd} modes are marked (the trombone's "harmonic peak", i.e. half of the second resonance frequency, is also shown). Then, the acoustic pressure in the mouthpiece is plotted along with its RMS shape, for increasing (middle) then decreasing (bottom) f_{lips}.

In first mode simulations (Figs. 4(b) & 5(b)), the oscillation frequency is (almost) driven by the lips resonance frequency; playing in tune with such an instrument would be very hard.

As the number of modes grows, a frequency plateau appears, which shows a more "natural" behavior : the oscillation frequency on one regime is mostly driven by the acoustical resonance, allowing small changes by varying the lips' parameters. For the trombone, a curious hysteresis appears in the oscillation frequency : the playing ranges highly depends on the direction of the f_{lips} slope (Figs. 4(a) & 4(d)). Starting from 5 modes, a maximum on the RMS ratio appears, particularly clear for descending slopes (Figs. 4(a) & 5(a)).

3.3 Comparison of normalized frequencies

For each simulation of the first and second regime, the oscillation frequency f_{osc} is calculated as the median frequency played (f_{mean} in Table 2), for upward and downward slopes of f_{lips}. The ratio between this frequency and the acoustic mode's resonance frequency allows comparisons between different resonators. The results are displayed in Figure 2 and reported in Table 2.

To stick with [1] measurements, similar analysis have also been made considering the oscillation frequency as the one where the RMS pressure of the signal reached its maximum (not shown here). This method show similar patterns; Moreover, it does not seem very relevant for simulations with a low number of modes, which do not show clear RMS peaks (Figs. 4(b) and (c), 5(b) and (c)).

Saxhorn's first regime (f_{mode} = 57Hz)				
mod	f_{mean}	f_{norm}	f_{mean}	f_{norm}
	Up		Down	
1	103,5 Hz	1,67	82,92 Hz	1,34
2	89,07 Hz	1,44	74,9 Hz	1,21
3	87 Hz	1,40	79 Hz	1,27
5	84,45 Hz	1,36	75,75 Hz	1,22
10	99,52 Hz	1,6	86,5 Hz	1,4
15	88,3 Hz	1,42	83,85 Hz	1,35
Trombone's first regime (f_{mode} = 38Hz)				
mod	f_{mean}	f_{norm}	f_{mean}	f_{norm}
	Up		Down	
1	88,12 Hz	2,32	55,3 Hz	1,46
2	77,02 Hz	2,02	60,65 Hz	1,60
3	72,82 Hz	1,91	59,97 Hz	1,58
5	69,4 Hz	1.83	61,08 Hz	1,60
10	68,5 Hz	1,80	59,87 Hz	1,57
18	67,62 Hz	1,78	58,20 Hz	1,53
Saxhorn's second regime (f_{mode} = 114Hz)				
mod	f_{mean}	f_{norm}	f_{mean}	f_{norm}
	Up		Down	
1	145,4 Hz	1,28	139 Hz	1,22
2	135,5 Hz	1,19	124,9 Hz	1,10
3	130,5 Hz	1,14	128 Hz	1,12
5	135 Hz	1,18	127 Hz	1,11
7	131,8 Hz	1,16	126,5 Hz	1,11
Trombone's second regime (f_{mode} = 111Hz)				
mod	f_{mean}	f_{norm}	f_{mean}	f_{norm}
	Up		Down	
1	154,5 Hz	1,39	132,8 Hz	1,20
2	138,9 Hz	1,25	129,8 Hz	1,17
3	136,9 Hz	1,23	128,3 Hz	1,16
5	136,4 Hz	1,22	127,8 Hz	1,15
9	136,9 Hz	1,23	127,8 Hz	1,15

Table 2: Oscillation frequencies from simulations of the 2 first regimes of both studied instruments. Note that for 2^d regime, only the modes which are harmonics of the second one are accounted

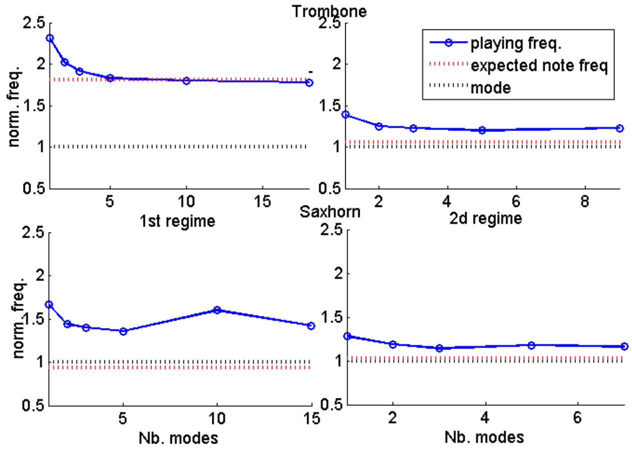

Figure 2: Simulated (normalized) frequencies for different number of modes taken into account. Dotted lines : frequencies of the expected note (red) and the acoustic mode (black). Top left : trombone 1^{st} regime; Top right : saxhorn 1^{st} regime; Bottom left : trombone 2^d regime; Bottom right : saxhorn 2^d regime

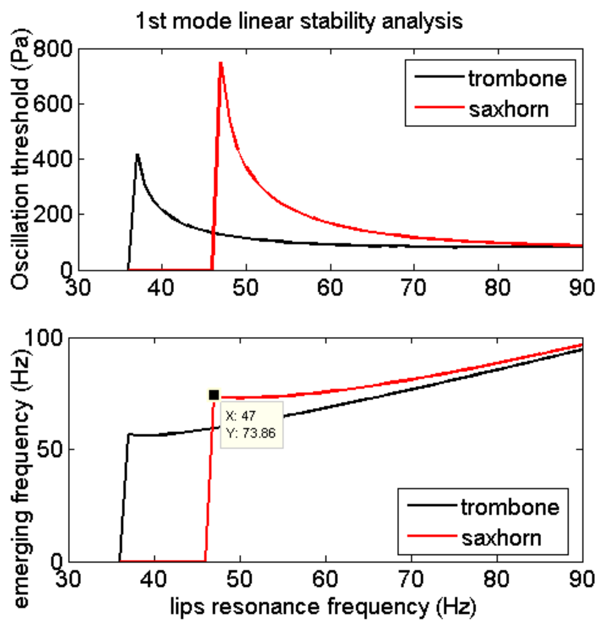

Figure 3: Results of Linear Stability analysis on first modes of the trombone (black) and the saxhorn (red). Top plot: the pressure thresholds of these modes (for different lips resonance frequencies). Bottom: the oscillation frequency at threshold.

4 Discussion

Observing the Figure 2, each instrument seems to have its own pattern with respect to the increase of N, for both regimes : trombone's oscillation frequency monotonously decreases to a fixed value, while saxhorn's one has no monotonicity.

For the trombone's first regime, the oscillation frequency decreases towards the "musically expected" note, somehow comforting the idea of the importance of higher modes for the "pedal note" regime of oscillation. Concerning the normalized frequencies values, this regime is clearly aside the three others, with a really higher f_{osc}/f_{mode} ratio, (circa 2).

For all simulations, the oscillation frequency is well above the acoustical resonance frequency; a well known limitation of the 1-DOF outward lips model [10] can explain this partially. Thus, the oscillation frequency of the saxhorn's first regime stays around a quite high mean value of 92Hz (normalized : 1.48 ±0.19), well above the expected note (B_b0 at 58 Hz). This is consistent with Linear Stability Analysis results (Fig. 3) showing minimum emergence frequencies of 74Hz (normalized: 1.19). The gap between the oscillation frequency and the acoustical resonance is particularly large on the first regime. The low quality factor Q_n of both instrument's first mode (see Table 1) could explain their large frequency ratio. Although the trombone's result matches experimental recordings (playing the B_b0 in tune), the saxhorn's oscillation frequency doesn't (Playing

E_b1 instead of B_b0).

The trombone's first regime, which we are particularly interested in, shows a peculiar behavior : the f_{osc}/f_{mode} ratio for this trombone regime is very high (2.3 to 1.8). When performing a linear stability analysis, the first mode's first emergence frequency is 56.6 Hz (norm. 1.49), corresponding to the "pedal note" phenomenon, with an oscillation frequency well above the acoustic resonance. This high f_{osc}/f_{mode} ratio appears even with a 1-mode input impedance for upward slopes of f_{lips}. This deserves further in-depth analysis, since the production of pedal notes is historically described as being the result of the collaboration between the higher modes (closest to the harmonic series)[11][12].

The results of downward slopes are a bit different: on trombone's first regime, the oscillation frequency decreases well below the resonance frequency of mode 1, which is intriguing (and pulls the median frequency down). This might be due to the effects of the dynamic parameters of the lips (varying f_{lips}) but has to be confirmed by further static simulations.

5 Conclusion

These simulations of sound production on the trombone and the saxhorn, using an outward model for the lips, reveal interesting results concerning the lowest playable notes on brass instruments (namely the 1st and 2nd registers). First of all, the trombone model is able to produce a Bb0 right in tune (a pedal note), though the 1st resonance frequency of the trombone is far below. This result had already been achieved in [1] with another numerical approach. Moreover, though it is commonly admitted in the litterature that the production of a pedal note relies on the cooperation of higher acoustic modes of the instrument, numerical experiments reveal that the reduction of the number of acoustic modes taken into account does not prevent the trombone model from playing a periodic regime with a f_{osc}/f_{mode} ratio close to 2, even when a single acoustic mode is considered. A second interesting result is that, regarding the 2nd register, both the trombone and the saxhorn models have comparable normalized playing frequencies, which is expected since input impedances have similar characteristics from their 2nd resonance frequencies.

However, questions arise from these numerical simulations. For example, the fact that the first register played by the saxhorn model is almost 6 semi-tones too sharp is questioning. This could be a limitation of the outward model. Following this hypothesis, one could wonder if what we called a pedal note in our numerical simulations is not a simple instability due to the coupling between the mechanical mode of the lips and the first acoustical mode of the trombone. The normalized frequency, larger for the trombone than for the saxhorn, could be explained by a quality factor 50% lower for the trombone. Thus, further work is required to confirm that the outward model is able to produce a pedal note.

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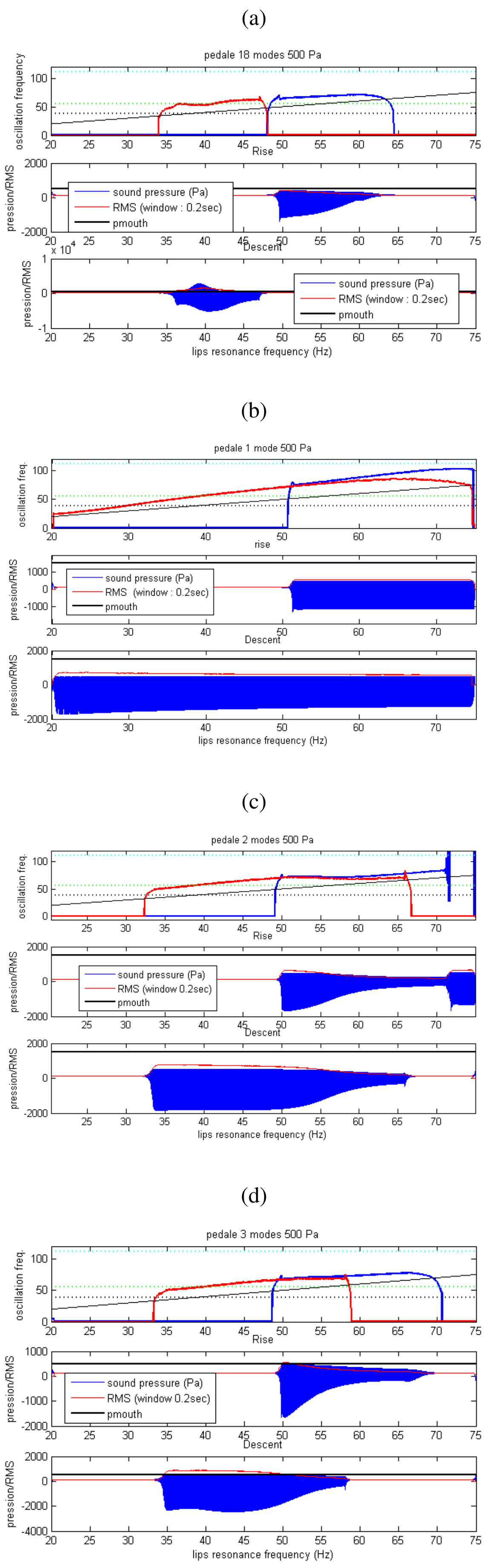

Figure 4: Trombone first regime simulations, $p_{mouth} = 500Pa$ with respectively 18 (a), 1 (b), 2 (c) and 3 (d) mode(s) taken into account. Top plot : f_{osc} for rising (blue) and decreasing f_{lips} (red), first (black) and second (cyan) acoustic modes, half of the second acoustic mode (green). Middle/top plots : radiated sound pressure and RMS envelope during f_{lips} rise/decrease.

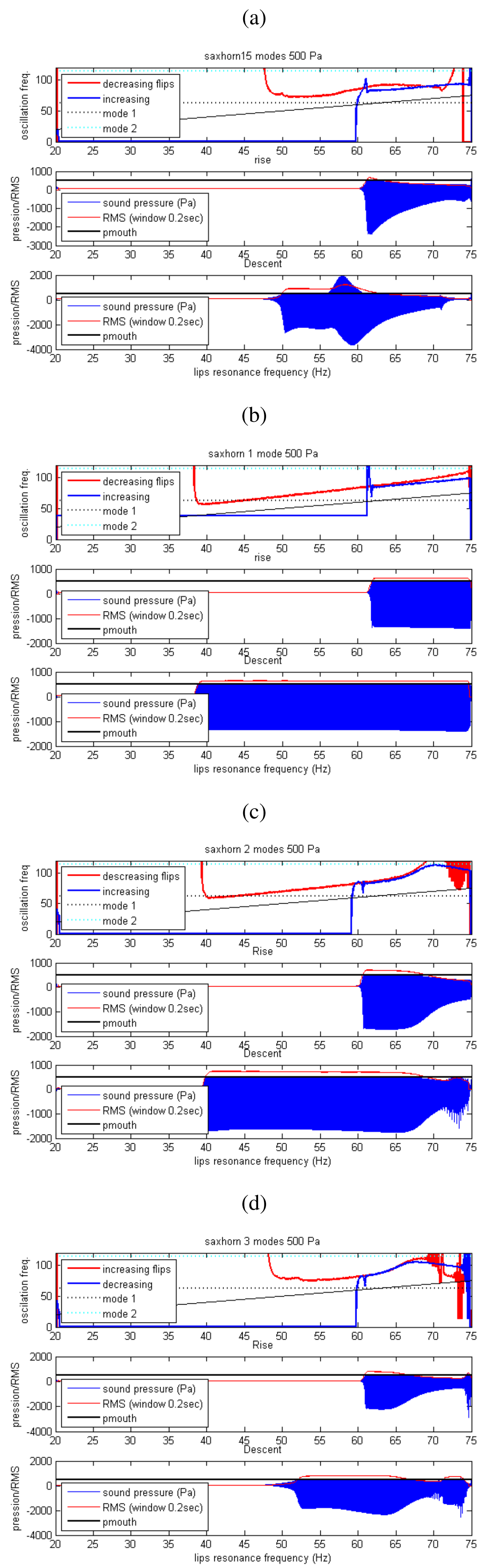

Figure 5: Saxhorn first regime simulations, $p_{mouth} = 500Pa$ with respectively 15 (a), 1 (b), 2 (c) and 3 (d) mode(s) taken into account. Top plot : f_{osc} for rising (blue) and decreasing f_{lips} (red), first (black) and second (cyan) acoustic modes. Middle/top plots : radiated sound pressure and RMS envelope during f_{lips} rise/decrease.