Assessing the perceived strike notes and tuning properties of two historical carillons based on the identified modes and empirical psychoacoustic pitch criteria

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Assessing the strike pitch of carillon bells is a difficult task, as this topic is connected with physical and subjective aspects, which cannot be easily departed. The perceived notes may pertain to actual modal frequencies but, as is well known, they often arise as virtual pitches. Furthermore, even if definite trends can be extracted from the results of subjective panel tests for evaluating the importance of the bell partials on the perceived pitches, results obtained along the carillon tessiture often display a significant dispersion. Therefore, criteria for assessing the bell tuning properties rely mostly on empirical data and are still a matter of debate. In this paper we address some of these issues, following our recent quantitative analysis of the two carillons of the Mafra National Palace, which represent the largest surviving 18th century carillons in Europe. Here, using a developed polyreference modal identification technique, we extend our previous modal identification results to include the bells higher frequency modes. Hence, for each carillon bell - which plays a separate note of the instrument -, we obtain charts displaying the frequency relationships between a set of its most prominent partials. Then, the topic of estimating the perceived pitch and tuning features of these musical instruments is addressed. We develop an optimization strategy for weighting the sets of identified modal frequencies with respect to several popular empirical criteria for strike notes, thus obtaining optimal estimations of the perceived bell pitches for the low register bells of the carillons. Following this evaluation, the perceived tuning features of the carillons are presented.

1 Introduction

The emitted tone of a bell when struck comprises a large number of partials which all combine together to create a resultant pitch. In spite of many efforts devoted to bell sounds, what the perceived pitch is after striking still constitutes an unclear issue which appears essential for assessing the tuning of carillons, where several bells sound in succession to play melodies and chords. Surprisingly, and as extensively reported in the literature, neither a bell partial is required in the sound spectrum where a pitch is perceived [1]. The actual confusion regarding the so-called *strike pitch* of a bell may certainly be attributed to several side-effects, both acoustical and psychological, among which the inharmonicity of the higher partials as well as the human auditory system may play a major role.

For a traditional minor-third bell, it is however commonly accepted that the strike pitch should coincide with the second lowest bell partial, referred as to the *prime* or *fundamental* by campanologists, even if such perception does not seem to have a direct relation with this partial [2]. Actually, for bells with mistuned partials, it is well known that the strike pitch often does not correspond to any bell modal frequencies [3]. Of particular interest, these confusing observations raise the question what criterion should be used for acoustical studies of the tuning of carillon bells.

Recently, in the framework of an interdisciplinary research project concerned with the restoration of the largest surviving 18th century carillon in Europe (Mafra, Portugal), the present authors developed a systematic approach for assessing the tuning properties of carillons and proved its effectiveness to reveal both acoustical and musicological features [4]. Based on an extensive modal identification of the bells comprising the instrument, the technique uses optimization strategies to assert properly the reference tone and musical temperament to which bell founders tuned their carillon. The results presented in [4] were obtained on the basis that the perceived pitch of bell stems from either one or a combination of its first five physical modes. By ignoring any perceptual features, such an attempt may seem crude at first sight but remarkably, one useful aspect of the proposed methodology is the possibility to handle easily different tuning criteria, either based on physical or perceptual concepts.

To develop a feel about the influence of the strike pitch on the tuning diagnosis of a carillon, we here extend our previous analysis by testing several empirical strike note criteria to a selection of bells of the two Mafra carillons. We focus on bells of lower pitch, with nominal frequencies in the range 400-1000 Hz, for which our modal identifications now include higher partials which are of strong perceptual importance, namely the *twelve* and *upper octave* [5–7]. We then compare three hypothesis for the determination of the strike pitch, assuming that the pitch heard of a bell corresponds to: *a*) half the frequency of *nominal* (Rayleigh's octave rule); *b*) a difference tone between *nominal* and *twelve*; and *c*) a virtual pitch at half the (missing) fundamental frequency created by the *nominal*, *twelve* and *upper octave*. After an overview on the perception of bell sound, the experimental modal identification and the optimization procedures which enables the estimation of the perceived reference pitch and musical temperament of a carillon are presented. The methodology is then applied to a selection of bells of the low register of the two carillons of Mafra, and results are discussed.

2 Overview on the perception of bell tones

The strike note of bells has been subject of great discussion in the literature. When a bell is struck, the perceived pitch, commonly know as strike note, may not correspond to any of the physical frequencies present in the sound spectrum. Despite the influence of carillon's tessiture in its perception, it is commonly accepted that, except for the small bells, the strike pitch is a virtual pitch effect typically found near one octave below the nominal [2]. Regarding the dominance region for its occurrence, it is commonly established that the most contributing frequencies are located between around 400-2000 Hz.

It was in the beginning of the 17th century that Jacob Van Eyck first mentioned the existence of a dominant tone in bells, which he named strike. According to him, all other tones could be referred to the strike by trained hearing. For example, if the strike is C_5, the desired series is: C_4-C_5-Eb_5-G_5-C_6 [8]. However, it was only in the end of the 19th century that the first scientific studies about this subject arose, raising a growing interest in several prominent researchers. Nevertheless, they have shared different perspectives about the subject, namely about how to calculate the strike pitch and what partials are associated to it. Summarily, four main theories can be identified.

One of the main theories on the strike pitch suggests that the strike note is an octave below the 5fth partial (*nominal*). This hypothesis, commonly known as the“octave rule”, was first proposed by Lord Rayleigh [3], the first to publish a systematic investigation on the strike note, in a study where he realized that the perceived pitch of several bells did not correspond to any of the experimentally measured frequencies. Later, other experimenters shared the same belief, namely Arts [9], Van Heuven [10] and Lehr [11]. In our days, this rule is still used in practice by most of the bell founders.

Nevertheless, other perspectives arise from different investigations. Some believe that the strike note lies close to a frequency given by the difference between the fifth and seventh partials [12]. Others associate it to a residual pitch, a missing fundamental originated by the *nominal*, *twelfth,* and *upper octave*, which normally have frequency ratios of approximately 2:3:4 [13–16]. More recently, Terhardt et al. [17] developed a new theory that takes in account spectral component patterns and the virtual-pitch theory, combining them in order to calculate the perceived strike pitch.

3 Experimental modal identification

3.1 In-field vibrational measurements

The tested bells were manually struck on their original support with an impact testing hammer, at several locations. For each bell, a mesh of 32 test locations regularly spaced near the rim was defined and impact excitation was performed on all of the points. The vibrational radial responses were recorded with three piezoelectric accelerometers, coupled to charge amplifiers, and the acquired time signals were 12 s long. The accelerometers were glued on the outer rim of the bell, in the same horizontal plane, at 3 positions. A Siglab/Spectral Dynamics acquisition board ensures the analog digital conversion. Care was taken to roughly control the frequency content of each impact excitation, and because of the large size of the bells, a specific instrumented impact hammer has been designed to ensure a proper excitation of the low frequencies. For each bell, a total of 96 impulse responses functions were analysed. Ten bells of the lower octave of the two Mafra's carillons were tested.

3.2 Modal parameter identification

Modal identification was achieved by implementing a sophisticated MDOF algorithm, called the Eigensystem Realization Algorithm [18]. The technique has been recognized as being very effective for the modal identification of complex systems. In particular, it provides good estimates of the modal parameters for structures presenting repeated eigenfrequencies due to the specifity of being a general polyreference multi-input/multi-output approach. The algorithm works in the time domain and is based on a state space formulation of the system dynamics. In essence, it attempts to identify a linear mathematical model to match the impulse responses of the structure by combining a set of free decay responses in the form of a generalized Hankel matrix and then uses a singular value decomposition to estimate the minimum order of the mathematical model. The last step of the algorithm consists in computing the eigenvalues of the chosen minimum model from which the modal parameters of the system are extracted.

Mathematically, the analytical model considered for the impulse response h_{ij} measured at j from an excitation at location i is given by:

$$h_{ij}(t) = \sum_{r=1}^{2R} \mathsf{A}_r^{ij} e^{\lambda_r t} \tag{1}$$

where λ_r are the complex eigenvalues and A_r^{ij} are the complex modal amplitude coefficients, R being the order of the original system. The modal frequencies ω_r and modal damping value ζ_r are extracted from the λ_r noting that:

$$\lambda_r = -\zeta_r \omega_r + j\omega_r \sqrt{1 - \zeta_r^2} \qquad (j^2 = -1) \tag{2}$$

Finally, the modeshapes of the system stem from the knowledge of the modal amplitude coefficients A_r^{ij} at each location along the rim, which can be either computed from the ERA realization matrices or by least-square fit with respect to a set of measurements.

In practice, the presence of noise in the input data perturbs the identifications and in general, it manifests through the identification of nonphysical modes. Consequently, the model order should be systematically overestimated and this makes delicate the estimation of the model size. To overcome such a difficulty, a stability diagram was implemented as part of the ERA algorithm. Tracking the estimates of the modal parameters, as a function of the model size, is a useful tool to assist in the selection of the system modes: indeed, the physical modes tend to stabilize at low model order whereas nonphysical modes do not stabilize at all during the process because of the random nature of noise. In addition, the identified modeshapes of the bell vibrations were also used as an indicator for the selection of the physical modes. Finally, the overall success of the estimation procedure is achieved by comparison of the synthesized and measured impulse response functions and transfer functions. To illustrate the satisfactory behaviour of the ERA-based modal identification technique, Figure 1 plots an example of a synthesized impulse response computed using 20 modes, compared with its respective measurement.

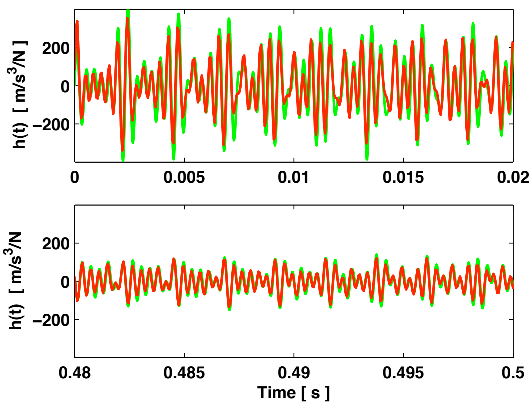


Figure 1: Details of the measured (green) and reconstructed (red) impulse response functions. 20 modes were identified.

4 Optimisation-based strategy for reference tuning estimation

Estimating the reference tuning and temperament of historical carillons is a rather delicate problem due to the less-than-perfect tuning of some bells forming the instrument. As presented in [4], the problem is addressed by devising optimisation strategies, and consists of adjusting a reference frequency to best fit a musical scale to a set of frequencies, either physical or virtual, stemming from the frequencies of the bells' partials. Mathematically, it corresponds to the minimization of an error ε_T which quantifies the overall detuning of the instrument with respect to a given temperament T, expressed as:

$$\varepsilon_T = \min \| \{\mathcal{S}_T\}\,\mathsf{F}_0 - \{\mathcal{F}_{meas}\} \| \qquad (3)$$

where $\{\mathcal{S}_T\} = \{s_1, s_2, \ldots, s_n\}_{n=1,\ldots,13}$ is a vector containing the musical intervals s_n of a given temperament T, F_0 is the reference pitch of the carillon to be adjusted, $\{\mathcal{F}_{st}\} = \{f_1, f_2, \ldots, f_N\}^t$ is a vector of the strike pitch of the carillon bells and $\|\,.\,\|$ is a suitable norm. In practice, the reference pitch F_0 is estimated in the least-square sense and solution is provided according to:

$$\mathsf{F}_0 = \{\mathcal{S}_T\}^{+}\{\mathcal{F}_{st}\} \qquad (4)$$

where the symbol $^+$ denotes the Moore-Penrose pseudoinverse [19]. By sequentially testing several temperaments in Eq.(3), a set of optimal reference pitches and corresponding fitting errors is obtained, and finally, the minimization of the set of errors provides the most plausible temperament as well as its corresponding reference pitch, to which the carillon is thought to be tuned.

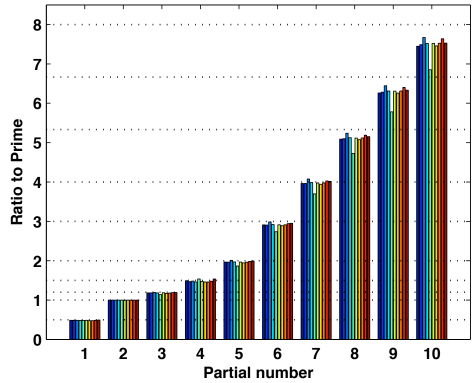
5 Internal tuning of the studied bells

Before testing any strike pitch theories for the tuning diagnosis of the Mafra's carillon, the proper relative tuning of each bell was analyzed in a systematic manner. We present here our modal identification results which include, for each bell, ten bell partials known to strongly contribute to the strike pitch sensation [14]. The frequency ratios of these partials are given in Table 1.

From the identified modal frequencies, one can analyze the relative tuning of individual bell by plotting partials in a manner which highlights the deviations from perfect tuning. A global view of the internal tuning of the bells of the low register of the two northern and southern Mafra's carillons is given in Figure 2, where tuning errors are referred to the identified physical *prime*. Because one partial is constituted by a pair of modes, internal deviations were computed using the mean frequency of the two modal components. Interestingly, Figure 2 attests a large difference in tuning qualities between the bells forming the two carillons, especially for the highest partials. Depending on the intensity of these partials in the bell sound, the influence of these higher modes may well contribute to clear audible differences between the two carillons, a result which supports the thesis of the poor tuning of the northern carillon (upper plot), built by N. Levache, which has already been evidenced by previous studies focusing on the first five physical modes only [4, 20]. Also, notice for the highest three partials the slight stretching of the partial series for the most of the bells from the southern carillon, cast by the well-known founder W. Witlockx (1669-1733), which has also been found in other church and carillon bells [2, 5].

		Ratio to prime r_n		
Partial numbers n	Partial names	Just	Equal	1/4 Meantone
1	Hum	0.500	0.500	0.500
2	Prime	1.000	1.000	1.000
3	Third	1.200	1.189	1.196
4	Quint	1.500	1.498	1.495
5	Nominal	2.000	2.000	2.000
6	Twelfth	3.000	2.997	2.991
7	Upper octave	4.000	4.000	4.000
8	Upper fourth	5.334	5.339	5.350
9	Upper sixth	6.667	6.727	6.687
10	Triple octave	8.000	8.000	8.000

Table 1: Relative frequency ratio of important partials for carillon bells.



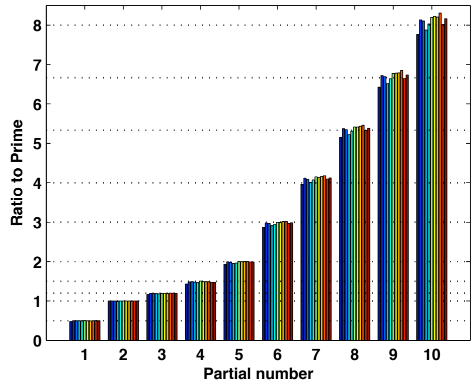


Figure 2: Relative tuning of important bell partials for the northern (up) and southern (bottom) carillons with respect to the Just temperament. Each bar refers to one bell.

6 Analysis of the Mafra's carillons tuning based on psychoacoustic pitch criteria

6.1 The Witlockx carillon

The southern tower of the Mafra National Palace contains one of the rare carillons built by Witlockx which is still in condition to be played. As presented in [4], its original bells have partial frequencies nearly in the ratios expected for traditional minor-third bells, especially for the low and middle registers. It therefore appears as a reliable carillon for examining how the strike pitch of bells may affect the carillon tuning diagnosis.

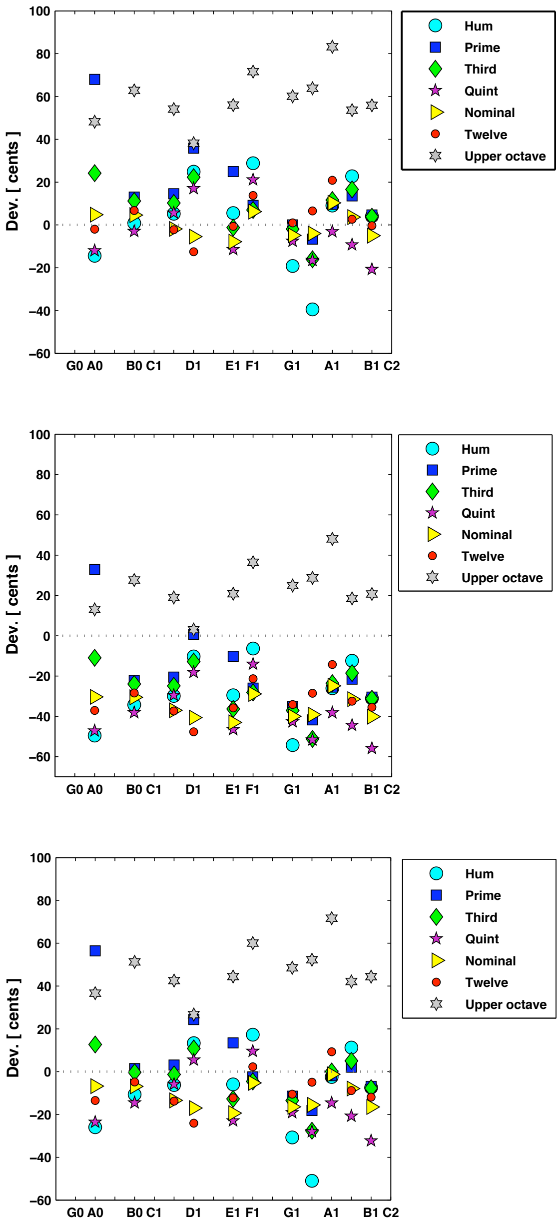


Figure 3: Witlockx carillon. Tuning deviations from 1/4-comma meantone temperament computed according to different strike pitch definitions. Up: half the nominal frequency. Middle: missing fundamental from the nominal, twelve and upper octave. Bottom: physical prime. Optimal reference pitch are given in Table 2.

In Figure 3, the tuning deviations obtained by considering two hypothesis for the determination of the strike pitch are displayed. The upper plot pertains to computations based on the octave rule - known to provide a good approximation in well-tuned bells [2] -, while the lower plot has been obtained by optimizing the perceived pitch of tthe carillon F_0 to the missing fundamental of the set of bells. If several missing fundamentals may occur in some very large bells [5], the missing fundamental considered here is the pitch created by the near-harmonic series formed by the *nominal*, *twelve* and *upper octave*, whose frequency ratios, with respect to the strike pitch, are close to 2:3:4 (see Table 1). To estimate the missing fundamental, a least-square fit of the identified frequencies of the three partials is used. Unlike the average distance between successive partials, this approach provides a means of accounting for the possible misalignment of the three partials. The pitch of the missing fundamental of a bell is thus obtained as:

$$\mathcal{F}_{st} = \{r_n\}^{+}\{f^{p_n}\} \tag{5}$$

where $\{r_n\} = \{r_5, r_6, r_7\}^t$ is a vector containing the bell partial frequency ratios according to a temperament, and $\{f^{p_n}\} = \{f^{p_5}, f^{p_6}, f^{p_7}\}^t$ is a vector of the identified modal frequencies of the partials *nominal*, *twelve* and *upper octave* respectively.

As displayed in Figure 3 for the 1/4 comma meantone temperament, the octave rule results, not surprisingly, in an overall good tuning of the *nominals*. Indeed, since only the *nominal* appears in the definition of the strike pitch according to this criterion, the optimization process adjusts the perceived frequency F_0 by minimizing the tuning errors of these bell partials only. Unlikely, for the partials discarded in the computations, large dispersions are obtained, especially for the *upper octaves* which are sharp by more than a quarter-tone for some notes. This is due to the less-than-perfect internal frequency relationships achieved by the founder between *nominals* and *upper octaves* (see Figure 2). For the missing fundamental criterion, one observes a net improvement of the tuning of upper octaves over the musical scale. Again, this shows the well-behaviour of the optimization process which now accounts for three partials, including the *upper octave*, for minimizing the overall detuning of the set of bells. Also displayed in Figure 3 are the tuning errors relative to F_0 stemming from an optimization based on the physical *prime* of the bells. Now, by comparing the results in Figure 3, notice that the dispersions of the bell partials over the musical scale only differ by a shift, i.e by the computed reference frequencies. This is easily explained as the choice of the criterion for determining the carillon's pitch does not affect the internal frequency relationships between the bell partials, which are governed by the tuning system, i.e the musical temperament. Consequently, performing different optimizations on the perceived pitch only change the values of the reference tuning F_0 (see Table 2 for the estimated F_0). The variations observed in the reference pitch can be easily related to the weight of the partials used in the definition of the strike note.

Because the influence of the physical *prime* remains unclear in relation to the qualities of bell sound, we report in Figure 4 the frequency deviations between: 1) the physical *prime* and the perceived reference pitch of the carillon. 2) the strike note of each bell and the perceived reference pitch of the carillon, and for each bell, 3) the physical *prime* and the strike pitch. Besides the smooth relative tuning observed between the *prime* and the reference pitch F_0 when one assumes the strike pitch to be one octave below the nominal, notice the proximity in frequency between the physical *prime* and the perceived strike pitch of the bells for tones above F_1. From the musical point of view, this closeness may strengthen the perceptual definition of the strike note of these bells. Also, notice that for the lower tones, the slight frequency difference between the physical *prime* and the perceived strike pitch may contribute to the occurence of beats in the sound of the bells. Finally, by comparing the

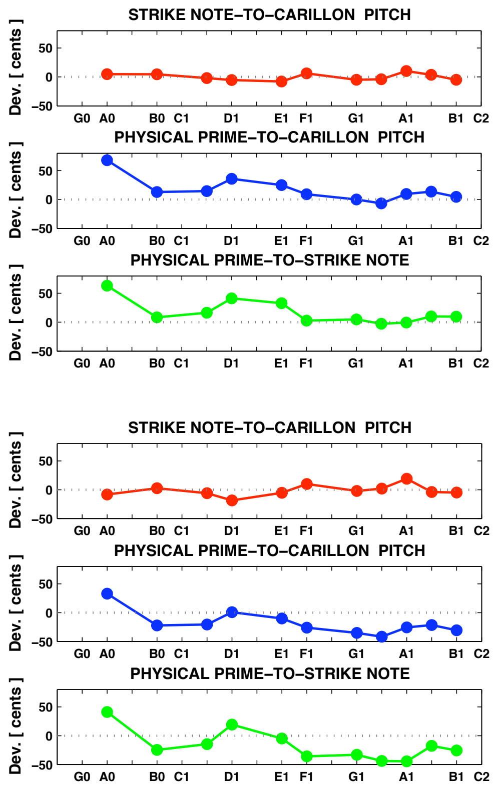


Figure 4: Witlockx carillon. Relative frequency differences between physical *prime* and perceived pitch of the carillon (red), strike note and perceived pitch of the carillon (blue), and physical *prime* and strike pitch (green). Up: half the nominal frequency. Middle: missing fundamental.

results obtained for the two perceptual criteria, one should observe that the relation *physical prime-to-strike note of a bell* changes according to the hypothesis considered for the determination of the strike pitch of a bell.

Regarding the tuning of the bells to an intended temperament, Figures 5 and 6 give the global tuning deviations of each partial, identified by its own color, in terms of statistical indicators computed over the low register of the instrument, for different temperaments. These plots offer a means of comparing, objectively, several temperaments to which the carillon would have been tuned for a given strike pitch criterion. Note, however, that tuning deviations are plotted with respect to different reference frequencies since the adjustement of F_0 depends on the assumed temperament. What is readily apparent from Figures 5 and 6 is that all temperaments produce a similar tuning deviation pattern once a strike pitch criterion has been defined. For the octave rule, the partials *hum*, *nominal* and *twelve* appear accuratly in tuned with the computed F_0 while the *upper octave* is raised by as much as 60 cents in average above its expected value for a well-tuned carillon. When we now turn to consider the missing fundamental criterion, optimizations result in a more homogeneous dispersion among the partials but actually any of the partials is perfectly in tune with the reference pitch. With such a result, one understands that any lower bell partial would convey the sensation of pitch of the strike, which would certainly results in an ambiguous sensation of pitch of the carillon.

Noting that the relative tuning between *nominal* and *twelve* is relatively good (see Figure 3), results for the difference tone criterion - which are not presented here - are nearly similar to those obtained for the octave rule. As a conclusion, it can be said that the presented results confort the conclusions presented in [4] where no preferential musical temperament for the Witlockx carillon has been determined. The main reason is that errors in tuning are larger than the stringent precision required to tune a carillon according to a given temperament.

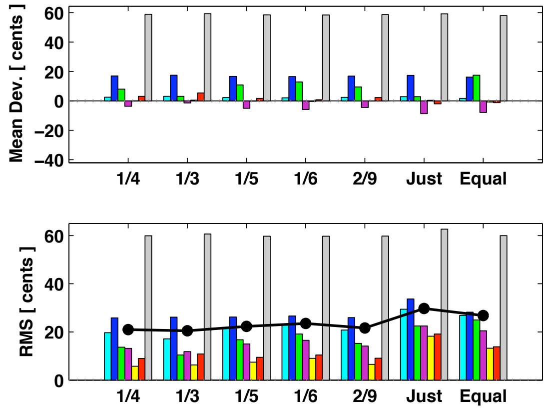


Figure 5: Witlockx carillon. Octave rule. Mean (up) and standard deviations (bottom) of the detuning of individual partials with respect to the perceived pitch of the carillon. Black dots in the lower plot stand for the mean of the standard deviations of the partials.

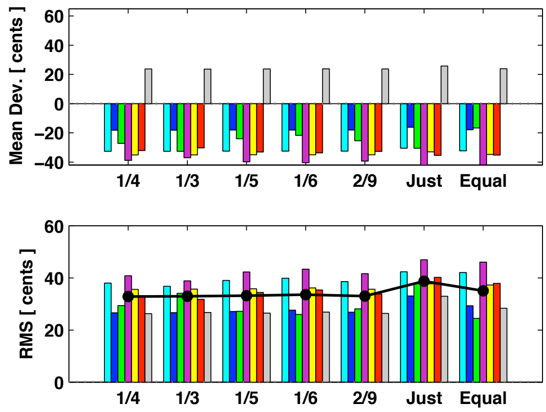


Figure 6: Witlockx carillon. Missing fundamental. Mean (up) and standard deviations (bottom) of the detuning of individual partials with respect to the perceived pitch of the carillon. Black dots in the lower plot stand for the mean of the standard deviations of the partials.

6.2 The Levache carillon

Figure 7 pertains to bells cast by N. Levache and shows the tuning deviations from the computed reference strike pitch of the set of bells, when one assumes the strike pitch to be one octave below the *nominal*. As seen, there is an important dispersion in the results (note the use of different scale for the y-axis compared to Figure 3), which means that bells are neither internally nor externally tuned with respect to the computed F_0. This confirms what could have been expected through a close examination of the relative frequencies of the bells partials displayed in Figure 2. Finally, Table 2 attests the difficulty to estimate accuratly the tuning of a carillon when the internal tuning of the collection of bells is not precise.

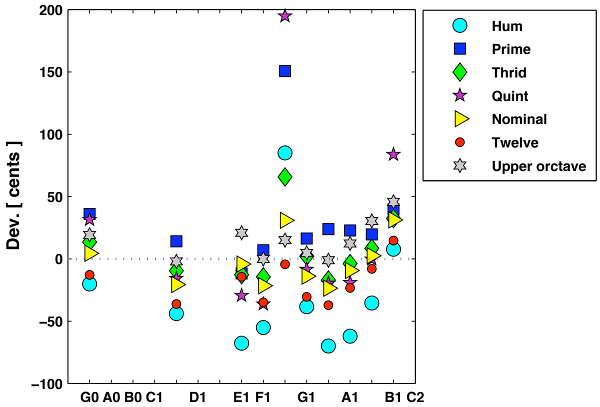


Figure 7: Levache carillon. Tuning deviations from 1/4-comma meantone temperaments. Octave rule. Optimal reference pitch computed is 427.6 Hz.

	Reference tuning F_0 (Hz)			
	Criterion (a)	Criterion (b)	Criterion (c)	Physical modes
Northern	427.6	411.8	428.7	435.7
Southern	395.5	394.9	403.6	396

Table 2: Reference frequencies of the set of bells from the Mafra's carillons computed for different criteria. Tuning is based on the 1/4-comma meantone temperament.

7 Conclusions

In this paper, we extend our previous analysis of the tuning properties of the Mafra's carillons by addressing the important perceptual strike note of the bells, which is a subjective tone heard when a bell is struck. From the modal identifications of the bells vibrational properties, we compare several approaches for the determination of the strike note, by focusing on the bells of the low register of the carillons. If the results of our optimization confirm the overall tuning qualities of the two carillons, they highlight that if the choice of a given criterion affect the estimation of the reference tuning of the carillons, it does not change however the dispersion in tuning observed along the musical scale, which is actually govern by the bells internal tuning. As illustrated for the Witlockx carillon, examining the tuning of a carillon with respect to the physical bell partials or a given perceptual criteria results in very small difference for the reference tuning of the musical instrument.

A popular view since the early 50s [21–23] is that, although not present in the response spectra, "virtual fundamentals" are connected with strong periodicities in the corresponding autocorrelation functions. Recent theories on pitch account for the amplitudes of partials, beyond their frequency information. This seems sound from a physical point of view, and we believe that the much used older concepts should be revised according to recent views on pitch from psychoacoustics. Our future work will follow such lines.

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