Mechanical Property Relationships in Sitka Spruce Soundboard Wood

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As a musical instrument construction material, wood is both musically and aesthetically pleasing. Easy to work and abundant, it has traditionally been the material of choice. It is not, however, without its challenges. As manufacturing of musical instruments continues to increase and supplies of suitable wood decrease, a need to maximize and optimize the use of available timber arises. Historically, distinct mechanical properties of wood have been compiled using separate samples of the same species. For any species, a great emphasis has been placed on describing average properties from a given region. Nonetheless, due to the great variation in wood properties even within a controlled region, manufacturing processes require direct measurements of the mechanical properties in order to construct acoustically consistent musical instruments. In this paper, non-destructive mechanical property tests have been developed so that they can all be performed on a single wooden specimen. In this way, relationships between mechanical properties of clear straight-grained quartersawn timber can be investigated. Measured properties include Young's modulus in the longitudinal and radial directions using a three point bending test, shear modulus using a two point square plate twist test and Poisson's ratios using a tension test, both in the longitudinal-radial plane, density and moisture content. Commonly used North American Sitka spruce of three different grades is studied. Relationships between various mechanical properties and other simplifications are proposed. These relationships are shown to reduce the number of measurements required by musical instrument builders wishing to construct acoustically consistent instruments.

1 Introduction

Wood as an engineering material has always presented challenges within the variability of its mechanical properties. Its highly variable nature is due to its growth which is at the mercy of its climactic and environmental surroundings [1]. Structural engineers have overcome these challenges by choosing the values for the mechanical properties of various species of wood to be the 5th percentile of the statistical bell curve associated with the given species' mechanical properties [2]. Although generally over-designed, this method works well when coupled with redundant structural design.

When using wood for its acoustic abilities, the 5th percentile cannot be used because acoustic properties are directly related to mechanical properties. Thus an exact knowledge of any given specimen's mechanical properties is an asset. Although, these can be measured, it is typically not economically feasible or convenient to do so on a per-specimen basis during manufacturing [3].

It has been observed in the guitar manufacturing industry that the soundboard's radial stiffness (Young's modulus, E_R) can be used as a measure of acoustic quality [4]. However, no scientific backing currently exists for this approach. Many studies have given the average range for a number of mechanical properties of wood, where measurements were made on separate pieces of wood of the same species and then averaged [1], [5]–[7]. McIntyre and Woodhouse discussed the measurement of the elastic and damping constants by interpreting the frequencies and Q-factors of the lowest modes of vibration of a wooden plate [8], [9]. Some studies have even taken a microstructure approach to mechanical properties [10], [11]. Nonetheless, very little is known with regards to the mechanical property relationships within the same wooden specimen.

Furthermore, a recent study has demonstrated the possibility of compensating for different mechanical properties of a wooden brace-plate system by adjusting the dimensions of the brace [12]. In doing so, it is possible to render the brace-plate system's acoustic properties consistent. Therefore, during manufacturing, a need exists for gathering mechanical property information. Any possible simplification to this procedure would be an asset.

Thus, in this study, a series of non-destructive tests are used in order to verify the mechanical properties of a number of wooden specimens. These tests measure Young's modulus in both the radial and longitudinal directions (E_L and E_R), as well as the shear modulus in the L-R plane (G_{LR}), the major and minor Poisson's ratio (ν_{LR} and ν_{RL}), the moisture content (MC) and finally the density (μ), based on the specimen mass at the given moisture content, rather than the typically measured specific gravity determined from oven dry wood.

In all cases, measurements are made on clear, straight grained and quartersawn Sitka spruce. This wood is chosen due to its common use in industry. Wood having been quarter sawn, and having grain as perpendicular to the surface of the plate as possible, has been chosen in order to limit grain angle as a variable during analysis. The Sitka spruce soundboards were obtained from Stewart-Macdonald (www.stewmac.com). Three bookmatched specimens of each grade (AA, AAA, and AAAA, based on their own grading system) were measured, for a total of nine samples. The soundboards were prepared using standard luthier techniques by gluing two bookmatched boards together and ensuring a perfect joint. This simplifies the analysis by creating symmetry in the mechanical properties of the soundboards.

2 Materials

Non-destructive testing is preferred so that the specimens can be used for future investigations after mechanical property measurements have been made.

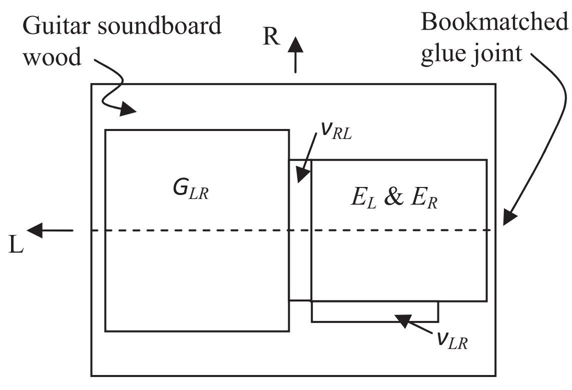

Figure 1: Sectioned soundboard.

In many tests for mechanical properties, damage can easily occur. Therefore, the tests, as well as the dimensions of the wooden plate have been carefully considered. Dimensions of the plate were chosen so that their natural frequencies can be measured and compared to those of a previous study [12]. Since this plate is rectangular and the square plate twist test [13] requires a square plate, the test plate was divided into sections as shown in Figure 1. The square plate marked with G_{LR} was used for the two point square plate twist test. Strips marked with ν_{LR} and ν_{RL} were used for tension tests. The rectangular plate section marked with E_L and E_R was used for the three point bending test and finally the remaining sections were used to measure moisture content and density.

3 Methods

3.1 Mechanical Properties

All mechanical properties were measured using non-destructive testing based partially on ASTM standard test methods for wood based materials, ISO standards for fibre-reinforced composite materials and on Static Test Methods for Composites [13]. Other references will be addressed in sequence. Great pain was taken to ensure that the best approach was chosen in each case.

3.1.1 Young's Modulus

Young's moduli in both the longitudinal and radial directions were measured using a three point bending test. A modified version of ASTM D3043-2011 for wooden structural panels, method A, was used [14]. Dimensions of the test rig were modified to account for those of the plate specified in section 2. Tarnopol'skii and Kincis [13] recommend that $s/h > 40$ for accurate measurements of Young's modulus, where s is the span between supports and h the thickness of the plate. Furthermore, although s and h values as given in the results section satisfy the recommended value, greater span length also increases the stringency with which the supports must be designed. The three point bending test rig setup is shown in Figure 2.

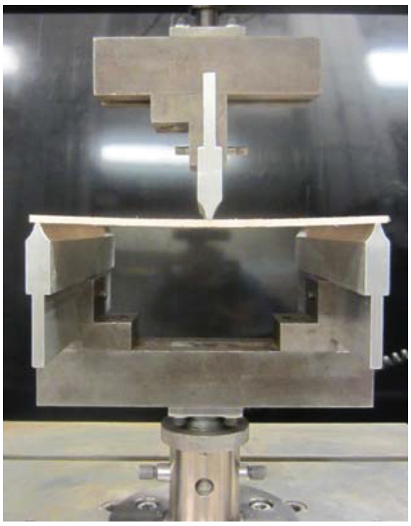

Figure 2: Three point bending test rig.

Based on an applied force of P and a flexural displacement of w, the Young's modulus, E_x, can be calculated from equation (1):

$$E_x = \frac{Ps^3}{48Iw_{\max}} \quad (1)$$

where I is the second moment of area of the cross section of the plate such that $I = bh^3/12$ and b is the depth of the plate.

Supports were designed in accordance with the study conducted by Ogorkiewicz and Mucci [15] which tested six different support types on fibre-plastic composite plates. Their conclusion was that supports having a supporting edge diameter of less than 3.5 mm but no smaller than 2.4 mm prevented specimen indentation of soft material and had a negligible effect on elastic modulus values due to span shortening during bending. Thus, a support of 3.2mm (1/8in) diameter was used. These tests were conducted using a span of 176 mm, thus it can be assumed that larger spans have even less of an effect. Roller supports were not considered because of the negligible friction effect at the supports.

3.1.2 Shear Modulus

Shear modulus, for the purpose of this study, was only measured in the *L-R* plane. Since non-destructive testing is sought, the two point square plate twist method was used. A modified version of ISO 15310-1999 for fibre-reinforced composites was used [16]. Although ASTM D3044-2011 for wooden structural panels [17] was also considered, it has been reported that the ISO method is easier to use and produces better results [18]–[20]. Dimensions of the test rig were modified to account for those of the plate specified in section 2. Tarnopol'skii and Kincis [13] recommend that $25 \le l/h \le 100$ and ISO recommends measurements be taken between $0.1h \le w_{load} \le 0.3h$, both of which were satisfied in this study. The span at the points of application (S) measured 328.34 mm. The two point square plate twist rig is shown in Figure 3.

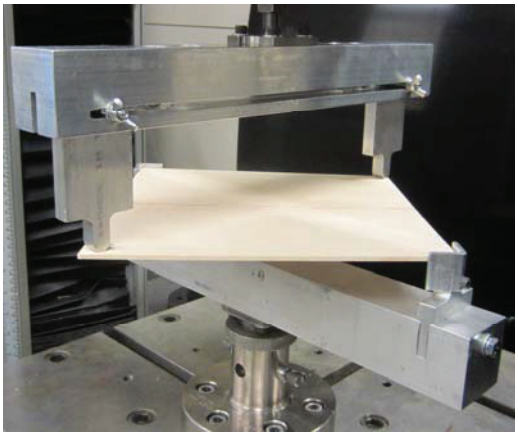

Figure 3: Two point plate twist test rig.

Under the applied load, the shear modulus can be calculated as,

$$G_{xy} = \frac{3Pl^2}{2wh^3} K \quad (2)$$

where l is the side dimensions of the plate and K is a correction factor to account for the application of the load at a position other than the corners. K can be calculated from the point of application span S and the diagonal dimension of the plate D using equation (3):

$$K = 3\left(\frac{S}{D}\right)^2 - 2\left(\frac{S}{D}\right) - 2\left(1-\frac{S}{D}\right)^2 \ln\left(1-\frac{S}{D}\right) \quad (3)$$

3.1.3 Poisson's ratios

Both the major and minor Poisson's ratios were measured in the *L-R* plane. To do so, two tension tests were used. The first applied the load in the grain's longitudinal directions, or parallel to the grain, while the second applied the load in the radial direction, or perpendicular to the grain. The tension tests were based on those proposed in [21]. Since ultimate strength was not measured, test specimens having a constant cross-section could be used. The test rig setup is shown in Figure 4.

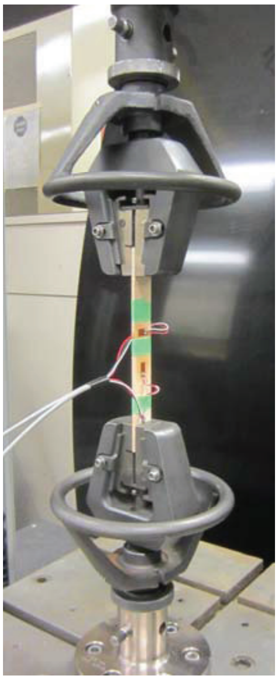

Figure 4: Tension test rig.

Micro-Measurements foil strain gauges (model CEA-06-250UW-120) with a gauge factor of $GF = 2.1$ were used to measure strain in the axial and transverse directions. Two strain gauges per direction were placed at at 0° and 90° to the load direction. Their layup can also be seen in Figure 4. Thus, strain in both the axial and transverse directions were calculated from

$$\varepsilon = \frac{\Delta R / R_G}{GF} \quad (4)$$

where R_G is the value of the strain gauge's undeformed resistance and ΔR, the change in resistance. Having found the strains, Poisson's ratios were calculated using the typical formula:

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_{tranverse}}{\varepsilon_{axial}} \quad (5)$$

3.1.4 Moisture Content

All wood used during the material testing was kiln dried to a moisture content of 6% and then air dried for a year. However, since wood's moisture content has such a large impact on its mechanical properties, a measure must be taken. An Electrophysics model MT808 pin type moisture meter was used having a stipulated accuracy of 0.1% between the ranges of 4-10%. Moisture measurements were made throughout the various property measurements in order to ensure consistent values.

3.1.5 Density

Density was measured at the given moisture content in order to ensure accurate subsequent frequency calculations. The density measurements were partially based on ASTM D2395-07 for wood-based materials [22], but without using the mass of the wooden specimen when oven dry. The specimen's volume was calculated from

$$V = L_L L_R h \quad (6)$$

and the density was then calculated as

$$\mu = \frac{m_{MC}}{V} \quad (7)$$

where m_{MC} is the mass of the specimen at the given moisture content.

3.2 Natural Frequencies

Natural frequencies were calculated using the measured mechanical properties and then compared to those calculated using material property simplifications. The theoretical natural frequencies can be calculated analytically using the typical plate equation [23]:

$$\omega_{m_x,m_y} = \frac{\pi^2}{\sqrt{\mu h}} \sqrt{\frac{m_x^4 D_x}{l_x^4} + \frac{2m_x^2 m_y^2 \left(D_{yx} + 2D_k\right)}{l_x^2 l_y^2} + \frac{m_y^4 D_y}{l_y^4}} \quad (8)$$

where

$$D_x = \frac{E_x h^3}{12\left(1-\nu_{xy}\nu_{yx}\right)}, D_y = \frac{E_y h^3}{12\left(1-\nu_{xy}\nu_{yx}\right)},$$
$$D_{yx} = \frac{\nu_{xy} E_y h^3}{12\left(1-\nu_{xy}\nu_{yx}\right)}, D_k = \frac{G_{xy} h^3}{12} \quad (9)$$

Natural frequencies were calculated for a plate measuring 3.29 mm in thickness and having side dimensions of $l_L = 240$ mm and $l_R = 180$ mm.

4 Results

The tests were conducted using an Instron machine model 4482. All tests were conducted on each specimen in sequence before moving on to the next test. Moisture content was verified on a regular basis in order to ensure consistent results.

Table 1: Experimentally measured mechanical properties of Sitka spruce.

Specimens	Moisture MC (%)	Grains ψ (grains/in)	Density μ (kg/m^3)	Shear G_{LR} (MPa)	Young's Modulus E_R (MPa)	E_L (MPa)	Poisson's Ratio ν_{RL}	ν_{LR}
Standard [1]	12	N/A	403.20	696.96	849.42	10890.00	0.04	0.372
1	9.1	17.61	404.86	954.27	1149.15	9875.45	0.077	0.533
2	8.5	16.83	387.26	653.60	962.83	12727.46	0.036	0.352
3	8.2	22.21	400.99	661.32	856.58	11984.12	0.052	0.411
4	8.4	14.45	400.45	734.94	849.70	11908.77	0.050	0.394
5	8.85	17.35	512.67	1027.95	1085.76	11393.27	0.056	0.491
6	8.3	19.16	396.16	711.74	717.91	13406.83	0.014	0.343
7	7.9	14.45	395.70	737.41	923.26	12412.55	0.063	0.383
8	8.25	24.02	433.05	823.81	796.81	12490.79	0.043	0.478
9	8.3	15.02	414.72	687.80	721.79	13069.35	0.040	0.396
Average	8.42	17.90	416.21	776.98	895.98	12140.96	0.048	0.420
Average (without 1, 5)	8.26	18.02	404.05	715.80	832.70	12571.41	0.042	0.394

4.1 Mechanical Properties

Results of the mechanical property tests are presented in Table 1 above. Specimens marked as standard, are the average values given by the US Forest Products Laboratory [1]. Specimens 1-9 are those that were specially prepared for this study and their average values are calculated in the second last row of Table 1 marked as "Average". It is interesting to note that all the experimentally calculated values, including their average are found to be close to those standard values. The one exception is the Young's moduli in the longitudinal direction (E_L) which is rather low compared to our experimentally obtained values. However, the Forest Products Laboratory also tabulates the average value of E_L for Canadian Sitka Spruce imported into the US as 12320 MPa, much closer to our values.

Specimens 1,4,7 are grade AA, specimens 2,5,8 are grade AAA and specimens 3,6,9 are grade AAAA. From the experimentally-obtained mechanical properties presented in Table 1, it is clear that there exists no direct relationship between the grading scheme offered by Stewart-Macdonald and the actual mechanical properties of the wooden specimens. This is immediately obvious from the very wide scatter of results. It is also not immediately obvious what relationships exist between any of these properties. However, a further in-depth study reveals certain possible simplifications.

The first observation that can be made is that specimens 1 and 5 vary considerably from their peers. Thus, these specimens were discarded. No obvious physical differences were observed in these two specimens except for the rather coarse grain of specimen 5 (thick latewood lines) compared to the rest. Further investigation is required to determine possible causes. With specimens 1 and 5 discarded, the average mechanical properties of the experimental specimens were recalculated. It becomes clear that there does not exist a large variation is both the major and minor Poisson's ratio. Thus, it is postulated that the average values can be used as a simplification. However, the average values obtained from the Forest Products Laboratory appear to be low for such instrument-grade wood. Therefore, we propose to use ν_{LR} = 0.394 and ν_{RL} = 0.042. This simplification will be verified by calculating the natural frequencies of a wooden plate having these properties. Furthermore, a simple relationship can be found between μ, G_{LR}, E_R and E_L. where the former properties are divided by the latter. These ratios are found in Table 2.

Table 2: Mechanical property ratios with regards to E_L.

Specimens	μ / E_L	G_{LR} / E_L	E_R / E_L
Standard [1]	0.037	0.064	0.078
4	0.034	0.062	0.071
7	0.032	0.059	0.074
2	0.030	0.051	0.076
8	0.035	0.066	0.064
3	0.033	0.055	0.071
6	0.030	0.053	0.054
9	0.032	0.053	0.055
Average	0.032	0.057	0.066

Although the ratios in Table 2 are not exactly the same, there is less variation in the ratios than what was expected based on current knowledge. Consequently, by using the average ratios for each material property and measuring only E_L, it would be possible to calculate μ, G_{LR} and E_R from $\mu = 0.032E_L$, $G_{LR} = 0.057E_L$ and $E_R = 0.066E_L$ for each specimen with a maximum error of 9%, 14% and 24% respectively. Although the errors are not negligible, of more interest to an instrument maker would be the effects of these calculated values on the natural frequencies. Thus, a frequency analysis was performed.

4.2 Natural Frequencies

Results of the fundamental natural frequency calculations are presented in Table 3 below. Values were calculated using direction L as x and R as y in equations (8) and (9).

Table 3: Fundamental frequencies of a spruce plate measuring 240 × 180 × 3.29 mm.

Specimens	Frequencies (actual) rad/s	Frequencies (with ratios) rad/s	Error %	Frequency (with ν(avg) and ratios) rad/s	Error %	Frequency (with ν(avg) and actual E_R) rad/s	Error %
4	1180.4	1194.8	-1.2	1193.3	-1.1	1201.0	-1.7
7	1213.3	1195.8	1.4	1193.3	1.7	1205.8	0.6
2	1222.8	1187.9	2.9	1193.3	2.4	1207.8	1.2
8	1183.2	1202.0	-1.6	1193.3	-0.9	1189.0	-0.5
3	1172.4	1196.8	-2.1	1193.3	-1.8	1201.2	-2.5
6	1181.5	1183.7	-0.2	1193.3	-1.0	1172.4	0.8
9	1160.1	1193.1	-2.8	1193.3	-2.9	1175.1	-1.3

Table 4: Partial frequencies using mechanical property simplifications (rad/s).

Specimens	Approx. m_L=1, m_R=2	Actual	Error %	Approx. m_L=2, m_R=1	Actual	Error %	Approx. mL=2, mR=2	Actual	Error %
4	2350.3	2310.9	-1.7	3904.1	3815.9	-2.3	4804.1	4721.8	-1.7
7	2380.4	2395.4	0.6	3906.9	3923.9	0.4	4823.3	4853.3	0.6
2	2392.9	2421.8	1.2	3908.1	3992.2	2.1	4831.3	4891.1	1.2
8	2273.5	2267.2	-0.3	3897.0	3822.9	-1.9	4755.8	4732.7	-0.5
3	2351.6	2294.8	-2.5	3904.2	3817.9	-2.3	4804.9	4689.6	-2.5
6	2165.1	2178.0	0.6	3887.3	3943.0	1.4	4689.6	4726.0	0.8
9	2183.2	2152.2	-1.4	3888.9	3861.9	-0.7	4700.5	4640.6	-1.3

In Table 3, the first frequency column represents the fundamental natural frequencies calculated using all the actual measured properties of Table 1. The second frequency column uses the mechanical properties calculated using the ratios of Table 2 and the actual Poisson's ratios. The third frequency column is similarly calculated using the ratios of Table 2 and the average Poisson's ratios of $ν_{LR}$ = 0.394 and $ν_{RL}$ = 0.042. Finally, the last frequency column is the same as the previous column but uses the actual values of E_R. It is clear that using the simplifications does not have a large effect on the natural frequencies, since the largest error is 2.9%. Using the average values of Poisson's ratio does increase the error, but only slightly. The advantages of using these values far out-weigh the disadvantages, especially since Poisson's ratios are the most time consuming to measure experimentally. Finally, using the actual values of E_R improves the natural frequency calculations and since these values are not more difficult to obtain than E_L, it is a recommended procedure. Unfortunately, it is generally desired to find natural frequencies within a 1% margin of the actual values [24], therefore the study is ongoing.

A consideration of the lowest three partial frequencies above the fundamental reveals a similar trend regarding error in the frequency estimations. These are tabulated in Table 4 for frequencies calculated using ratios of μ/E_L, G_{LR}/E_L, average values of $ν_{LR}$ = 0.394 and $ν_{RL}$ = 0.042 and actual values of E_R.

4.3 Discussion

Calculations were performed using D_{RL} rather than D_{LR} of equation (9) since it was found that errors were reduced by using this approach. It was found that the reduction in error stems from the fact that ν_{LR} is more consistent than ν_{RL}. Furthermore, an additional frequency analysis was performed by varying individual mechanical properties within the range of those experimentally obtained. In doing so, it was found that the values at the limit of the range for all properties significantly affected the natural frequencies, with the exception of Poisson's ratio. The maximum variation in frequency by varying Poisson's ratio was 1.5%. Thus, the use of a constant average values for Poisson's ratio as a simplification is justified.

Finally, grain density (ie. number of grain line per inch) was not a good indication of any mechanical property. Further work needs to be performed in order to experimentally measure the natural frequencies of the wooden plates for which the mechanical properties of Table 1 were obtained and to verify the results of the frequency calculations.

5 Conclusion

The goal of this study was to determine the existence of relationships between mechanical properties of clear straight-grained and quartersawn Sitka spruce. It was shown that using average values for the major and minor Poisson's ratios of $ν_{LR}$ = 0.394 and $ν_{RL}$ = 0.042 as well as using the relationships $\mu = 0.032E_L$, $G_{LR} = 0.057E_L$ and $E_R = 0.066E_L$ is a good approximation for calculating natural frequencies with an error not exceeding 2.9%. Results can be improved by using actual values of E_R. It is clear that further improvements must be made and this study is ongoing.

If wooden musical instrument manufacturers are to improve the acoustical consistency of their instruments, then knowledge of the mechanical properties of the wood is required. A reduction in the number of mechanical properties that require direct measurement is desired. Currently only a measure of E_R is taken. It is suggested that a measure of E_L also be taken. A simple three point bending rig could easily accommodate both measurements.

Acknowledgments

The authors would like to acknowledge the generous support provided by the Natural Science and Engineering Research Council of Canada.

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