



Collisions in Drum Membranes: a preliminary study on a simplified system

A. Torin^a and M. Newton^b

^aUniversity of Edinburgh, Room 1606, JCMB, King's Buildings, Mayfield Road, EH9 3JZ Edinburgh, UK

^bUniversity of Edinburgh, Acoustics and Audio Group, EH9 3JZ Edinburgh, UK

a.torin@sms.ed.ac.uk

Collisions play an important role in musical acoustics. Extensive study has already been performed on this subject for a number of systems, with notable examples being the hammer-string collision in the piano, and the mallet-membrane collision relevant in drums.

This paper uses experiments and modelling to investigate collisions in the snare drum. This is an interesting percussion instrument, as two collision mechanisms feature prominently. Initial excitation usually results from collision between a striker, such as a mallet, and the upper ('batter') membrane. Ensuing membrane and cavity vibrations then give rise to secondary collisions between the lower membrane and a number of metal wires ('snares') that are tensioned across it. These collisions are crucial for the sound qualities of the snare drum, and so a proper understanding of their nature is essential for accurate sound synthesis models.

In this preliminary paper, a simplified experimental setup will be studied, consisting of a drumskin with a single snare stretched across it. The system can be excited either by plucking the string, or by striking the membrane with a mallet. A setup involving a high speed camera and a laser vibrometer will allow the motion of the snare and membrane to be captured.

Experimental results will be used in comparison with a novel numerical model that describes both the mallet-membrane and string-membrane collisions. The interactions are described by a non-linear force expressed in terms of a power law, similar to one used in the past for modelling the hammer-string collision in pianos.

1 Introduction

Many musical instruments rely on collisions for the production of sound, and percussion instruments are probably the best example of this. Among the various components of this family, drums occupy a prominent position. Experimental research on drums is abundant, and a comprehensive review can be found in [1]. The collision mechanism between a drumstick and the drumhead has been studied in [2, 3], while investigation of the snare-membrane interaction in a snare drum has been reported in [4] and, more recently, in [5].

Another useful approach to musical acoustics is numerical simulation based on physical models, and a comprehensive review of the various techniques adopted can be found in [6]. Finite difference methods [7] are but one of the possibilities, and their use is now widespread. This technique has been applied to the simulation of timpani drums [8, 9], nonlinear double membrane drums [10] and snare drums [11, 12]. Other methods have also been successfully used for the modelisation of collisions in percussion instruments [13].

Simulation of collisions using finite difference methods is a difficult task, prone to numerical instabilities. One possible approach is the use of penalty based methods [14], in which some spurious interpenetration between the components is allowed. Several numerical schemes for the simulation of collision exist, but their stability is difficult to prove [15]. Energy methods can be applied in order to derive a stable scheme, but they lead to a nonlinear equation to be solved in the loop [16]. Existence and uniqueness of a solution in this cases is guaranteed by a recent result [17] in the lumped case, and has been extended to a variety of different configurations in [16].

In this paper, we will investigate the behaviour of a snare-like system composed of a single headed drum with a single metal string positioned across and in contact with the drum membrane. This is a simplified, and more controllable version of a snare drum, where multiple snares are in contact with the lower membrane of a double headed drum. Despite its simplicity, this system exhibits similar perceptual features and therefore constitutes an ideal test case for controlled laboratory experimentation. We will report preliminary experimental results and we will compare these with physics-based numerical simulations.

2 Experimental setup

2.1 A snare-like experimental model

The experimental work presented in this paper describes a preliminary investigation into the nonlinear contact dynamics of the snare drum. In order to make the experimental setup as controllable as possible, and to facilitate the comparison with a numerical simulation, a simplified snare-like system was constructed. The objective was to isolate the most important feature of the snare drum, namely the interaction between a snare(s) and a membrane, and to study this system in a carefully controlled and repeatable manner.

The 'drum' (membrane) element of the snare-like system was comprised of an Irish bodhrán, a single headed drum made from a 16-inch (≈ 40 cm diameter) piece of goat skin stretched over a circular wooden rim. The 'snare' was made from a single metal string, securely positioned across the diameter of the membrane, and, at rest, in contact with the membrane along its whole length. The snare was secured to the membrane by pinning it down on either side of the drum rim, under an approximately tensionless condition (though one experiment did involve application of additional snare tension, see section 4). Choice of the string density and thickness were the primary control variables for the various experiments carried out. Experiments were performed with different strings to compare their behaviours. Results are reported for a copper-wound piano string (measured density $\rho_s = 7800$ kg/m³, cross sectional radius $R_s = 0.78$ mm), and a 9-gauge electric guitar D string ($\rho_s = 7041$ kg/m³ and $R_s = 0.29$ mm).

The snare-like system was positioned horizontally over a table, supported on the back side (i.e. the non-snare side) on thin rubber strips, to prevent it from moving (see Figure 1). This support system was designed to be as minimal as possible, to ensure that the membrane was very close to being in an acoustic free-field on either side (to aid in matching the computational model conditions; see section 3.1).

2.2 Integrated experimental setup

It is well known that the snare drum's unique acoustic characteristics result from motion of and contact between the snare(s) and membrane. It was therefore necessary

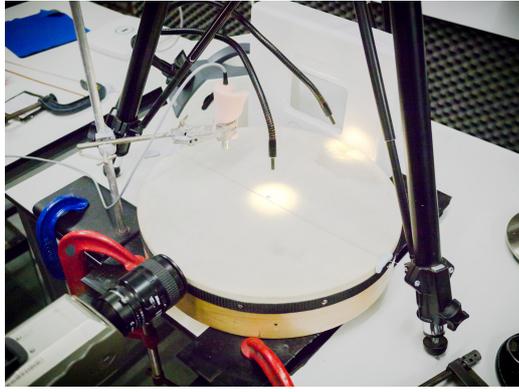


Figure 1: Picture of the experimental setup used, including the drum, high speed camera and microphone. The vibrometer is mounted on the tripod above the table shown in the picture. Swan neck lamps are used to illuminate the string and clear the background.

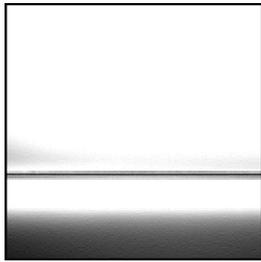


Figure 2: A single frame from the high speed digital video camera (HSC), showing the snare running left-to-right across the lower portion of the image. The frame is approximately 1cm in width. Careful choice of lighting ensures that the snare stands out from the background (white polyfoam) and foreground (membrane and rim), to aid ease of tracking at the image processing stage.

to extract data from the snare-like system outlined in 2.1 describing the motion of both the snare and membrane under playing conditions (labels match those of the model; see Table 1). Furthermore these displacement signals needed to be temporally synchronised with each other for a given drum strike, in order to allow a proper examination of their contract dynamics.

The experimental system, shown in Figure 1, combined a laser doppler vibrometer (LDV; Polytec OVF-5000) to record the membrane displacement, a high speed digital video camera (HSC; Vision Research Phantom v4.11) to track the motion of the snare, with a near field microphone (Bruel & Kjaer Type 4134) to record the acoustic field approximately 8 cm above the membrane. The LDV and microphone signals were sampled into a Bruel & Kjaer PULSE system, and the HSC was controlled via a separate, dedicated PC. System synchronisation was achieved via a Berkley Nucleonics 500 triggering unit. The system was excited into a playing condition by striking the membrane with a regular drumstick approximately 13 cm from the centre. Although a calibrated striking device was not available, care was taken to deliver, where necessary, a consistent striking force and position. Acquired data suggest that this was tolerably well achieved (see section 4.1).

The LDV and HSC were positioned to record the motion of the central portion of the membrane and snare respectively.

The LDV was placed directly above the drum, targeted on a small ($\approx 4\text{mm} \times 4\text{mm}$) square of reflective tape. The snare was positioned to run directly alongside this tape, and the HSC targeted to include this portion of the snare in the centre of the image frame. An example frame from the HSC is shown in Figure 2. The snare can be seen running left-to-right across the lower portion of the image, the width of which is approximately 1cm. The LDV was targeted at a point on the membrane lying approximately 2mm behind the central position along the snare.

2.3 Data analysis

A post-synchronisation workflow was required to produce signals describing $w_{exp}(x_0, y_0, t)$ and $u_{exp}(x_0, t)$. The first step was analysis of the HSC images for a given experimental run (i.e. strike repetition). Typical frame rates were 3703Hz, resulting in 2461 images per run. Images were ingested by a custom Matlab program, and the snare isolated from the surrounding image features. Its mean position in the image was then computed for each frame, and concatenation of all this data provided $u_{exp}(x_0, t)$, where x_0 indicates an averaged spatial position located at the centre of the drum. The second processing stage involved use of a global (hardware) triggering signal to align the HSC, LDV and mic signals.

3 Numerical simulation

The model adopted for the numerical simulation of the system is but a simplified version of the snare drum model presented recently in [12], and only a brief description will be given here. It is sufficient to remove one membrane, to reduce the height of the cylindrical shell and to consider only a single snare attached at the rim of the drum.

3.1 Physical model

The system under study can be schematically described as a circular membrane defined over a region C with radius R , with physical parameters listed in Table 1. Let $w(x, y, t)$ be the transverse displacement of the membrane at position $(x, y) \in C$ and time t . The equation of motion is:

$$\rho_m \partial_{tt} w = \mathcal{L}_m w + \mathcal{F}_a + \mathcal{F}_M + \mathcal{F}_s, \quad (1)$$

where \mathcal{L}_m is a linear operator defined as

$$\mathcal{L}_m = T_m \Delta_{2D} - \sigma_{0,m} \rho_m \partial_t + \sigma_{1,m} \rho_m \partial_t \Delta_{2D}, \quad (2)$$

and Δ_{2D} is the Laplacian operator. The final three terms in equation 1 are the external excitation force densities due to the air, the mallet and the string, respectively. Fixed (simply supported) conditions are applied at the boundaries.

The acoustic field is modelled by means of an acoustic velocity potential [18] obeying the wave equation. Coupling conditions must be enforced at the interface with the membrane, and absorbing boundary conditions must be applied at the walls of the finite computational box. See [11, 12] for details.

The snare is modelled as a stiff string with a single (transverse) polarisation. Let it be defined over a one dimensional domain \mathcal{D} along a diameter of the membrane,

and let $u(\chi, t)$ be the transverse displacement with $\chi \in \mathcal{D}$. The equation of motion can be written as:

$$\rho_s A_s \partial_{tt} u = \mathcal{L}_s u + \mathcal{F}_e, \quad (3)$$

where \mathcal{L}_s groups together all the linear operators,

$$\mathcal{L}_s = T_s \partial_{\chi\chi} - EI \partial_{\chi\chi\chi\chi} - \rho_s \sigma_{0,s} \partial_t + \rho_s \sigma_{1,s} \partial_t \partial_{\chi\chi}, \quad (4)$$

and \mathcal{F}_e represents the collision term arising from the interaction with the membrane.

Finally, the mallet is modelled as a lumped object with mass M , obeying Newton's equation

$$M \partial_{tt} z = f_M, \quad (5)$$

where z is the position of the mallet measured relatively to the membrane, and f_M is the collision force.

Membrane		Value
ρ_m	surface density (kg/m ²)	2600
T_m	tension (N/m)	3000
$\sigma_{0,m}$	freq. indep. loss coeff. (1/s)	20
$\sigma_{1,m}$	freq. dep. loss coeff. (m ² /s)	0.0005
Snare		Value
ρ_s	density (kg/m ³)	7800
R_s	cross section radius (mm)	0.18-0.29
A_s	cross section (m ²)	πR_s^2
T_s	tension (N)	0.2 - 2
E_s	Young's modulus (N/m ²)	2×10^{11}
I_s	second moment of area (m ⁴)	$\pi R_s^4 / 2$
$\sigma_{0,s}$	freq. indep. loss coeff. (1/s)	2
$\sigma_{1,s}$	freq. dep. loss coeff. (m ² /s)	0.001

Table 1: List of physical parameters used in the model.

3.2 Modelling collisions

A possible approach for the modelisation of contact forces is to use of penalty methods [14]. A low degree of mutual interpenetration between objects is allowed, and repulsive forces are introduced which depend on this quantity. This technique has its origins in Hertz's law of contact (see [14] for a historical review), and has been extensively used in acoustics simulation for a wide variety of systems [19, 13].

A detailed description of an energy conserving numerical implementation of contact forces for a wide class of different systems of interest in musical acoustics can be found in [16] and for the particular case of a snare drum in [12]. This novel approach relies on the solution of a scalar nonlinear equation for the mallet-membrane interaction, and a formally similar but vectorised equation for the snare-membrane collision. Existence and uniqueness of a solution in the former case is guaranteed by the results in [17] for a lumped/lumped system, which can be easily extended to the lumped/distributed case. For the distributed/distributed case of the snare membrane interaction, these two properties have been proved in [16]. In both cases, the solution of the nonlinear equation is calculated with an iterative Newton-Raphson algorithm. Although convergence is not formally proved, simulations do converge to a solution for a wide variety of parameters for both types of collision.

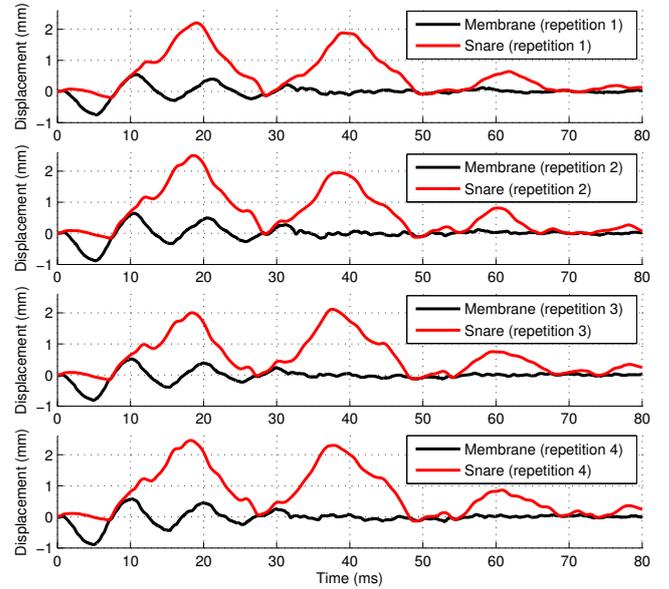


Figure 3: Experimental data showing snare and membrane displacements for four repetitions of the same snare-membrane configuration (piano string as snare; see section 4.1).

4 Results: Experiment and Simulation

The objectives of this paper are twofold. Firstly, to confirm whether the experimental snare model and data acquisition system permitted repeatable and accurate simultaneous measurement of snare and membrane motion (section 4.1). And secondly, to use the experimental snare model to investigate the physical plausibility of the numerical model outlined in section 3 (section 4.2).

4.1 Consistency of repeated experimental measurements

The uncalibrated nature of the striking mechanism used for the experiments (see section 2.2) meant that it was necessary to check that repeatable results could be obtained for a given membrane-snare configuration. Figure 4 shows four repetitions using the copper-wound piano string as a snare. The snare and membrane displacements and interactions appear to be within the same 'regime' between striking repetitions for a given snare-membrane configuration. This suggests that the overall experimental setup is suitable for a preliminary exploration of snare-membrane contact dynamics.

4.2 Analysis of experimental and simulated snare-membrane interaction

The parameter space of physically plausible experimental and numerical configurations, and the resulting range of snare-membrane dynamics, is very large. Presented here is a preliminary analysis based upon a selection of configurations that serve to highlight the apparent strengths and weaknesses of the numerical model, when compared with experimental data. Hints as to the relevance of such comparisons are also provided. In all cases the analysis is based upon the motion of coincident points at the midpoint of the snare and the centre of the membrane.

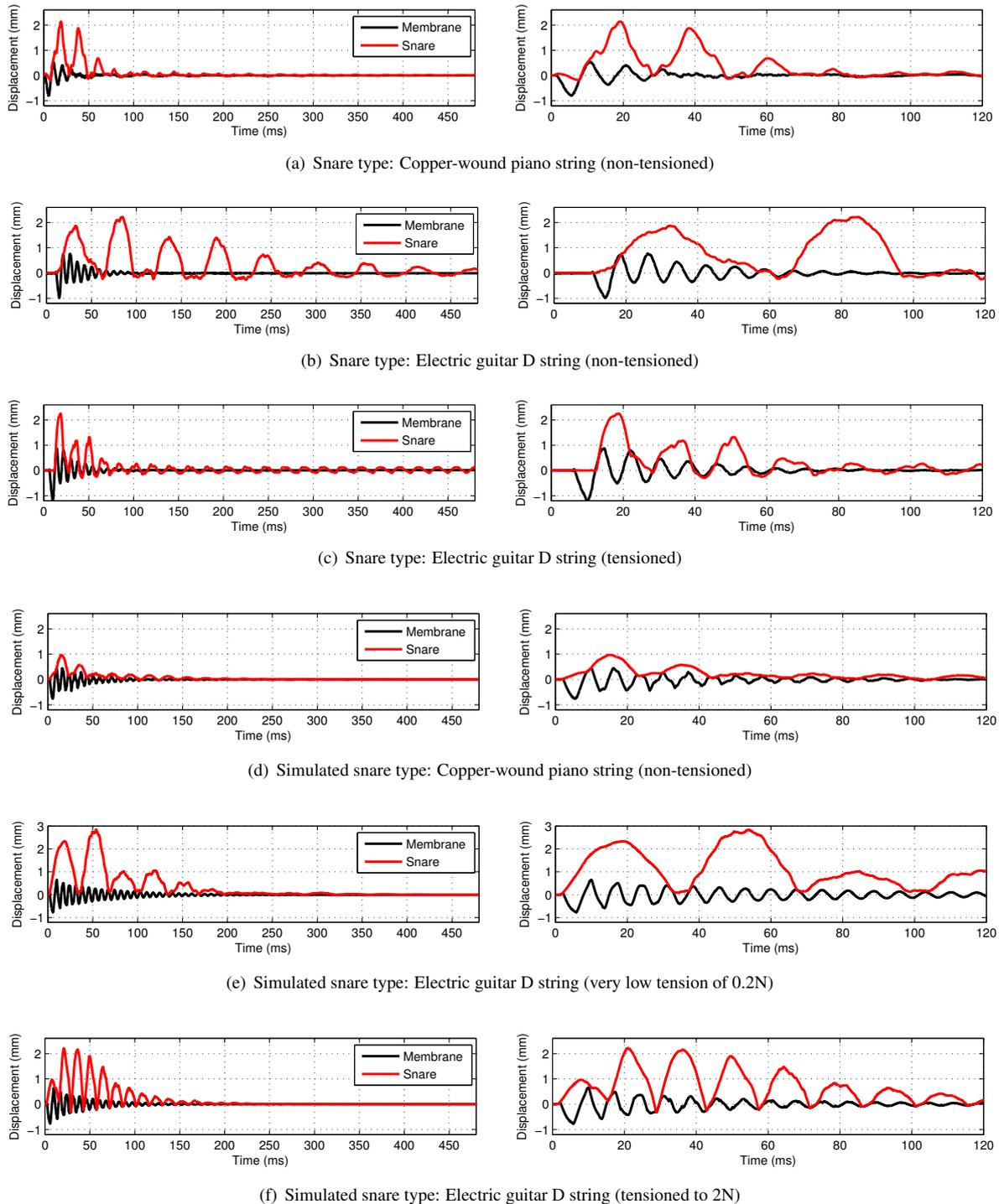


Figure 4: Figs (a) - (c) show experimental snare and membrane displacements for four different snare types (see section 4.2). The striking force has been matched as closely as possible between the four repetitions (see sections 2.2 and 4.1). Figs (d) - (f) show companion modelled data for snares that respectively model each of the three experimental conditions.

Figures 4(a)-(c) show experimentally measured snare and membrane displacements for three independent snare configurations. Figure 4(a) used a copper-wound piano string as a snare, held under a non-tensioned condition. Figures 4(b) and (c) were recorded using a D-string 'snare' from a set of 9-gauge electric guitar strings. In (b) the snare was held to the membrane under a non-tensioned condition, while in (c) the snare was held under tension (the precise level of tension was not measurable in the current setup; this remains as a future refinement). The snare in (a) possessed a higher linear mass density and stiffness than the snare used for (b) and (c). Figures 4(d)-(f) are companion

numerical simulations of snare-membrane interaction for three numerical model configurations that are based on the three experimental trials.

Considering the whole set of experimental and modelled data, it is clear that the range of peak membrane and snare displacements are broadly similar, falling in the range of around 1 mm and 2-3 mm respectively. In all cases the snare is seen to periodically 'bounce' on the membrane, as well as to move in unison with it (e.g. at 30 ms, 40 ms, 55 ms and so on in Figure 4(c), and 30 ms, 45 ms, 55 ms in Figure 4(f)). The precise timing and amplitude of the experimental and modelled snare motion, and of the fundamental membrane

frequency, are not particularly close. However, this is not surprising given that it was difficult to match the physical properties of the membrane between model and experiment (mylar was used in the model, as is normal for snare drums, while the experimental model used a leather bodhrán).

More convincing are the responses to deliberate changes in system properties, such as the non-tensioned and tensioned D-string snare. The experimental results show the tensioned snare undergoing approximately twice as many bounces during the first 120 ms, when compared to the non-tensioned case. Likewise, the numerical model exhibits considerably more bounces for the tensioned versus (near) non-tensioned case. Note that the increase in tension is of an order of magnitude between the two cases, yet the increase in apparent snare ‘bounce frequency’ is only about double, highlighting the nonlinear nature of the snare-membrane interaction, which is well captured by the model.

Also evident from both the experimental and modelled data is the sensitivity of the snare-membrane system to the value of physical parameters. Experimentally, the piano string collided six times with the membrane during the first 120 ms (Figure 4(a)), when held under no tension. The less massive and considerably less stiff D-string snare, however, underwent just two collisions (also under a tensionless condition). A similar pattern is seen in the modelled data (Figures 4(d) and (e)). The snare displacement amplitude is also seen to decay fastest for the piano string snare in both the experimental and numerical data. Additional behaviours, such as the second snare bounce sometimes reaching a larger amplitude than the first, are also seen to occur in both experiment (4(b)) and model (4(e)).

The preceding discussion suggests that various physically-observed snare-membrane behaviours are captured quite well by the computational model. However, without a more precise quantification of the experimental system parameters, the snare tension and membrane properties in particular, it is not possible to draw any stronger conclusions about the accuracy of the model. It is clear, however, that the model does not properly capture the decay of the membrane, which is considerably faster for the experimental data. This behaviour may be due to a mismatch between the experimental and modelled membrane properties, or it may point to the presence of a non-elastic collision mechanism between the snare and membrane, which is not included in the present model.

5 Final remarks and future work

In this paper, the behaviour of a simplified system consisting in a single membrane drum and a snare has been investigated, both experimentally and with numerical simulation. The setup adopted is to be considered as a preliminary attempt towards the study of the snare membrane collisions in a snare drum. In order to make the system more controllable, and to allow a more quantified comparison between experiment and model, several improvements need to be made. First of all, a complete analysis of the physical parameters of the drum at hand needs to be performed, in order to have meaningful quantities to use in the simulation. Second, a more advanced tensioning mechanism to hold the snare in the contact with the drumhead is necessary. In the present case, only wires with a reasonably large cross

section could be studied, as thinner ones could not be kept in a fixed position. Despite these accepted limitations, it is nevertheless clear that the numerical model exhibits a range of behaviours that are also observed in the experimental setup.

Acknowledgments

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