Mechanical Analysis of the Voicing Process in the Harpsichord

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The harpsichord has been widely used between the 17th and 18th centuries before being replaced by the *piano-forte*. With the 20th century started a new interest for the harpsichord. Studies have analysed the design parameters of the instrument to improve it or to restore its original characteristics. Few studies have investigated the relationship between the voicing process and the feeling of a touch on the keyboard. The main goal of voicing a harpsichord is to give the instrument a homogeneous response over its whole tessitura. One stage of the process consists in carving the plectrum to give it a unique shape to produce the desired sound. This shape is important since it affects the plectrum bending during its contact with the string. This paper presents an investigation of how the plectrum shape is related to the musician sense of touch. Studying this phenomenon needs a complete model of the plucking mechanism. It must take into account the finger action on the key and link it to the string movement until its release. A combination of rigid body mechanics and finite element analysis allows to build the model. An experiment is carried out with a robot to compare the simulation results to reality. It will show the model prediction follow the same tendencies as the experimental data and also that a real haptic feedback occurs in the harpsichord.

1 Introduction

The harpsichord was a typical instrument of the Renaissance period. It was the top instrument of the 17th and 18th centuries before being replaced by the *piano-forte*. In the beginning of the 20th century people found a new interest for this instrument. Musicians, instrument makers and scientists started to build new harpsichords and restore old ones. One goal was to restore the original sounds of the instrument. Another question tackles the problem of making new harpsichords : the techniques have evolved since 17th and the materials have changed. How does these modifications impact the sound properties of a harpsichord? Previous studies [1, 2] have investigated the influence of general design parameters on the instrument quality, such as strings material, length and thickness or soundboards dimensions and ribs placement. Modal analysis have been performed on historical instruments to understand their soundboard behaviour [3]. Other studies concentrated on the plucking mechanism of the instrument. The harpsichord mechanism is composed of the keyboard, the jacks and the plectrums. At the end of each key lays the jack. On each jack is mounted a plectrum that plucks the string when the musician strikes a key. The plectrum was first made of bird quill but nowadays it is usually made of delrin. The first models reduced the musician action on the key to the jack rising at a constant speed. The plectrum was represented as a set of rigid rods linked together with torsional springs [4] and later approached by a cantilever beam [5, 6, 7, 8]. All these models used one or more *identical* beams, which does not take into account the *complex shape* of the plectrum resulting from the voicing process. This process is an important step in a harpsichord making. It consists in tuning the instrument to give it a homogeneous response over its whole tessitura. Harpsichord makers can do so by carving the plectra to tune each note. The resulting shape of the plectrums can be the key to understand the instrument acoustical identity. A more recent model proposed to combine two cantilever beams with different sections to approach a real plectrum shape [9]. It was a slight but noticeable improvement because it showed the plectrum has to be taken into account in the model. In this paper we characterize the haptic feedback experienced by the musicians on the keyboard. We will link experimentally and with a model the effort applied on the key to the plectrum shape. Instead of using theoretical solution for beam bending we will use a finite-element (FE) model. We will introduce a rigid body model of the key mechanism to investigate the evolution of the effort applied on it. A finger robot [10] will be used to control the key presses. This paper is organised as follows : first, we describe the plucking model, the key and string dynamics and compare it to another model. Then we describe the experimental setup and detail which quantities are measured. Eventually we compare the model to experimental results.

2 Description of the model

The model we describe in this section is an assembly of three parts that simulate the plectrum bending, the force exerted on the key and the string movement.

2.1 Playing action decomposition

Figure 1 presents a simplified drawing of a harpsichord key mechanism. We can decompose the musician action when playing the instrument into three steps. It begins when he presses the key with his finger. First the jack rises and grabs the string. Second the plectrum starts to bend. Third the string slides on it. All these steps repeat until the string have reached the tip of the plectrum and is released. The effort applied by the musician is denoted $\vec{F}_{f1}$, the key rotation is marked α and the plectrum deflection angle is denoted ϕ_0. The string coordinates are named (x_s, y_s). We denote $\vec{F}_{sp}$ the force exerted by the string on the plectrum. We call $\vec{F}_{s2}$ the bending reaction at the clamped end of the plectrum. The straightforward relationship between $\vec{F}_{s2}$ and $\vec{F}_{sp}$ will be exposed later in this paper.

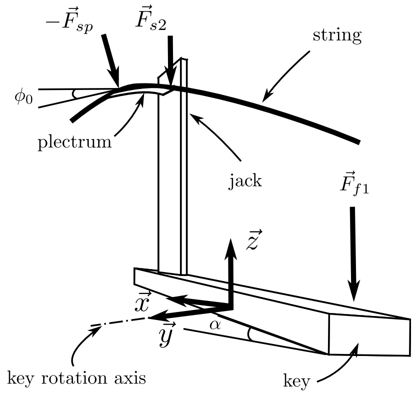

Figure 1: Simplified key mechanism. The reference frame is denoted $(\vec{x}, \vec{y}, \vec{z})$. The angles α and ϕ_0 are measured relatively this reference system. $\vec{F}sp$ is the effort applied by the string on the plectrum at the plucking point (x_s, z_s).

A more detailed view of each part of the model is described in the following sections.

2.2 Key model

We use rigid body mechanics to model the key mechanism presented in figure 1. There are two main solids in the mechanism : the key lever and the jack. They are linked to the reference frame by a balance pin at the key rotation axis and by the register which is not represented here. The balance pin is modeled as a pin joint and the register is modeled as a slider. The link between the key and the jack is modeled as a contact point. The plectrum in this part of the model is represented as a force $\vec{F}_{s2}$ applied on the jack at its clamped end. Rigid body mechanics help us drawing a set of equations that link together α and its derivatives, $\vec{F}_{f1}$ and $\vec{F}_{s2}$. In that way, we are able to predict the evolution over time of the key pressure $\vec{F}_{f1}$ knowing the other variables.

In our model we suppose the jack speed is constant, which sets the evolution of $\alpha, \dot{\alpha}$ and $\ddot{\alpha}$. We still need the evolution of $\vec{F}_{s2}$ to predict the variations of $\vec{F}_{f1}$. A realistic plectrum bending model can provide this missing information.

2.3 Plectrum model

A FE model is used to predict the plectrum bending and find the force $\vec{F}_{s2}$ on its clamped end. It is designed and solved using the free software *Cast3m*. It is the main improvement proposed in our model because previous studies used rigid rods or beams to represent the plectrum. Finite elements models allows to take into account the real geometry of the plectrum when it has been carved by the instrument maker. Figure 2 presents the photograph of a plectrum taken with a microscope and the corresponding FE plectrum on its right.

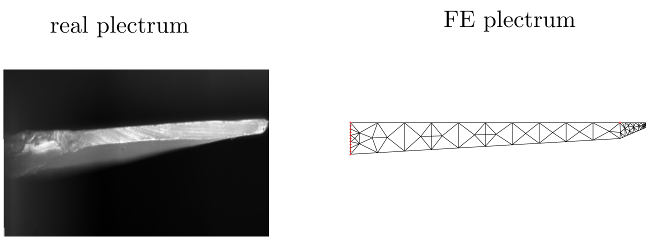

Figure 2: Plectrum modelisation using *Cast3m*. The image on the left is a photograph of a real plectrum took with a microscope at the Musée de la musique and the image on the right is the equivalent FE model created with Cast3m. The meshing is coarse here for readability but is much thiner in reality.

We assumed in this paper that the width of the plectrum is constant so we used a FE planar stress model. We also suppose the plectrum to be in the elastic approximation. It means the plectrum recovers its initial state when the load that have caused the deflection is suppressed. The plectrum then undergoes successive quasi-static deformations. It is then easy to links the force applied by the string $\vec{F}_{sp}$ at the plucking point to the force $\vec{F}_{s2}$ on its clamped end by eq. 1.

This model will help to predict the string movement over time, though we need some more hypothesis which will be described in the next subsection.

$$\vec{F}_{sp} + \vec{F}_{s2} = \vec{0} \tag{1}$$

2.4 String model

We consider an infinitesimal element of the string. This element has a mass ∂m. We suppose the string element to be in quasi-static approximation. This hypothesis can be considered true while the string element velocity V_s stay far below the string oscillation velocity $V_{osc} \approx \frac{4hf\sin^2(\pi\beta)}{\pi\beta(1-\beta)}$, where h is the string displacement, β is the plucking point ratio and f is the string fundamental frequency.

We suppose the string element to be linked to its rest position by a spring and to be sliding on the plectrum. We do not involve any friction effect in our model. The force exerted on this element are the spring force and the contact force with the plectrum. The direction of the force applied by the plectrum on the string element is supposed to be orthogonal to the plectrum shape at the plucking point.

The spring stiffness K depends only on the string length L, its tension T and the plucking ratio $\beta = \frac{L_p}{L}$ where L_p is the distance between the string clamped end and the plucking point (eq. 2).

$$K = \frac{TL}{\beta(1-\beta)} \tag{2}$$

The application of Newton's second law to the string element gives the set of equations 3 that predicts the string movement on the plectrum.

$$\begin{cases} \partial m \frac{\partial^2 x_s}{\partial t^2} &= -\|\vec{F}_{sp}\| \sin\phi_0 - K(x_s - x_0) \\ \partial m \frac{\partial^2 z_s}{\partial t^2} &= \|\vec{F}_{sp}\| \cos\phi_0 - K(z_s - z_0) \end{cases} \tag{3}$$

The initial position of the string is denoted (x_0, z_0). All parameters value for the model and the experiments are detailed in table 1. In the next subsection we will present the model implementation and compare its results to those presented in [8].

2.5 Implementation and models comparison

The work presented in [8] is the most recent approach to model the harpsichord key mechanism. It is the first model using continuum mechanics to simulate the plectrum bending. An improvement have already been made in [9] toward the introduction of the plectrum shape in the model. It consists of using two cantilever beams with different sections rigidly linked together. The results obtained with this model are found to be closer to experimental results. We suggest the use of a FE model to improve the prediction accuracy.

Our model is designed to predict the effort $\vec{F}_{f1}$ applied by the musician on the key. We suppose here the inputs are the jack speed V_j and the string initial position (x_0, z_0). The effort applied to the plectrum by the string $\vec{F}_{sp}$ is then deduced from the string model. It can be used to find the plectrum deflection via the FE model. It also gives the effort at its clamped end $\vec{F}_{s2}$ as well as its deflection angle ϕ_0 needed for the model update. In the end the touch feeling of the musician represented by $\vec{F}_{f1}$ can be computed. The model implementation is presented in figure 3.

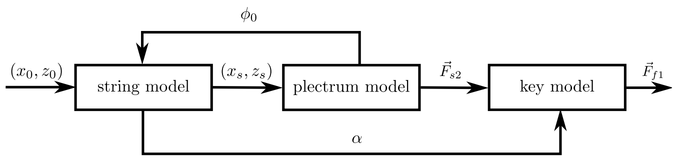

Figure 3: Implementation of the key mechanism model. Each box represents a part of the model and which link together the variables.

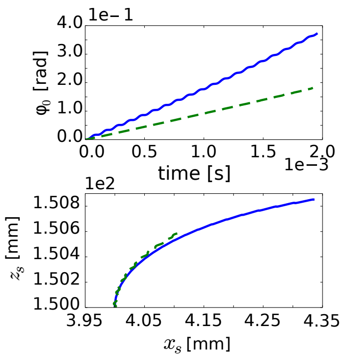

Figure 4: Simulation results. The plain line corresponds to the model presented in [8] and the dashed line corresponds to our model. The top left graph presents the evolution over time of the plectrum deflection angle ϕ_0, the bottom left graph presents the string trajectory $z_s = f(x_s)$ and the top right graph presents the evolution over time of the effort $\|\vec{F}_{s2}\|$.

To compare our model to the previous ones we plotted on figure 4 various simulation results. The parameters we used for our simulations are detailed in table 1. The string velocity V_s has been computed from simulation results and found to be 0.03 m.s^{-1}. It is thus smaller than V_{osc} and equivalent to V_j which validates the hypothesis made in subsection 2.4. We used a beam-like plectrum for this validation with the characteristics of the real plectrum. Results shown in figure 4 are close from each other in first approximation. $F_{s2} = \|\vec{F}_{s2}\|$ and ϕ_0 are smaller for our model but follow the shape of those predicted by the previous model. They are also of the same order of magnitude. We plotted the string trajectory until it is released from the plectrum. In our simulation, the string is released much faster.

The simulation results are close from the results provided by the former model. An experimental comparison with the simulation results lacked in the previous studies. This is the topic of the following section.

3 Experimental comparison

We carried out experiments on a harpsichord to validate our model results. We used a robot to control the key deflection and the jack velocity. We start presenting the robot and then present the experimental protocol and parameters.

3.1 Experimental setup

Figure 5 presents the robot on the harpsichord. It is a two pin joints planar robot. We can control the end effector

Parameter	Value	Description
V_j	0.072 m.s^{-1}	jack velocity
V_{osc}	0.46 m.s^{-1}	string oscillation velocity
E	2.9×10^9 MPa	plectrum Young modulus
ν	0.35	plectrum Poisson ratio
ρ	1247 kg.m^{-3}	plectrum density
L_s	4.5 mm	plectrum length
W	0.75 mm	plectrum thickness
h	0.48 mm	plectrum height
(x_0, z_0)	$(L_s$, 150) mm	string initial position
f	365 Hz	string frequency
L	460 mm	string length
L_p	108 mm	plucking point position
T	66.95 N.m^{-2}	string tension
∂m	4.45×10^{-4} mg	string element mass

Table 1: Simulation and experimental parameters. This table is composed of three sets of parameters. The first set is used to validate the quasi-static movement hypothesis made in subsection 2.4, the second set is related to the plectrum model and the third is related to the string model.

trajectory (x_p, z_p) and its velocity to reproduce the finger gesture [10].

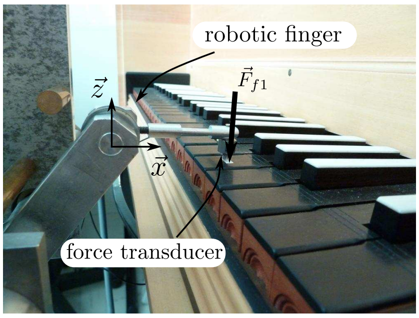

Figure 5: Robot mounted on the harpsichord. The force transducer is mounted on the end effector of the robot. The robot trajectory is done in the zOx plane.

We generated its trajectory to ensure the jack a constant velocity along the $\vec{z}$ axis during the key depression. It can be seen in figure 6. We used a speed of $V_j = 0.072$ m.s^{-1} and chose to give the jack an elevation of 10 mm from its rest position. All these conditions give the end effector a curved path in order to keep a constant step along $\vec{z}$ direction between two successive positions. It is also designed to keep the direction of the force transducer orthogonal to the key surface. The trajectory is then not only along $\vec{z}$ direction but also along $\vec{x}$ direction to follow the key rotation.

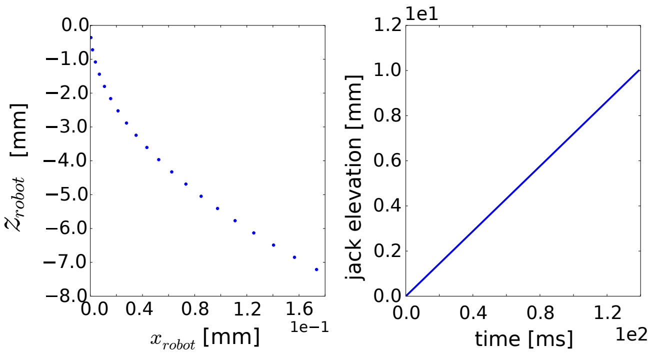

Figure 6: Robot trajectory. The left graph presents the robot end effector trajectory in the zOx plane. The right graph presents the jack supposed position over time. The slope 0.072 m.s^{-1} and the maximal elevation is 10 mm.

The experiment is conducted on the 37$^{\text{th}}$ key, which is a G4 note which fundamental frequency is 365 Hz. We used a laser doppler vibrometer PDV-100 from Polytech to record the key deflection velocity. A force transducer B&K 8203 with 2692-C Nexus conditioning amplifier is used to monitor the effort applied by the robot on the key. The transducer was mounted on the end effector of the robot as it can be seen in figure 5. The laser beam is focused at 103 mm from the end of the key.The pressing point is located at 14 mm of the end of the key.

3.2 Experimental results

Figure 7 show the key deflection which is composed of three phases. The first phase begins at 0 ms when the plectrum is not in contact with the string. It rises with the key rotation and the effort applied to the key is constantly increasing. The second phase begins at 27 ms when the slope changes. It is the moment when the plectrum grabs the string. This instant is easy to detect on the effort graph because of the peak at 27 ms. The key deflection slope is lower in the second part than for the first part. After a transient phase the effort increases again until 47 ms with the second slope change. It marks the beginning of the third phase when the string has been released from the plectrum. The slope is nearly the same as for the first part and the effort diminishes.

We suppose the effort applied by the string to the plectrum causes the jack to slow down in the second part. This may explain the slope change. We can observe small oscillations which we call a transient phase. They may be the string oscillations at the beginning of the contact with the plectrum in the second phase. Further investigation will be done to confirm this hypothesis. The force increase at the end of this part corresponds to our model results presented in figure 8.

Experimental investigation of the relationship between the key deflection and the force applied by the musician has shown that a real haptic feedback occurs in harpsichord playing.

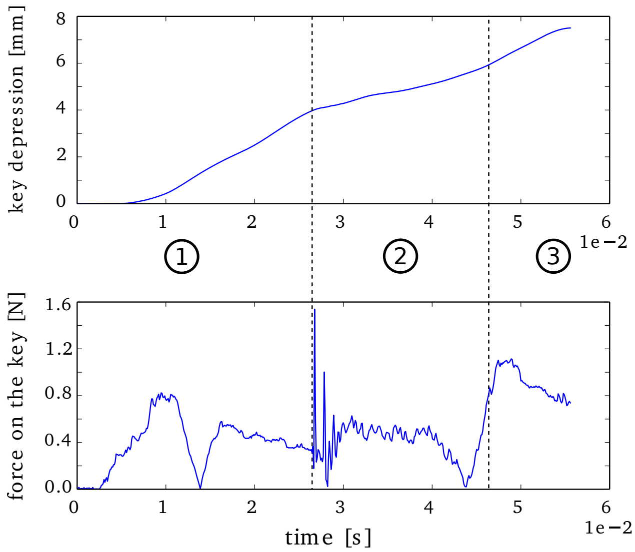

Figure 7: Experimental data. The top graph shows the key deflection evolution over time and the bottom graph is the evolution of the effort applied on the key over time. The graphs can be split into three parts indicated on the graph.

3.3 Comparison between model and experimental results

Figure 8 shows the results obtained for the prediction of the variables $\|\vec{F}_{f1}\|$ and the key deflection. Our results are not consistant with the measurements, but the global shape of the force over deflection graph is respected. There is 60% error between the slope of the model and the slope of the experimental curve. The magnitude of the movement is underestimated for the model. The duration of the string contact phase is slightly longer than in reality.

The explanation for such discrepancies may be an incorrect model parameters estimation. The Young modulus for the plectrum is not exactly known and the key mechanical

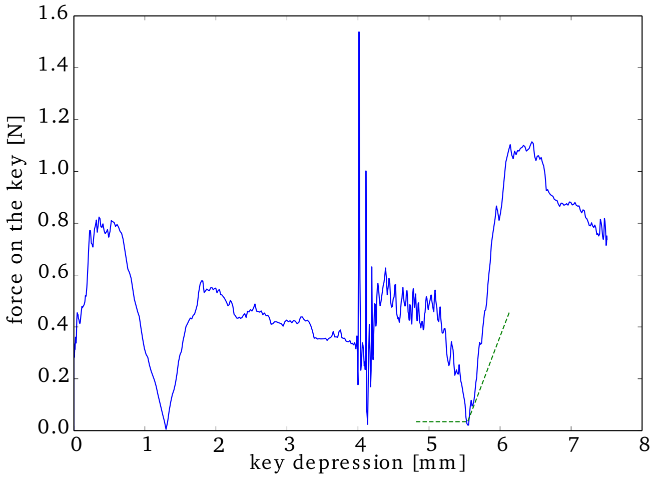

Figure 8: Comparison between experimental data to simulation results. The graph presents the relationship between F_{f1} and the key deflection over time. The plain line presents experimental data and the dashed line presents the simulation results. The same set of parameters have been used for the experiment and the simulation. The curve retiming is done relatively to the beginning of the second phase described by figure 7.

characteristics where estimated numerically for simplified geometries. With an accurate measurements of these parameters we hope improving our model prediction.

4 Conclusion

We propose in this study to take the plectrum shape into account in the harpsichord plucking mechanism model. We used a finite element model instead of beam theoretical equations to reach this goal. We added a rigid body model of the key to link the force applied by the musician on it to the plectrum bending. The simulation predictions are consistant with the results in the literature but there are noticeable differences concerning the string displacement. We acquired experimental data with a robot on a harpsichord to verify our predictions. The robot was used to control finely the key deflection and thus ensured to be closed to our simulation hypothesis. A comparison between the measurements and the model showed the same tendencies. We hope to correct the discrepancies in the future with a precise mechanical parameters identification.

The experimental process have shown a clear haptic feedback on the keyboard. Future work after the model tuning will investigate the relationship between the plectrum shape and this feedback. We would like to understand how small geometry variation can modify the instrumentalists touch since they qualify it themselves as hard, soft or medium. We will also study the influence of the plectrum shape on the initial conditions of the string free oscillations.

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