Harmonics generation in flute-like instruments

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This paper presents a preliminary study about the control of the spectral content by recorder players. The variation of the spectral content has been identified to be due to a change in the mouth volume. This is interpreted as following. A tuning of the vocal tract resonances may maximize the acoustic coupling that occurs with the resonator. This induces jet velocity fluctuations that strongly modify the instability mechanisms of the jet, a crucial feature of the auto-oscillation. A simplified model of jet velocity instabilities is proposed and yields promising results. The symmetrical properties of the jet perturbations are identified to be crucial for the strength of the different harmonics.

1 Introduction

The sound of different flute-like instruments may be differentiated through the time-variant spectral content. The relative strength of the harmonics as well as the noise generated by turbulence are characteristics of each instrument. They may vary from the attack transient to the quasi-steady part of the sound. They are part of the perceptive information process that discriminates one flute from another and flutes from other instruments.

The strength of the different harmonics depends among others on geometrical parameters that have to be controlled either by the instrument maker or the musician. From a physical point of view, the strength depends on the amplitude of the acoustic modes of the resonator and on the non-linear element, required by the auto-oscillation process. Any change in the symmetrical properties of the non-linear element will modify the energy distribution among the harmonics. This is the case of the offset between the flue exit and the labium that can either be controlled by the instrument maker (i.e. recorder) or the musician (i.e. transverse modern flute).

In the case of a recorder, the geometry of the excitation element of the instrument is fully adjusted by the maker. Actually, because of the non-linear element, the variation of the strength of the harmonics depends on the amplitude of oscillation and thus on the jet mean velocity. The spectral enhancement due to the non-linear saturation is is however strongly related to a sensation of loudness. The controllability of the spectral content seems poor in the case of a fixed labium/channel geometry.

2 Controllability of the acoustic coupling

The pressure $p = \langle p \rangle + p'$ within the upstream cavity may vary faster than the flautist initially intended to control due to the acoustic coupling between with the internal pressure p_{ac} within the recorder. In first approximation, this coupling may be described by a linearisation of the unsteady Bernoulli's law combined with the (linear) mass conservation yielding the transfer function between both pressures [1, 2, 3]:

$$H(\omega) = \frac{P'(\omega)}{P_{ac}(\omega)} = \frac{-j\omega\langle u \rangle S_j/V + \omega_0^2\left(1 - \langle u \rangle^2/c_0^2\right)}{\omega_0^2 - \omega^2 + j\gamma\omega} \quad (1)$$

with the parameters

$$\omega_0^2 = \frac{c_0^2 S_j}{V l_e} \text{ and } \gamma = \frac{\langle u \rangle}{l_e} \quad (2)$$

and where V is the mouth volume, c_0 is the speed of sound, S_j is the cross section of the flue exit, l_e is the equivalent channel length and $\langle u \rangle$ is the jet mean velocity estimated by the steady Bernoulli's law. The fluctuations of the upstream pressure occurs in parallel with the fluctuations of the jet velocity . An equivalent transfer function can be found [3]. The pressure fluctuation p' within the mouth then depends on the parameters of the transfer function and more precisely on the geometrical parameters of the supply system. For instance, the length of the channel l_e will affect the damping coefficient γ and the efficiency of the coupling. This is a parameter set by the recorder maker. However other parameters can be controlled by the flautist and may have an influence on the coupling. For instance varying the mouth volume V will change the resonance frequency ω_0 and then the phase and the gain of the coupling.

3 Experimental observation

For the purpose of this study, an alto recorder (model Aesthé made by the Canadian recorder maker Jean-Luc Boudreau) has been modified to allow the measurement of both pressures in the mouth of the flautist an within the recorder. The mouth sensor is a Honeywell (model 176PC14HG1) plugged on a capillary tube that pass through the mouthpiece to end within the mouth. The pressure within the recorder is measured by means of a B&K pressure sensor (model 4938) mounted flush into the bore of the recorder at a close distance of the labium.

The bore sensor has a bandwidth corresponding to the audible frequency (20Hz – 20kHz), whereas the mouth sensor has a wider bandwidth, including lower frequencies. The mean variation of the mouth pressure can thus be measured as well as the higher frequency components due to the acoustic coupling. Fourier transforms of both pressure signals allows to estimate the gain $|P'(\omega_p)/P'_{ac}(\omega_p)|$ and the corresponding phase of the coupling at the oscillating pulsation ω_p, corresponding to the transfer function of Eq. (1).

Experienced flautists and recorder teachers agreed to participate to the measurements. Several conditions have been tested. Among others, the flautist was told to play an isolated note and to try to modify the timbre of the note. A flautist affirmed being able to varying his mouth volume while playing a note. Figure 1 shows the gain of coupling and the phase of coupling as a function of time for this measurement. The variation of volume claimed by the flautist is correlated to the amplitude and the gain of coupling. For a "large" mouth volume ($t < 10$ s) the coupling is weak (-60 dB) and the phase close to $-\pi$. For a "small" mouth volume (11 s< t <13 s) the coupling is stronger (-20 dB) and the phase is also close to $-\pi$. When varying from the large to the small volume and vice versa, the amplitude and phase continuously vary from their asymptotic values, and go back to their initial value. Note that the phase pass through almost zero between the two mouth configurations.

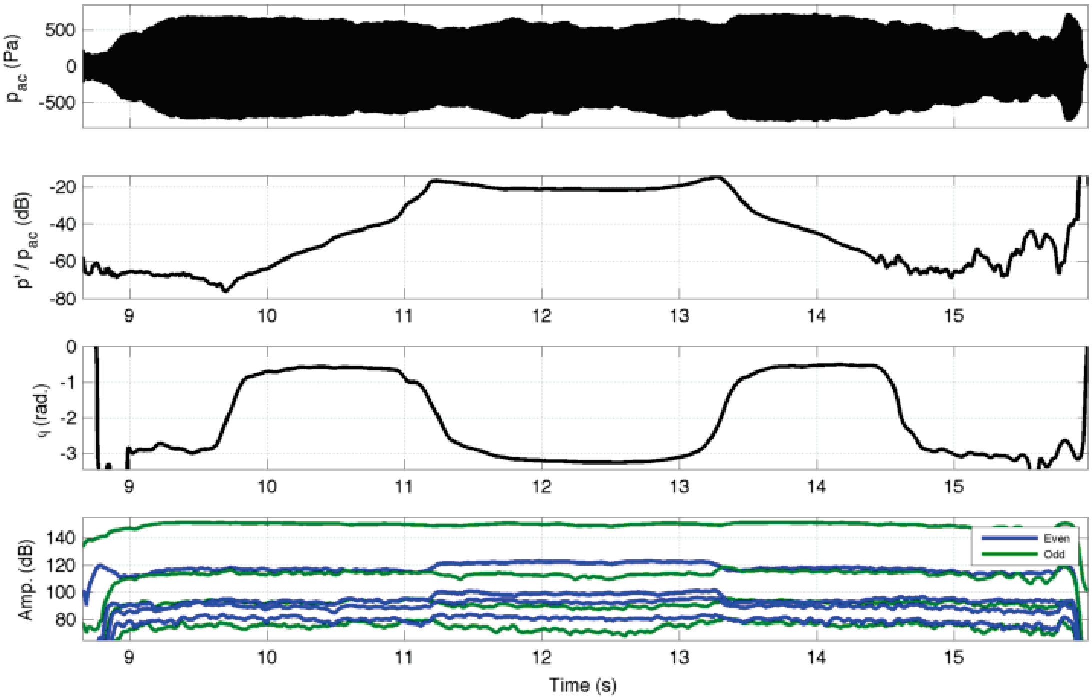

Figure 1: A flautist affirms varying his mouth volume on an isolated note. From top to bottom: internal pressure signal, amplitude of coupling, phase of coupling and strength of the 8 first harmonics of the internal pressure.

The acoustic coupling is controllable by the flautist.

Along with the variation of the coupling, the spectral content is also modified. Figure 1 also shows the evolution of the strengths of the 8 first harmonics (estimated by a short time Fourier transform) as function of the time. If the amplitude of the fundamental is almost constant, the relative strength of the higher harmonics strongly depends on the coupling condition such that the odd and even harmonic strength differs from the weak to the strong coupling.

The spectral content dependency on the coupling condition has already been observed and quantified with an artificial mouth [2, 3]. More precisely, the even/odd harmonics strength depends on the phase of coupling only, the amplitude of coupling having an effect on the efficiency of the phenomena. The higher the amplitude of coupling, the more pronounced the difference between odd and even harmonics.

The present experimental study confirms that the results obtained by controlling an artificial mouth can actually be obtained by skilled flautists. A clear effect of the phase of coupling on the spectral content has been highlighted. To interpret this variation of the spectral content, a short digression must be done about the effect of the coupling on the jet instabilities.

4 Pulsating jet and jet instabilities

As the jet is at the heart of the auto-oscillation process, any change in the sound production, as the spectral content of a quasi steady note, will be investigated through a change in the hydrodynamic of the jet itself.

The description of the jet velocity fluctuation proposed in section refsec:controllability is one dimensional. A more accurate description would focus on the unsteady modification of the velocity profile along the channel and at the flue exit. However, in the present and preliminary study, the effect of the jet velocity fluctuation is assumed to only affect the kind of perturbation that growth downstream the jet. The jet velocity fluctuations are assumed to represent a symmetrical excitation of the jet instability mechanisms, as opposed to the anti-symmetrical excitation of the transverse acoustic field that surround the jet within the window of the recorder. Both kind of excitations are assumed to provide energy into two unstable modes. This is presented in this section.

The linear stability theory of infinite plane jets with a symmetrical velocity profile provides two unstable solutions [4]. One has anti-symmetrical properties whereas the other is symmetrical (see Fig. 2). Without any external excitation, the anti-symmetrical mode (on the left side) is more unstable than the symmetrical one (on the right side). Besides, the two modes are convected a different phase velocity (roughly a factor of 2).

In the recorder, the “natural” excitation due to the transverse acoustic field that surrounds the flue exit yields to the selection of the anti-symmetrical mode only. This has been observed by several authors [5, 6]. The presence of jet velocity fluctuation may induce the rise of the symmetrical mode overlaying with the anti-symmetrical mode due to the transverse acoustic excitation, at least for the early and linear stage of the instabilities.

The existing modelling will be modified to include the modulation of the jet width $b(t)$ that now accounts for the jet velocity fluctuations due to the acoustic coupling between the mouth and the instrument.

5 Towards a control of the spectral content

As for any self-sustained instrument, the modelling of recorders requires a non-linear element that saturates the oscillation and stabilizes the so-called auto-oscillation

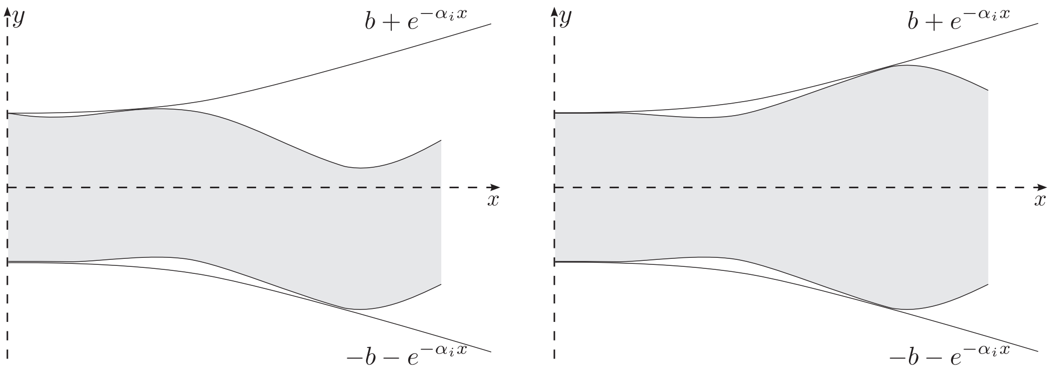

Figure 2: Shapes of the symmetrical (right) and anti-symmetrical (left) unstable modes of jet.

process. The Jet-Drive model, that consists in writing the flux injection from both side of the labium as a pressure difference Δp that drives the resonator, depends on the jet centreline $\eta(t)$ and the jet width $b(t)$ at the labium through [7]:

$$\Delta p = A\frac{d}{dt}\left[b(t)f\left(\frac{\eta(t)-y_0}{b(t)}\right)\right] \tag{3}$$

where y_0 is the offset between the channel axis and the labium, f is a function that depends on the jet velocity profile and where A is a constant of proportionality of no interest for the present purpose. For a Bickley profile $U(y) = u_0 \mathrm{sech}^2(y/b_0)$, as usually assumed [5, 7], the function f becomes

$$f(y) = \tanh(y). \tag{4}$$

The source term in Eq. (3) combine with Eq. (4) allows to discuss the spectral content of the source and finally the filtered sound. This has been done by Fletcher & Douglas [8] and is briefly presented in the next section.

5.1 Symmetry and spectral content

For an harmonic oscillation of the jet at the pulsation ω, without symmetrical component ($b(t) = b_0$), the jet centreline motion at the labium can be written in a linear approximation as

$$\eta(t) = a\sin\omega t, \tag{5}$$

where a is the amplitude of the motion. Combining Eqs. (3), (4) and (5) yields to the simplified expression of the source term

$$\Delta_p = A\frac{d}{dt}\left[b_0\tanh\left(\frac{a\sin(\omega t)-y_0}{b_0}\right)\right]. \tag{6}$$

The spectral content of this non-linear composition of basic trigonometric functions can be computed by means of a Fourier transform. This is done with an arbitrary value of the amplitude of the jet motion $a = 2b_0$. The same results as Fletcher & Douglas [8] are found (see Fig. 3 from [8] with original caption). As the tanh function is anti-symmetrical for the case , the spectral content of the source term is only made of odd harmonics. Varying the offset modifies the symmetrical properties and thus modifies the odd/even distribution (see Fig. 3). The spectral content of the radiated sound corresponds to the one of the source term filtered and radiated by the resonator. This is confirmed by the measurements of Fletcher & Douglas [8]. In this paper, they observe a slight shift of the curves because of the jet deflection. Nevertheless, the theoretical results remain in substance the same. When adjusting the offset, the recorder maker has a direct impact on the sound production.

5.2 Consequences in the case of a pulsating jet

In the case of a pulsating jet, the jet width is allowed to vary while the jet is oscillating. Besides due to the different phase velocity of both hydrodynamic modes, the oscillation of the jet centreline and the jet width are phase shifted:

$$\eta(t) = a\sin\omega t \ \text{ and } \ b(t) = b_0 + b_0'\sin(\omega t + \phi), \tag{7}$$

where b_0' is the amplitude of the jet width modulation. Combining Eqs. (3), (4) and (7) yields the modified expression of the source term

$$\Delta_p = A\frac{d}{dt}\left[(b_0 + \sin(\omega t + \phi))\tanh\left(\frac{a\sin(\omega t)-y_0}{b_0 + b_0'\sin(\omega t + \phi)}\right)\right], \tag{8}$$

which now depends on the amplitudes of the jet width and the jet centreline b_0' and a, on the phase shift between both

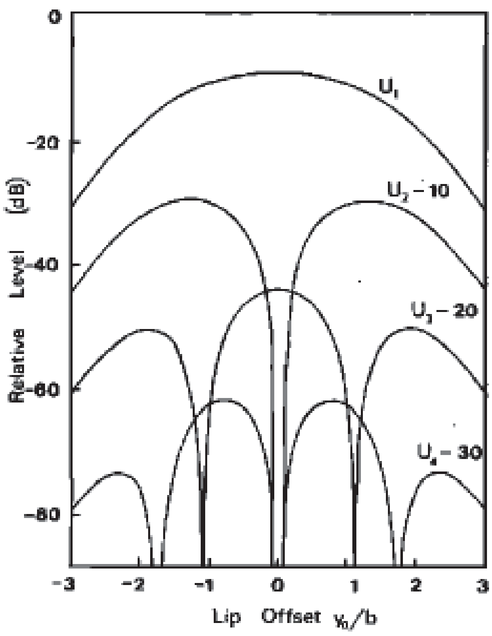

FIG. 3. Calculated relative pressure levels of the first four harmonics in the jet source function as functions of the relative lip displacement y_0/b, as given by (2). The relative jet displacement amplitude is $a/b = 2$. For clarity the curves for successive harmonics are displaced downwards by 10, 20, and 30 dB relative to the scale for the first harmonic.

Figure 3

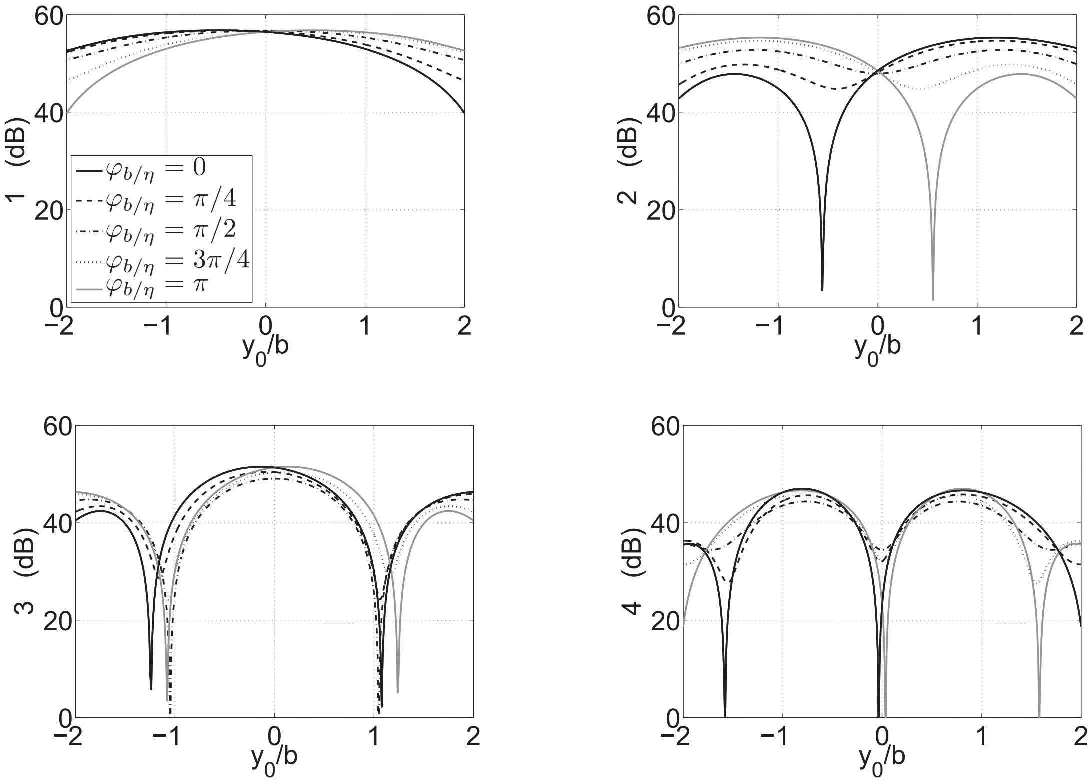

Figure 4: Four first harmonics of the modified source term (Eq. (8)) as function of the offset y_0 for different phase shifts ϕ.

of them ϕ and always on the offset y_0. Performing the same Fourier transform analysis as previously leads to the spectral content of the modified source term as function of both the phase shift and the offset (see Fig. 4). The distribution of harmonics depends on the symmetrical properties of the source term. As soon as the symmetrical properties of the jet are changed, the ones of the source term are and the spectral distribution is too.

6 Conclusion

In spite of the adjustments of the recorder maker that seems to predefine the sound, the player still has some possibilities to modify the sound properties. Among others, tuning the mouth resonance on the note, with a good phase relation, may strongly modify the spectral content in terms of odd/even harmonics distribution. This preliminary study calls for a more extensive and accurate study about the vocal tuning in flute-like instruments. Such studies should focus on the hydrodynamic aspects of the upstream coupling, and its consequences on the symmetrical properties of the jet and the source term.

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